| CH:COMPLEX NUMBER AND ANALYTIC FUNCTION | |
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| | **MODOLUS AND ARGUMENT OF COMPLEX NUMBER** |
| (1) | Find the principal value of $\arg i$ . [ $Arg\ i = \frac{\pi}{2}$ ] |
| (2) | Find the principal argument of $z = \frac{-2}{1+i\sqrt{3}}$. |
| (3) | Determine the modulus and argument of $Z^5$ Where $Z = 1+i\sqrt{3}$ [ $\arg\left(z^5\right) = -\frac{\pi}{3}$ ] |
| (4) | To calculate principal value of argument of following complex number a) $\sqrt{3}+i$ b) $-\sqrt{3}+i$ c) $-\sqrt{3}-i$ d) $\sqrt{3}-i$ [a) $\frac{\pi}{6}$ b) $\frac{5\pi}{6}$ c) $-\frac{5\pi}{6}$ d) $\frac{-\pi}{6}$ ] |
| (5) | Find the value of $\text{Re}(f(z))$ and $\text{Im}(f(z))$ at the indicate point Where $f(z) = \frac{1}{1-z}$ at $7+2i$ . |
| (6) | Is $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$ ? Justify. |
| | **SOLUTION OF QUADRATIC EQUATION** |
| (7) | Find the roots of the equation $z^2 + 2iz + (2-4i) = 0$ [ $z = 1+i$ or $z = -1-3i$ ] |
| (8) | Solve the Equation of $z^2 - (5+i)z + 8 + i = 0$ .[ $z = 3+2i$ or $z = 2-i$ ] |
| (9) | Find the roots of the equation $z^2 - (3-i)z + (2-3i) = 0$ [ $z = 2+i$ or $z = 1-2i$ ] |
| | **De Moirve's theorem & ROOTS OF COMPLEX NUMBER** |
| (10) | Find and plot the square root of $4i$ [ $\sqrt{4i} = \pm\left(\sqrt{2} + i\sqrt{2}\right)$ ] |
| (11) | Find and plot aii root of $\sqrt[3]{8i}$ . |
| (12) | Show that if $c$ is any $n^{th}$ root of Unity other than Unity itself , then $1 + c + c^2 + ......... + c^{n-1} = 0.$ |
| (13) | Find and plot all the roots of $(1+i)^{1/3}$ . |
| (14) | Find real and imaginary part of $(-1-i)^7 + (-1+i)^7$ .[ $\text{Re}\,al = -\sqrt{2}$ $\text{Im}\,g = 0$ ] |
| | **ELIMENTRY FUNCTIONS AND EXAMPIE.** |
| (15) | Define 1)Exponential function 2) Trigonometric function 3) Hyperbolic function 4) Logarithmic function 5)Inverse trigonometric and Inverse hyperbolic function 6)Relation between hyperbolic and trigonometric functions 7)Hyperbolic identity. |
| (16) | Prove that $\tan^{-1} z = \frac{i}{2}\log\frac{i+z}{i-z}$. |
| (17) | Define $\log(x+iy)$ Determine $\log(1-i)$. |
| (18) | Show that $\cos(i\bar{z}) = \overline{\cos(iz)}$ for all z. |
| (19) | Expand $\cosh(z_1 + z_2)$. |

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| (20) | Prove that $\left\|e^{(-2z)}\right\| < 1$ if and only if $\operatorname{Re} z > 0$. |
| (21) | Find all Solution of sinz=2. |
| (22) | Show that the set of values of $\log(i^2)$ is not the same as the set of values $2\log i$. |
| (23) | Find the principal value of $\left[\frac{e}{2}\left(-1-i\sqrt{3}\right)\right]^{3\pi i}$. |
| (24) | Find all root s of the Equation $\log z = \frac{\pi}{2}$. |
| | **FUNCTION OF COMPLEX VARIABLE** |
| (25) | Define 1) Limit of function 2)continuous function 3)Differentiable function. |
| (26) | Prove $\lim_{z\to 1}\frac{iz}{3} = \frac{i}{3}$ by definition. |
| (27) | Use the $\varepsilon-\delta$ definition of limit to Show that where $\lim_{z\to 3i}\left(3x+iy^2\right) = 9i$ Where $z = x+iy$. |
| (28) | Show that the limit of the function does not exist $f(z) = \begin{cases} \frac{\operatorname{Im} g(z)}{\|z\|} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$ |
| (29) | Find out and (given reason) Where $f(z)$ is continuous at $z = 0$ if $f(z) = \begin{cases} \frac{\operatorname{Re}(z^2)}{\|z\|} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$ . |
| (30) | Find the derivative of $\frac{z-i}{z+i}$ at $i$. |
| (31) | Show that $f(z) = z\operatorname{Im}(z)$ is differential only at $z = 0$ and $f'(0) = 0$. |
| | **ANALYTIC FUNCTION** |
| (32) | Define 1)Analytic function 2)Entire function<br>3)C-R Equation 4)Harmonic function. |
| (33) | State necessary and sufficient Condition for function to be analytics and prove that necessary Condition. |
| (34) | The function $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & , When\ z \neq 0. \\ 0 & , When\ z = 0. \end{cases}$ Satisfies C-R equation at the origin but $f'(0)$.<br>fails to exist. |
| (35) | Check Whether the function is analytics or not.<br>$f(z) = \bar{z}$. |
| (36) | Check Whether the function is analytics or not at any point. $f(z) = 2x + ixy^2$ |
| (37) | Check Whether the function is analytics or not at any point.<br>$f(z) = e^{\bar{z}}$ |
| (38) | Verify that $f(z) = z^2$ is analytic everywhere. |
| (39) | Check Whether the function is analytics or not.<br>$f(z) = z^{\frac{5}{2}}$ |

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| (40) | Check Whether the function $f(z)=\sin z$ is analytics or not. if analytic find it's derivative. |
| (41) | Find the all analytic function $f(z)=u+iv$ if $u-v=(x-y)(x^2+4xy+y^2)$. |
| (42) | Show that if $f(z)$ is analytics in a domain $D$ and $\lvert f(z)\rvert=k$ constant in $D$ then show that $f(z)=const$ in $D$. |
| (43) | Let a function $f(z)$ be analytic in a domain $D$ prove that $f(z)$ must be constant in $D$ in each of following cases.<br>a) if $f(z)$ is real value for all $z$ in $D$<br>b) if $\overline{f(z)}$ is analytic in $D$. |
| (44) | Define harmonic function. Show that $u=x\sin x\cosh y-y\cos x\sinh y$ is harmonic |
| (45) | Determine 'a' and 'b' such that $u=ax^3+bxy$ is harmonic and find Conjugate harmonic. |
| (46) | Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find a harmonic Conjugate $v(x,y)$. |
| (47) | Determine the analytic function whose imaginary part is $e^x(x\cos y-y\sin y)$. |
| (48) | Determine the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$. |