Assignment: 5 DIMENSIONAL ANALYSIS AND SIMILARITIES

1. State the dimensional homogeneity. Prove that the following equations are homogeneous equations:

(i) $Q = AV$ (ii) $T = 2\pi\sqrt{\frac{L}{g}}$ (iii) $V = \sqrt{2gH}$

2. Using Buckingham's π-theorem, show the efficiency η of a fan depends on density ϱ, dynamic viscosity μ of fluid, angular velocity ω, diameter D of the rotor and the discharge Q.

3. Using Buckingham's π-theorem, show that the velocity through a circular orifice given by

$$V = \sqrt{2gH}\ \emptyset\left[\frac{D}{H}, \frac{\mu}{\rho VH}\right]$$

Where H is head causing flow, D is the diameter of the orifice, μ is the coefficient of viscosity, ϱ is mass density and g is the acceleration due to gravity.

4. The frictional torque T of a disc of diameter D rotating at a speed of N in a fluid of viscosity μ and density ϱ in a turbulent flow is given by,

$$T = D^5N^2\rho\ \varphi\left[\frac{N}{D^2N\rho}\right]$$

Prove this by Buckingham's π-theorem.

5. The resistance R to the motion of completely submerged body depends on length of body, velocity of flow, mass density and kinematic viscosity. Find the relation between R and other variables using suitable method.

6. The lift force FL on the air foil depends upon the mass density ϱ of the medium, velocity of flow V, characteristic length d, the viscosity μ and the angle of attack α. Obtain an expression for lift force by using Buckingham's π-theorem.

7. Prove that the scale ratio for discharge for a distorted model is given as

$$\frac{Q_p}{Q_m} = (L_r)_H (L_r)_V{}^{1.5}$$

8. The resisting force R of a supersonic plane during flight can be considered as dependent on the length of aircraft l, velocity V, viscosity of air μ, air density ϱ and bulk modulus of air K. Express the functional relationship between these variables with the resisting force.

9. The efficiency of fan depend upon diameter of rotor, discharge of fluid, density of fluid, dynamic viscosity of fluid and angular velocity of rotor. Find the expression for efficiency in terms of dimensionless number.

10. The pressure difference Δp in a pipe of diameter d and length L due to viscous flow, depends on velocity v, viscosity μ and density ϱ. Using Buckingham's π-theorem, obtain an expression of Δp.

11. Show, using Buckingham's π-theorem, that the resistance (F) to the motion of a sphere of diameter D moving with a uniform velocity V through a real fluid of density ϱ and viscosity μ is given by

$$F = \rho D^2 V^2 \, \varphi \left[\frac{\mu}{\rho V D}\right]$$