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| CHAPTER: BASIC PROBABILITY |  |
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| (1) | Define Mutually Exclusive and Exhaustive events with a suitable example |
| (2) | Describe the sample space for the indicated random experiments. <br> (a) A coin is tossed 3 times. (b) A coin and die is tossed together. |
| (3) | Four card are labeled with A, B, C and D. We select and two cards at random without replacement. Describe the sample space for the experiments. |
| (4) | If probability of event A is $9 / 10$, what is the probability of the event "not A ". |
| (5) | If A and B are two mutually exclusive events with $\mathrm{P}(\mathrm{A})=0.30, \mathrm{P}(\mathrm{B})=0.45$. Find the probability of $\mathrm{A}^{\prime}, \mathrm{A} \cap \mathrm{B}, \mathrm{A} \cup \mathrm{B}, \mathrm{A}^{\prime} \cap \mathrm{B}^{\prime}$. |
| (6) | A fair coin is tossed twice. Find the probability of (a) Getting H exactly once. (b) Getting T at least once. |
| (7) | One card is drawn at random from a well shuffled pack of 52 cards. Calculate the probability that the card will be (a) An Ace (b) A card of black color (c) A diamond <br> (d) Not an ace. |
| (8) | Four cards are drawn from the pack of cards. Find the probability that (a) all are diamonds (b) there is one card of each suit (c) there are two spades and two hearts. |
| (9) | A box contains 5 red, 6 white and 2 black balls. The balls are identical in all respect other than color. <br> (a) One ball is drawn at random from the box. Find the probability that the selected ball is black. <br> (b) Two balls are drawn at random from the box. Find the probability that one ball is white and one is red. |
| (10) | If $\mathrm{P}(\mathrm{A})=1 / 3, \mathrm{P}(\mathrm{B})=3 / 4$ and $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=11 / 12$. Find $\mathrm{P}(\mathrm{AB})$. |
| (11) | If $A$ and $B$ are independent events, with $P(A)=3 / 8, P(B)=7 / 8$. Find $P(A \cup B), P(A B)$ and $P(B / A)$. |
| (12) | From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after the other without replacements, find the probability that first ball is white and second is black. |
| (13) | In a bolt factory, three machines A, B and C manufacture $25 \%, 35 \%$ and $40 \%$ of the total product respectively. Out Of these outputs $5 \%, 4 \%$ and $2 \%$ respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the Probabilities that it was manufactured by machine $\mathrm{A}, \mathrm{B}$ and C ? |

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| (14) | A company has two plants to manufacture hydraulic machine. Plat I manufacture $70 \%$ of the hydraulic machines and plant II manufactures $30 \%$. At plant I, $80 \%$ of hydraulic machines are rated standard quality and at plant II, $90 \%$ of hydraulic machine are rated standard quality. A machine is picked up at random and is found to be of standard quality. What is the chance that it has come from plant I? |
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| (15) | A microchip company has two machines that produce the chips. Machine-I produces $65 \%$ of the chips, but $5 \%$ of its chips are defective. Machine-II produces $35 \%$ of the chips, but $15 \%$ of its chips are defective. A chip is selected at random and found to be defective. What is the probability that it came from Machine-I? |
| (16) | The probability distribution of a random variable X is given below. Find a, $\mathrm{E}(\mathrm{X}), \mathrm{E}(2 \mathrm{X}+3), \mathrm{E}(\mathrm{X} 2+2), \mathrm{V}(\mathrm{X}), \mathrm{V}(3 \mathrm{X}+2)$ |
| (17) | The probability distribution of a random variable X is given below. <br> Find $\mathrm{k}, \mathrm{E}(\mathrm{X}), \mathrm{E}(4 \mathrm{X}+3), \mathrm{E}(\mathrm{X} 2), \mathrm{V}(\mathrm{X}), \mathrm{V}(2 \mathrm{X}+3)$. |
| (18) | A continuous random variable X has p.d.f. $f(x)=3 x^{2} ; 0 \leq x \leq 1$. Find ' b ' such that $\mathrm{p}(\mathrm{X}>\mathrm{b})=0.05$. |
| (19) | A random variable X has p.d.f. $f(x)=\mathrm{k} x^{2}\left(1-x^{3}\right) ; 0<x<1$. Find the value <br> of ' $k$ ' and hence find its mean and variance. |
| (20) | If mean and standard deviation of a random variable x are 5 and 5 respectively. Find $E(X)^{2}$ and $E(2 X+5)^{2}$. |
| (21) | The joint probability mass function of $(X, Y)$ is given by $P(x, y)=K(2 x+3 y)$ Where $\mathrm{x}=0,1,2$ and $\mathrm{y}=1,2,3$. Find the marginal probability of X . |
| (22) | A random variable X has p.d.f. $f(x)=\begin{array}{cl}3+2 x & ; 2 \leq x \leq 4 \\ 0 & ; \text { otherwise }\end{array}$. Find the standard deviation of the distribution. |
| (23) | Let $\mathrm{P}(\mathrm{X}=0, \mathrm{Y}=1)=1 / 3, \mathrm{P}(\mathrm{X}=1, \mathrm{Y}=-1)=1 / 3, \mathrm{P}(\mathrm{X}=1, \mathrm{Y}=1)=1 / 3$. Is it the joint probability mass function of X and Y ?. if yes, Find the marginal probability function of X and Y . |


| (24) | The random variables X and Y have the following joint probability distribution. What is the expected value of X and Y ? |  |  |  |
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|  |  | $\mathrm{Y}=0$ | $\mathrm{Y}=1$ | $\mathrm{Y}=2$ |
|  | $\mathrm{X}=0$ | 0.2 | 0.1 | 0.2 |
|  | $\mathrm{X}=1$ | 0 | 0.2 | 0.1 |
|  | $\mathrm{X}=2$ | 0.1 | 0 | 0.1 |
| (25) | Consider the joint density function for X and Y , $f(x, y)=x^{2} y^{3} ; 0<x<1 \&<y<x$, find the expected value of X . |  |  |  |
| (26) | The joint probability mass function of $(X, Y)$ is given by $P(x, y)=K(2 x+3 y)$ Where $x=0,1,2$ and $y=1,2,3$. Find the marginal probability of $X$. |  |  |  |



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| CHAPTER: SOME SPECIAL PROBABILITY DISTRIBUTIONS |  |
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|  | BINOMIAL DISTRIBUTION |
| (1) | Write assumption of Binomial Distribution. |
| (2) | Find the binomial distribution for $\mathrm{n}=4$ and $\mathrm{p}=0.3$. |
| (3) | $12 \%$ of the tablets produced by a tablet machine are defective. What is the probability that out of a random sample of 20 tablets produced by the machine, 5 are defective? |
| (4) | If 3 of 12 car drivers do not carry driving license, what is the probability that a traffic inspector who randomly checks 3 car drivers, will catch 1 for not carrying driving license. (Use binomial dist.) |
| (5) | What are the properties of Binomial Distribution? The average percentage of failure in a certain examination is 40 . What is the probability that out of a group of 6 candidates, at least 4 passed in examination? |
| (6) | The probability that in a university, a student will be a post-graduate is 0.6. Determine the probability that out of 8 students (a) None (b) Two (c) At least two will be post-graduate. |
| (7) | A dice is thrown 6 times getting an odd number of success, Find probability <br> (a) Five success (b) At least five success (c) At most five success. |
| (8) | A multiple choice test consist of 8 questions with 3 answer to each question (of which only one is correct). A student answers each question by rolling a balanced dice \& checking the first answer if he gets 1 or 2 , the second answer if he gets 3 or $4 \&$ the third answer if he gets 5 or 6 . To get a distinction, the student must secure at least $75 \%$ correct answers. If there is no negative marking, what is the probability that the student secures a distinction? |
| (9) | Obtain the binomial distribution for which mean is 10 and variance is 5. |
| (10) | For the binomial distribution with $n=20, p=0.35$. Find Mean, Variance and Standard deviation. |
| (11) | If the probability of a defective bolt is 0.1 Find mean and standard deviation of the distribution of defective bolts in a total of 400 . |
|  | POISSON DISTRIBUTION |
| (12) | In a company, there are 250 workers. The probability of a worker remain absent on any one day is 0.02 . Find the probability that on a day seven workers are absent. |
| (13) | For Poisson variant X , if $\mathrm{P}(\mathrm{X}=3)=\mathrm{P}(\mathrm{X}=4)$ then, Find $\mathrm{P}(\mathrm{X}=0)$. |
| (14) | In a bolt manufacturing company, it is found that there is a small chance of 1 500 for any bolt to be defective. The bolts are supplied in a packed of 30 bolts. Use Poisson distribution to find approximate number of packs, (a) Containing no defective bolt and (b) Containing two defective bolt, in the consignment of 10000 packets. |
| (15) | Potholes on a highway can be serious problems. The past experience suggests that there are, on the average, 2 potholes per mile after a certain amount of usage. It is assumed that the Poisson process applies to the random variable "no. of potholes". What is the probability that no more than four potholes will occur in a given section of 5 miles? |

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| (16) | 100 Electric bulbs are found to be defective in a lot of 5000 bulbs. Find the probability that at the most 3 bulbs are defective in a box of 100 bulbs. |
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| (17) | If a bank receives an average six back cheques per day what are the probability that bank will receive (a) 4 back cheques on any given day (b) 10 back cheques on any consecutive day |
|  | EXPONENTIAL DISTRIBUTION |
| (18) | The lifetime T of an alkaline battery is exponentially distributed with $\theta=0.05$ per hour. What are the probabilities for battery to last between 10 and 15 hours? What are the probabilities for the battery to last more than 20 hr ? |
| (19) | The time between breakdowns of a particular machine follows an exponential distribution with a mean of 17 days. Calculate the probability that a machine breakdown in 15 days period. |
| (20) | In a large corporate computer network, user log-on to the system can be modeled as a Poisson process with a mean of 25 log-on per hours. (a) What is the probability that there are no log-on in an interval of six min.? (b) What is the probability that time until next log-on is between $2 \& 3$ min.? (c) Determine the interval of time such that the probability that no log-on occurs in the interval is 0.90 ? |
| (21) | Accidents occur with Poisson distribution at an average of 4/week ( $\lambda=4$ ). (a) Calculate the probability of more than 5 accidents in any one week. (b) What is probability that at least two weeks will elapse between accidents? |
|  | GAMMA DISTRIBUTION |
| (22) | Suppose you are fishing and you expect to get a fish once every $1 / 2$ hour. Compute the probability that you will have to wait between 2 to 4 hours before you catch 4 fish. |
| (23) | The daily consumption of milk in a city, in excess of 20000 liters, is approximately distributed as a gamma variate with $r=2$ and $\lambda=1 / 10000$. The city has daily stock of 30000 liters. What is the probability that the stock is insufficient on a particular day? |
| (24) | The daily consumption of electric power in a certain city is a random variable X having probability density function $f(x)=\{19 x e-x 3, x>00, x \leq 0$. Find the probability that the power supply is inadequate on any given day if the daily capacity of the power plant is 12 million KW hours. |
|  | NORMAL DISTRIBUTION |
| (25) | The compressive strength ofthe sample of cement can be modelled by normal distribution with mean $6000 \mathrm{~kg} / \mathrm{cm} 2$ and standard deviation of $100 \mathrm{~kg} / \mathrm{cm} 2$. <br> (a) What is the probability that a sample strength is less than $6250 \mathrm{~kg} / \mathrm{cm} 2$ ? <br> (b) What is probability if sample strength is between 5800 and $5900 \mathrm{~kg} / \mathrm{cm} 2$ ? <br> (c) What strength is exceeded by $95 \%$ of the samples? $[\mathrm{P}(\mathrm{z}=2.5)=0.9938$, $\mathrm{P}(\mathrm{z}=1)=0.8413, \mathrm{P}(\mathrm{z}=2)=0.9772, \mathrm{P}(\mathrm{z}=1.65)=0.95]$ |
| (26) | In a photographic process, the developing time of prints may be looked upon as a random variable having normal distribution with mean of 16.28 seconds and standard deviation of 0.12 seconds. Find the probability that it will take (a) Anywhere from 16.00 to 16.50 sec to develop one of the prints; (b) At |


|  | least 16.20 sec to develop one of the prints; (c) At most 16.35 sec to develop one of the prints. $[\mathrm{P}(\mathrm{z}=1.83)=0.9664, \mathrm{P}(\mathrm{z}=0.66)=0.7454, \mathrm{P}(\mathrm{z}=0.58)=$ 0.7190] |
| :---: | :---: |
| (27) | A random variable having the normal distribution with $\mu=18.2 \& \sigma=1.25$, find the probabilities that it will take on a value (a) less than 16.5 (b) Between 16.5 and 18.8. $[\mathrm{F}(0.48)=0.3156, \mathrm{~F}(-1.36)=0.0869]$ |
| (28) | In AEC company, the amount of light bills follows normal distribution with standard deviation $60.11 .31 \%$ of customers pay light-bill less than Rs.260. Find average amount of light bill. |
| (29) | Weights of 500 students of college is normally distributed with average weight 95 lbs. \& $\sigma=7.5$. Find how many students will have the weight between 100 and 110 . |
|  | CHEBYSHEV'S INEQUALITY |
| (30) | A random variable X has a mean 12, variance 9 and unknown probability distribution. Find $\mathrm{P}(6<\mathrm{X}<18)$. |
| (31) | Determine the smallest value of ' $k$ ' in the chebyshev's inequality for which the probability is at least 0.95 . |
| (32) | A random variable X has mean 10, variance 4 and unknown probability distribution. Find ' c ' such that $\mathrm{P}\{\|\mathrm{X}-10\| \geq \mathrm{c}\} \leq 0.04$ |
| (33) | A random variable $X$ has pdf $f(x)=e-x, x \geq 0$. Use chebyshev's inequality to show that $\mathrm{P}\{\|\mathrm{X}-1\|>2\} \leq 14$ and also find the actual probability. |

