	UNIT : LAPLACE TRANSFORM
	Define Laplace transform: Let $f(t)$ be a given function defined for all $t \geq 0$,then the Laplace transform of f(t)is denoted by $L\{f(t)\}$ or $\overline{f(s)}$ or $F(S)$,and is defined as $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$,provided the integral exist.
(1)	Prove that $L\{1\} = \frac{1}{s}$ and $\{sinh\, at\} = \frac{a}{s^2-a^2}$.
(2)	Prove that $L\{e^{-at}\} = \frac{1}{s+a}, s > -a$.
(3)	Prove that $L\{t^n\} = \frac{n!}{s^{n+1}}$, n being positive integer.
(4)	Find the Laplace transform of $(t) = \begin{cases} 0, 0 < t < \pi \\ sin\, t\, , t > \pi \end{cases}$.
(5)	Find the Laplace transform of $(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 3, & when\ t \geq 2 \end{cases}$.
(6)	Find the Laplace transform of $t^3 + e^{-3t} + t^{\frac{1}{2}}$.
(7)	Find $L\{sin\, 2t\, cos\, 2t\}$.
(8)	Find the Laplace transform of $cos^2 2t$.
(9)	Find Laplace transform of $cos^2(at)$, where a is a constant.
(10)	Find the Laplace transform of $f(t) = sinh(\omega t)\, , t \geq 0$.
	FIRST SHIFTING THEOREM:- **If $L\{f(t)\} = F(s)$, then $L\{e^{at}f(t)\} = F(s-a)$.**
(11)	By using first shifting theorem, obtain the value of $L\{(t+1)^2 e^t\}$.
(12)	Find Laplace transform of $e^{-2t} sin^2 2t$, where a is a constant.
	MULTIPLIED BY t^n:- **If $L\{f(t)\} = F(s)$, then $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s); n = 1, 2, 3, \ldots$**
(13)	Find the value of $L\{t\, sin\, \omega t\}$.
(14)	Find the Laplace transform of $f(t) = t^2\, sinh\, at$.
(15)	Find the Laplace transform of $t^2\, sin\, \pi t$.

(16)	Find the Laplace transform of $t^2 \sin 2t$.
(17)	Find the Laplace transform of $t^3 \cosh 2t$.
	DIVISION BY t:- **If $L\{f(t)\} = \overline{f(s)}$, and if $L\left\{\frac{f(t)}{t}\right\}$ exists then** **Prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f(s)}\, ds$.**
(18)	Find $L\left\{\frac{\sin 2t}{t}\right\}$.
(19)	Find the Laplace transform of $\frac{1-\cos t}{t}$.
(20)	Find the Laplace transform of $\left[\frac{\sin wt}{t}\right]$.
	LAPLACE TRANSFORM OF THE INTEGRAL OF A FUNCTION:- **If $L\{f(t)\} = F(s)$, then $L\left\{\int_0^t f(u)\, du\right\} = \frac{F(s)}{s}; s > 0$.**
(21)	Find $L\left\{\int_0^t e^{-x} \cos x\, dx\right\}$.
(22)	Find $L\left\{\int_0^t \int_0^t \sin au\, du\, du\right\}$.
(23)	Find the Laplace transform of $\int_0^t e^{-u} \cos u\, du$.
	If $\overline{f(s)}$ is the Laplace transform of f(t) and a ≥ 0, then **prove that $L\{f(t-a)u(t-a)\} = e^{-as}\overline{f(s)}$.**
(24)	Find the Laplace transform of $e^{-3t}u(t-2)$.
(25)	Find the Laplace transform of $e^t u(t-2)$.
	LAPLACE TRANSFORM OF PERIODIC FUNCTION :- **$L\{f(t)\} = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t) dt; s > 0$, where f(t) is a periodic function with period p** **that is $f(t+p) = f(t)$.**
(26)	Find the Laplace transform of the half wave rectifier $f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ and $f(t) = f\left(t + \frac{2\pi}{\omega}\right)$.
(27)	Find the Laplace transform of $f(t) = \lvert \sin wt \rvert, t \geq 0$.
	CONVOLUTION:- **The convolution of f and g is denoted by f ∗ g** **And is defined as $f * g = \int_0^t f(u) g(t-u) du$.**

(28)	Find the value of $1 * 1$ where $*$ denote convolution product.
(29)	Find the convolution of t and e^t.
	INVERSE LAPLACE TRANSFORM
(30)	Find $L^{-1}\left\{\frac{6s}{s^2-16}\right\}$.
(31)	Find $L^{-1}\left\{\frac{s^3+2s^2+2}{s^3(s^2+1)}\right\}$.
(32)	Find $L^{-1}\left\{\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}\right\}$.
(33)	Find $L^{-1}\left\{\frac{5s^2+3s-16}{(s-1)(s+3)(s-2)}\right\}$.
(34)	Find $L^{-1}\left\{\frac{3s^2+2}{(s+1)(s+2)(s+3)}\right\}$.
(35)	Find $L^{-1}\left\{\frac{s^3}{s^4-81}\right\}$.
(36)	Find $L^{-1}\left\{-\frac{s+10}{s^2-s-2}\right\}$.
	FIRST SHIFTING THEOREM:- **If $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{F(s-a)\} = e^{at}f(t)$.**
(37)	Find $L^{-1}\left\{\frac{10}{(s-2)^4}\right\}$.
(38)	Evaluate :$L^{-1}\left\{\frac{3}{s^2+6s+18}\right\}$.
(39)	Find the inverse Laplace transform of $\frac{6+s}{s^2+6s+13}$,use shifting theorem.
(40)	Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\}$.
(41)	Find the inverse Laplace transform of $\frac{5s+3}{(s^2+2s+5)(s-1)}$.
	SECOND SHIFTING THEOREM:- **If $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{e^{-as}F(s)\} = f(t-a)H(t-a)$.**
(42)	Find the inverse Laplace transform of $\frac{se^{-2s}}{s^2+\pi^2}$.

(43)	Find the inverse Laplace transform of $\frac{e^{-4s}(s+2)}{s^2+4s+5}$.
	INVERSE LAPLACE TRANSFORM OF DERIVATIVES:- **If $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{F'(s)\} = -tf(t)$.**
(44)	Find $L^{-1}\left\{log\frac{s+a}{s+b}\right\}$.
(45)	Find $L^{-1}\left\{log\frac{s+1}{s-1}\right\}$.
(46)	Obtain $L^{-1}\left\{log\frac{1}{s}\right\}$.
(47)	Find the inverse transform of the function $ln\left(1+\frac{w^2}{s^2}\right)$.
	CONVOLUTION THEOREM:- **If $F(s)$ and $G(s)$ are the Laplace transform of the two functions $f(t)$ and $g(t)$, that are sectionally continuous and of order of $e^{\alpha t}$ (α is constant) as $t \to \infty$, then the Laplace transform of the convolution $f * g$ exists when $s > \alpha$ and it is $F(s)G(s)$, that is $L^{-1}\{F(s)G(s)\} = \int_0^t f(u)g(t-u)du$.**
(48)	Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$.
(49)	State convolution theorem and use to evaluate $L^{-1}\left\{\frac{1}{(s^2+\omega^2)^2}\right\}$.
(50)	State the convolution theorem on Laplace transform and using it find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$.
(51)	State convolution theorem and use to evaluate Laplace inverse of $\frac{a}{s^2(s^2+a^2)}$.
	APPLICATION OF LAPLACE TRANSFORM **Laplace transform of derivative:** if $f(t)$ is continuous for all $t \geq 0$, and $f'(t)$ is piecewise continuous then $L\{f'(t)\}$ exists, provided $\lim_{t\to\infty}[e^{-st} f(t)] = 0$ and $L\{f'(t)\} = sL\{f(t)\} - f(0) = s\bar{f}(s) - f(0)$. In general $L\{f^n(t)\} = s^n\bar{f}(s) - s^{n-1}f(0) - s^{n-2}f'^{(0)} - \cdots - f^{(n-1)}(0)$.
(52)	Solve by Laplace transform $y'' + 6y = 1, y(0) = 2, y'(0) = 0$.
(53)	Using Laplace transform solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$, where $x(0) = 0$, $x'(0) = 1$.

(54)	Solve the differential equation $\frac{d^2y}{dt^2} + 4y = f(t), y(0) = 0,\ y'(0) = 1$ by Laplace transform where (i) $f(t) = \begin{cases} 1, 0 < t < 1 \\ 0, t > 1 \end{cases}$ and (ii) $f(t) = H(t-2)$.
(55)	Solve the IVP using Laplace transform $y'' + 4y = 0, y(0) = 1, y'(0) = 6$.
(56)	Solve the simultaneous equation :Using Laplace transform $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$ given $x(0) = 1, y(0) = 0$.
(57)	Using Laplace transform solve the IVP $y'' + y = \sin 2t, y(0) = 2, y'(0) = 1$.
(58)	By Laplace transform solve, $y'' + a^2y = K \sin at$.
(59)	By using the method of Laplace transform solve the IVP : $y'' + 2y' + y = e^{-t}, y(0) = -1 \text{ and } y'(0) = 1$.