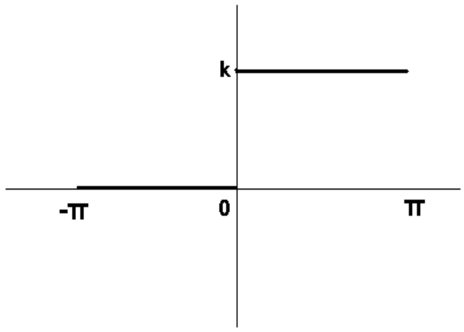
UNIT: FOURIER SERIES

(1)	Define with example: Periodic Function, Fundamental Period.
(2)	Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence deduce that $(i)\ \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ $(ii)\ \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ $(iii)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$
(3)	Find the Fourier series expansion of $f(x) = x; -\pi < x < \pi$
(4)	Find the Fourier series of $f(x) = x - \pi; -\pi < x < \pi$
(5)	Find the Fourier series of $f(x) = \frac{x^2}{2};\ -\pi < x < \pi$
(6)	Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$ hence prove that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$
(7)	Find the Fourier series for $f(x) = \lvert\sin x\rvert$ in $-\pi < x < \pi$
(8)	Find the Fourier series expansion of $f(x) = \sqrt{1 - \cos x}$ in the interval, $(i)\ -\pi < x < \pi$ $(ii)\ 0 \le x \le 2\pi$
(9)	Find the Fourier series of $f(x) = x + \lvert x\rvert; -\pi < x < \pi$
(10)	Find the Fourier Series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}; -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi};\ 0 \le x \le \pi \end{cases}$ hence prove $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$
(11)	Find the Fourier series expansion of the function $f(x) = \begin{cases} -\pi; -\pi \le x \le 0 \\ x;\ 0 \le x \le \pi \end{cases}$ Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$
(12)	Find the Fourier Series for the function $f(x)$ given by $f(x) = \begin{cases} -\sin \omega t; -\pi \le \omega t \le 0 \\ \sin \omega t;\ 0 \le \omega t \le \pi \end{cases}$ hence prove $\sum_{n=2,4,6,\ldots\ldots} \frac{1}{n^2-1} = \frac{1}{2}$
(13)	Obtain the Fourier Series for the function $f(x)$ given by $f(x) = \begin{cases} 0\ ; -\pi \le x \le 0 \\ x^2;\ 0 \le x \le \pi \end{cases}$ hence prove $1 - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots \ldots \ldots = \frac{\pi^2}{12}$
(14)	Find the Fourier series of the Function $f(x)$ when $f(x)$

	
	Function having arbitrary periods
(15)	Find the Fourier series for $f(x) = x^2$ in $-2 < x < 2$
(16)	Find the Fourier series to represent the function $f(x) = 2x - x^2$ in $(0, 3)$
(17)	Find the Fourier series for $f(x) = x^2$ in $(0, 1)$
(18)	Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2$
(19)	Find the Fourier series of the periodic function $f(x) = \pi \sin \pi x$ where $0 < x < 1$, $p = 2l = 1$
(20)	Find the Fourier series of the periodic function $f(x) = 2x$ where $-1 < x < 2$, $p = 2l = 2$
(21)	Expand $f(x) = x$ in $-l < x < l$ the Fourier series.
(22)	Find the Fourier Series for the function $f(x)$ given by $f(t) = \begin{cases} 0; & -L \le t \le 0 \\ E \sin \omega t; & 0 \le t \le L \end{cases}; f\left(\frac{2\pi}{\omega} + t\right) = f(t)$
	Half range Fourier series
(23)	Express $\sin x$ as cosine series in $0 < x < \pi$
(24)	Show that when $0 < x < \pi$, $\pi - x = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{\sin 2nx}{n}$
(25)	Find a cosine series for $f(x) = e^x$ in $0 < x < \pi$
(26)	Find hale-range cosine series for $f(x) = e^x$ in $(0, 1)$
(27)	Find the sine series $f(x) = 2x; 0 < x < 1$ $= 4 - 2x; 1 < x < 2$