UNIT:MATRICES, SYSTEM OF LINEAR EQUATIONS & EIGEN VALUE EIGEN VECTOR

Echelon & Reduced Echelon Form

1) Find reduced row Echelon Form & Rank of

$$A=\begin{bmatrix}1 & 3 & 2 & 2\\ 1 & 2 & 1 & 3\\ 2 & 4 & 3 & 4\\ 3 & 7 & 4 & 8\end{bmatrix}.$$

2) Find reduced row Echelon Form & Rank of

$$A=\begin{bmatrix}0 & 0 & -2 & 0 & 7 & 12\\ 2 & 4 & -10 & 6 & 12 & 28\\ 2 & 4 & -5 & 6 & -5 & -1\end{bmatrix}$$

Non-Homogeneous Linear System
By Gauss-Elimination (Echelon Form) &
By Gauss-Jordan Elimination (Reduced Echelon Form)

3) Solve by G.E

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -2\\ -x_1 + x_2 - 2x_3 &= 3\\ 2x_1 - x_2 + 3x_3 &= 6 \end{aligned}$$

4) Solve by G.E.

$$5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5.$$

5) Solve by G.J.

$$-2y+3z=1,\ 3x+6y-3z=-2,\ 6x+6y+3z=5.$$

6) Solve by G.J.

$$x+y+z=6,\ x+2y+3z=14,\ 2x+4y+7z=30.$$

Homogeneous linear system

7)
$$\begin{aligned} 2x_1 + 2x_2 - x_3 + x_5 &= 0\\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0\\ x_1 + x_2 - 2x_3 - x_5 &= 0\\ x_3 + x_4 + x_5 &= 0 \end{aligned}$$

8)
$$\begin{aligned} 3x - y - z &= 0\\ x + y + 2z &= 0\\ 5x + y + 3z &= 0 \end{aligned}$$

	Find A^{-1} by Gauss-Jordan Method
9)	$(i)\ A=\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix}$ $(ii)\ A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$
10)	$A=\begin{bmatrix}1&0&1\\-1&1&-1\\0&1&0\end{bmatrix}$ Ans: Not invertible (det A=0) (non- Singular)
11)	$A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$
	Rank of a Matrix **(A) By Determinant Method** **(B) By Echelon Form**
12)	$A=\begin{bmatrix}1&4&5&2\\2&1&3&0\\-1&3&2&2\end{bmatrix}$
13)	$A=\begin{bmatrix}1&2&3\\2&3&4\\3&4&5\end{bmatrix}$
14)	$A=\begin{bmatrix}3&4&5&6&7\\4&5&6&7&8\\5&6&7&8&9\\10&11&12&13&14\\15&16&17&18&19\end{bmatrix}$
	Non-Homogeneous Linear System. **Homogeneous Linear System.**
15)	What conditions must b_1, b_2 & b_3 satisfy in order for the system of equations $x_1+2x_2+3x_3=b_1$ $2x_1+5x_2+3x_3=b_2$ $x_1+8x_3=b_3$
16)	Investigate for what values of λ & μ the equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite numbers of solutions

Eigen value and Eigen vector

17) Prove that the characteristic equation of a 2×2 matrix can be expressed as $\lambda^2-\lambda\, tr(A)+\det(A)=0$

18) If $A=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 3\\ 0 & 0 & 2\end{bmatrix}$ then find Eigen values of A^T & $5A$.

(Use property to find λ) Ans : $[1,2,2\ \&\ 1,10,10]$

19) Find Eigen values & bases for the Eigen space of $A, A^{25}, 3A, A^{-1},\ A^T,\ A+2I$, $A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$

20) Check whether 7 & 1 are Eigen values of $A=\begin{bmatrix}1 & 6\\ 5 & 2\end{bmatrix}$ or not?

21) Check whether $X=\begin{bmatrix}1\\ 2\end{bmatrix}$, $Y=\begin{bmatrix}0\\ 1\end{bmatrix}$, $Z=\begin{bmatrix}1\\ -2\end{bmatrix}$ are Eigen vectors of $A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$ or not?

22) *Show that Eigen vectors of symmetric matrix* $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ *are Orthogonal & L.I.*

23) Find Eigen spaces of A, Is A is diagonalizable? Where $A=\begin{bmatrix}0 & 0 & 2 & 0\\ 1 & 0 & 1 & 0\\ 0 & 1 & -2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$

24) Verify Cayley Hamilton Theorem for given matrix , also find A^{-1} where $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$

$$A^{-1}=\frac{1}{3}\begin{bmatrix}2 & -1 & -1\\ 0 & 3 & 0\\ -1 & -1 & 2\end{bmatrix}$$

25) Find a matrix P that diagonalizes A & determine $P^{-1}AP$ to diagonalize $A=\begin{bmatrix}-1 & 4 & -2\\ -3 & 4 & 0\\ -3 & 1 & 3\end{bmatrix}$