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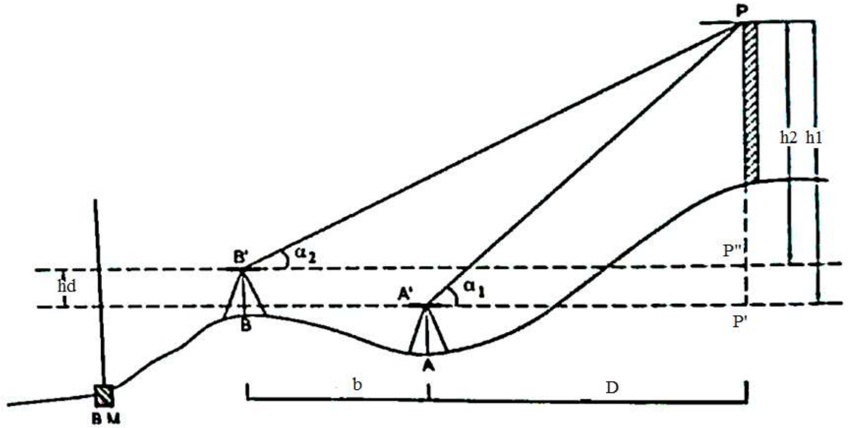
Module - 3
Trigonometric Leveling



Subject:- Surveying **Code:-3140601**	**Prepared by:** **Asst. Prof. Nutan Patel** **(Civil Department, ACET)**	COLLEGE OF ENGINEERING & TECHNOLOGY

Trigonometric Leveling

- Trigonometric leveling is the process of determining the different elevation of station from observed vertical angle and known distance.
- The vertical angle are measured by means of theodolite.
- The horizontal distance may either measured or computed.
- Relative heights are calculated using trigonometric formula.

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- If the distance between the instrument station and object is small, correction of earth curvature and reflection is not required.
- If the distance between the instrument station and object is large the combined correction = 0.0673 D^2, for earth's curvature and reflection is required, were D = distance in Km.
- If the vertical angle is +ve, the correction is taken as +ve.
- If the vertical angle is –ve, the correction is taken as –ve.

Methods of Observation

- There are two method of observation in trigonometric leveling.

1. *Direct method*
2. *Reciprocal method*

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1. Direct Method

- This method is useful where it is not possible to set the instrument over the station, whose elevation is to be determine.
- Ex: To determine the height of the tower.
- In this method the instrument is set on the station on the ground whose elevation is known.
- If the distance between two point is so large, combined correction = 0.0673 D^2 for earth curvature and refraction is required. (D in Km)

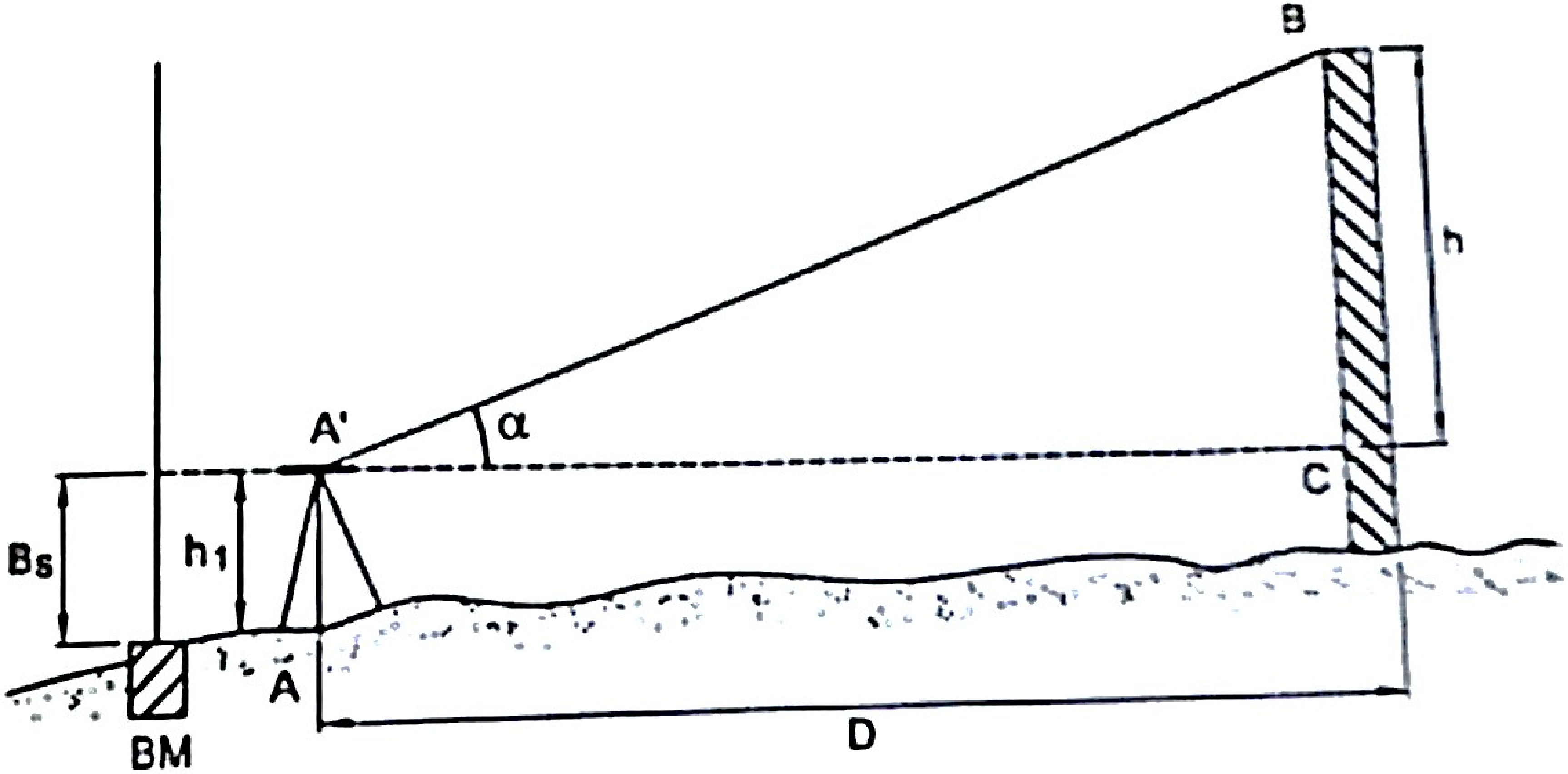
2. Reciprocal Method

- In this method the instrument is set on each of the two station alternatively and observation are taken.
- Difference in elevation between two station A and B is to be determine.
- First set the instrument on A and take observation of B then set the instrument on B and take the observation of A.

Method of determining the elevation of a point by theodolite

- There are main three cases to determine the R.L of any point.
- Case : 1 :- Base of Object accessible.
- Case : 2 :- Base of object inaccessible, instrument station in the vertical plane as the elevated object.
- Case : 3 :- Base of the object inaccessible , instrument stations not in the same vertical plane as the elevated object.

Case : 1 :- Base of Object accessible.



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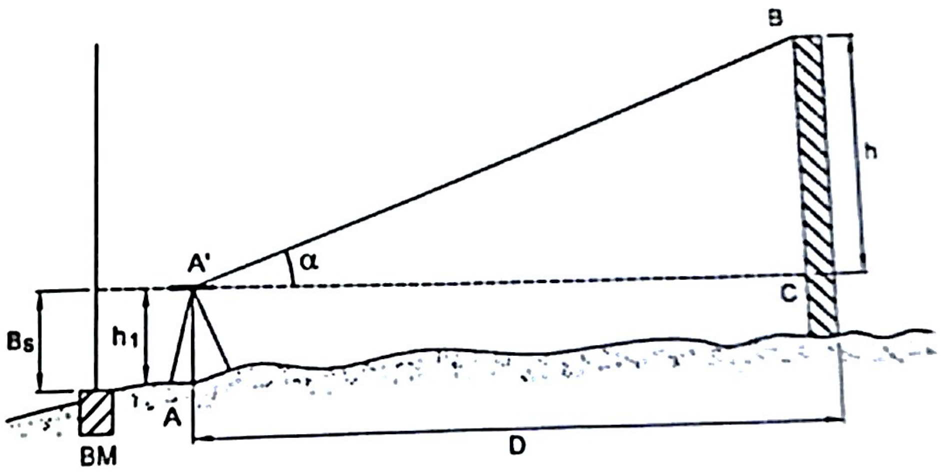
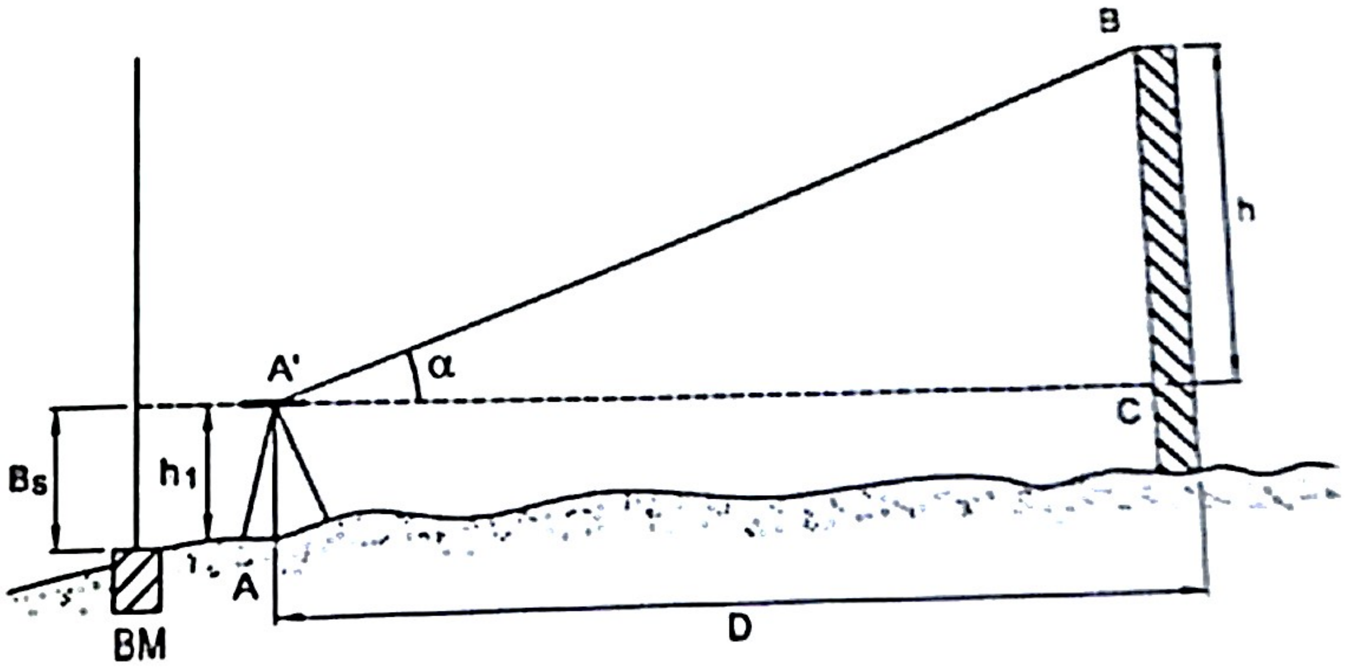


- Here
- A = Instrument station
- B = Point to be observed
- h = Elevation of B from the instrument axis
- D = horizontal distance between A and the base of object
- h1 = height of the instrument
- Bs = Reading of staff kept on BM
- α = Angle of elevation ∠BA'C



- From the fig :
- $\tan\alpha = \dfrac{h}{D}$
- $h = D\tan\alpha$
- R.L of B = R.L of B.M + Bs + h

  = R.L of B.M + Bs + $D\tan\alpha$
- If the distance between the two point is large so the curvature of the earth is considered.
- R.L of B = R.L of B.M + Bs + $D\tan\alpha + 0.0673\,D^2$

Case : 2 :- Base of object inaccessible, instrument station in the vertical plane as the elevated object.

- When it is not possible to measure the horizontal distance between the instrument station and the base of the object, this method is employed to determine the R.L of the object.
- There may be two case

A. Instrument axis at the same level

B. Instrument axis at the different level

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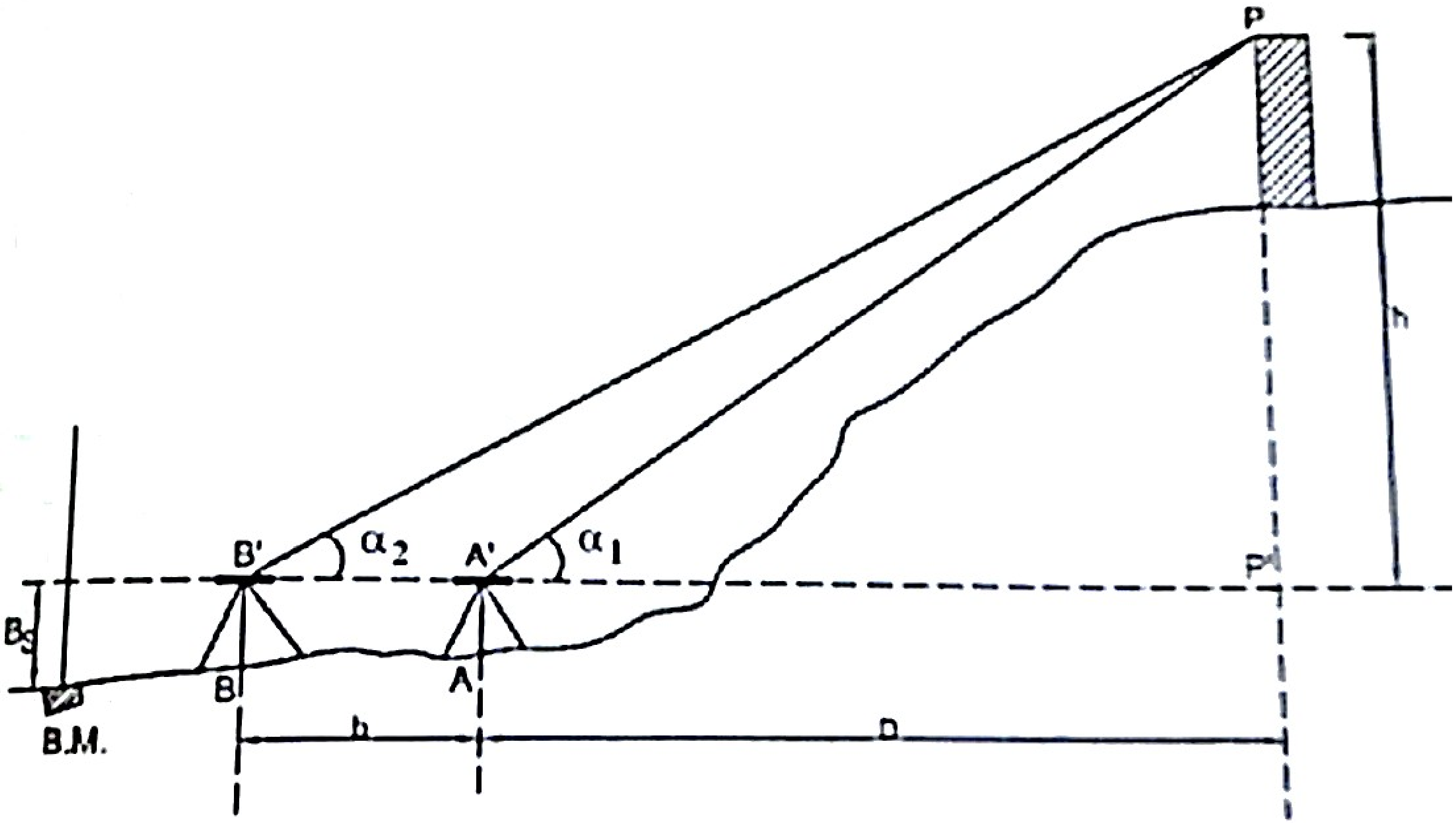
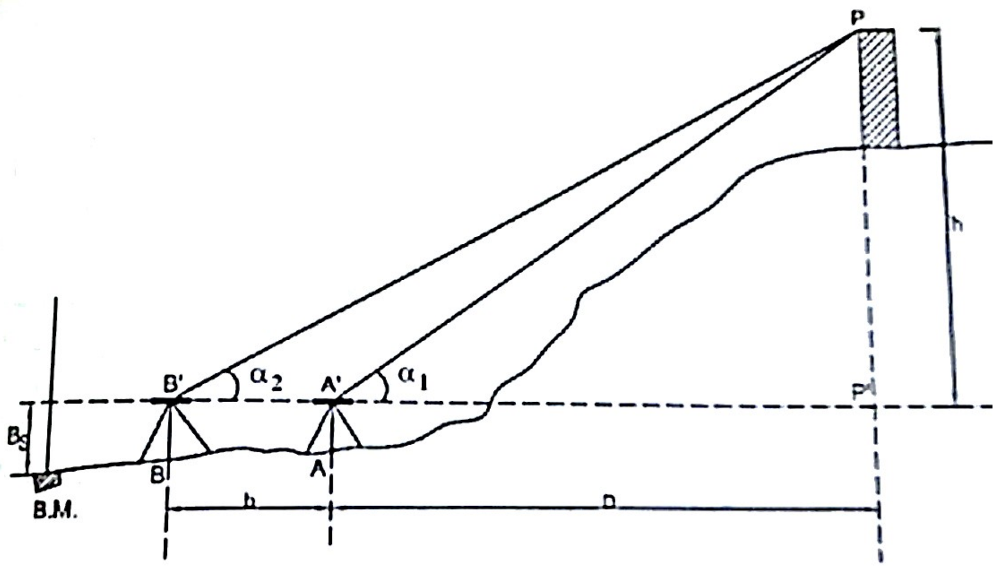
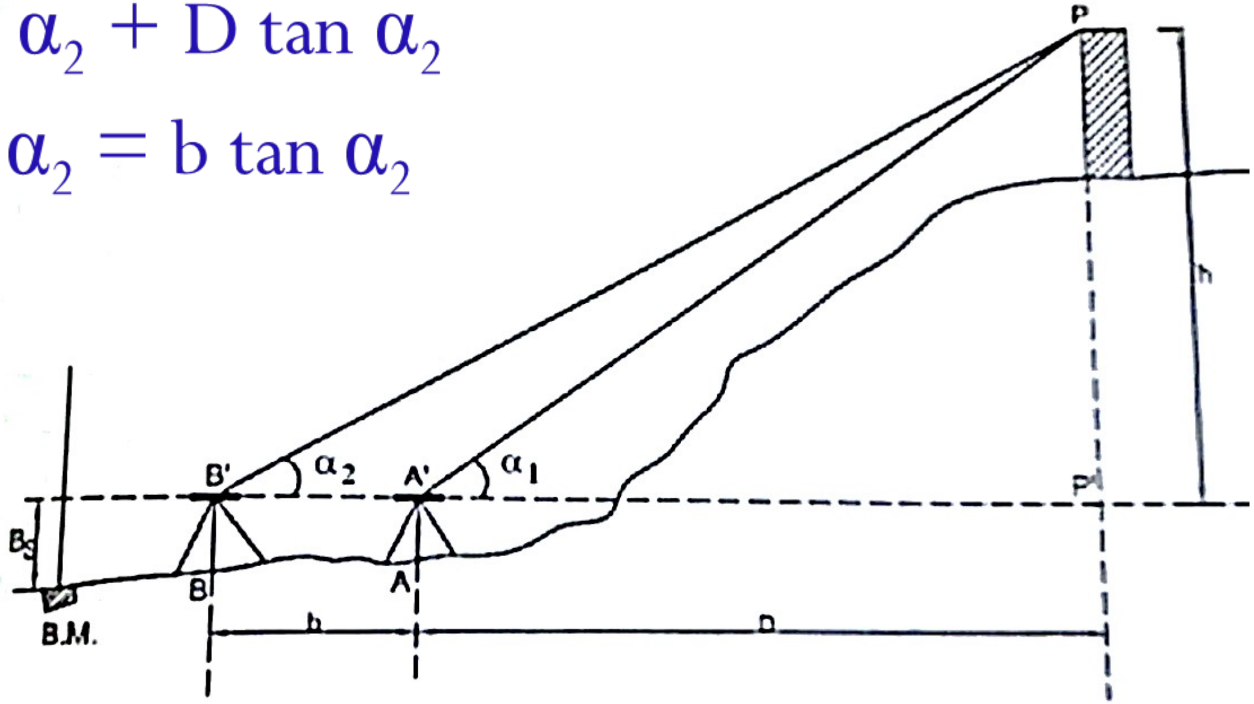
A. Instrument axis at the same level



- A, B = Instrument station
- h = Elevation of the top of the object from the instrument axis
- b = horizontal distance between A and B
- α_1 = angle of elevation from A to P
- α_2 = angle of elevation from B to P
- From Δ PA'P'
- $\tan \alpha_1 = \dfrac{h}{D}$
- $h = D \tan \alpha_1 \ldots(1)$



- From Δ PB'P'
- $\tan \alpha_2 = \dfrac{h}{(b + D)}$
- $h = (b + D) \tan \alpha_2$ ……………………………(2)
- Equating (1) and (2)
- $D \tan \alpha_1 = (b + D) \tan \alpha_2$
- $D \tan \alpha_1 = b \tan \alpha_2 + D \tan \alpha_2$
- $D \tan \alpha_1 - D \tan \alpha_2 = b \tan \alpha_2$



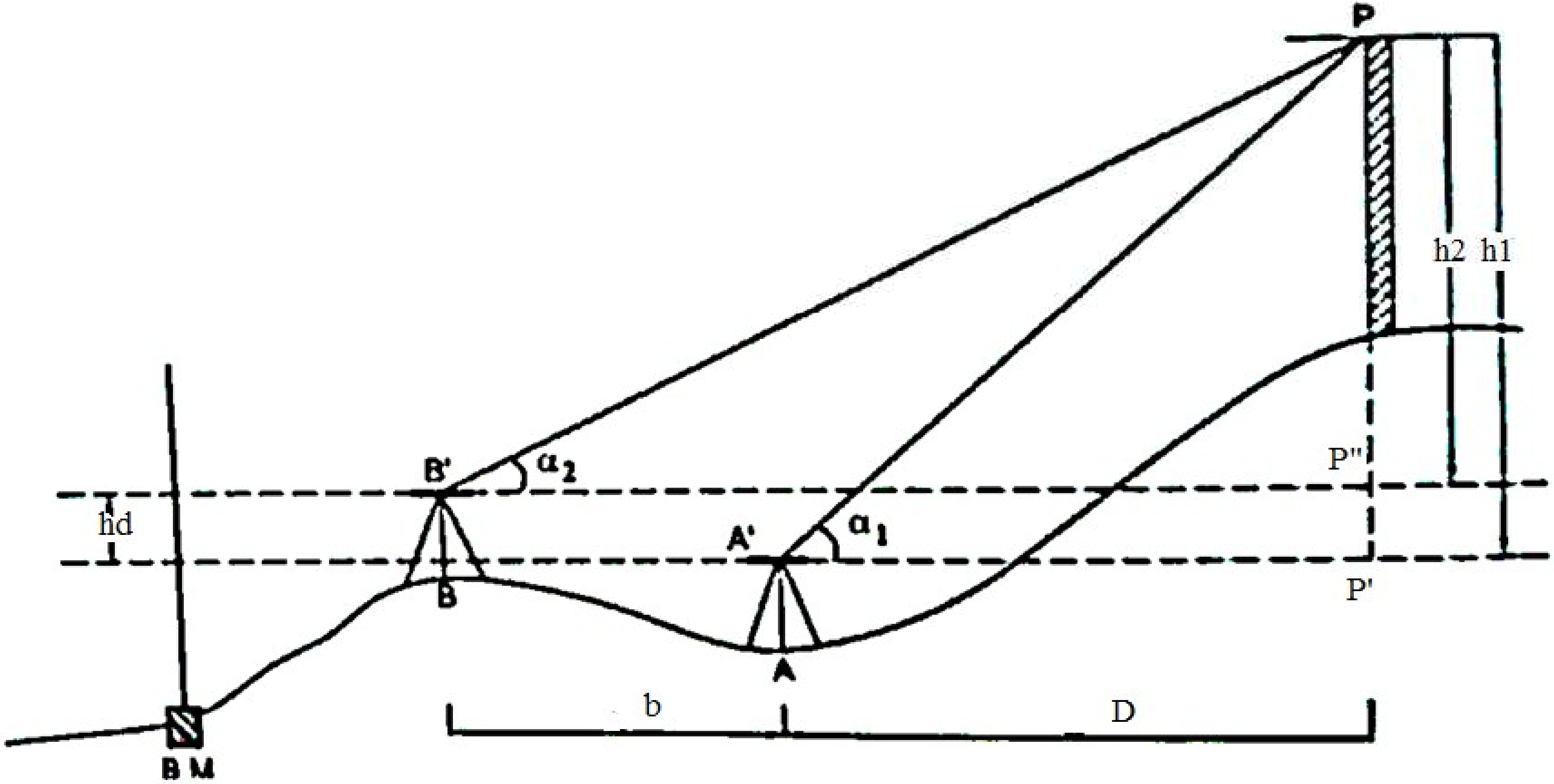
- $D \tan \alpha_1 - D \tan \alpha_2 = b \tan \alpha_2$
- $D (\tan \alpha_1 - \tan \alpha_2) = b \tan \alpha_2$
- $D = \dfrac{b \tan \alpha_2}{(\tan \alpha_1 - \tan \alpha_2)}$ ....(3)
- Put the value of D in equation (1)
- $h = D \tan \alpha_1$
- $h = \dfrac{b \tan \alpha_1 \tan \alpha_2}{(\tan \alpha_1 - \tan \alpha_2)}$
- R L of P = R L of B M + B + h

B. Instrument axis at different level

- In the field it is difficult to keep the height of the instrument at the same level.
- The instrument is set at the different station and height of the instrument axis in both the cases is taken by back sight on B.M.
- There are main two cases

1. Height of the instrument axis nearer to the object is lower.
2. Height of the instrument axis near to the object is higher.

1. *Height of the instrument axis nearer to the object is lower.*



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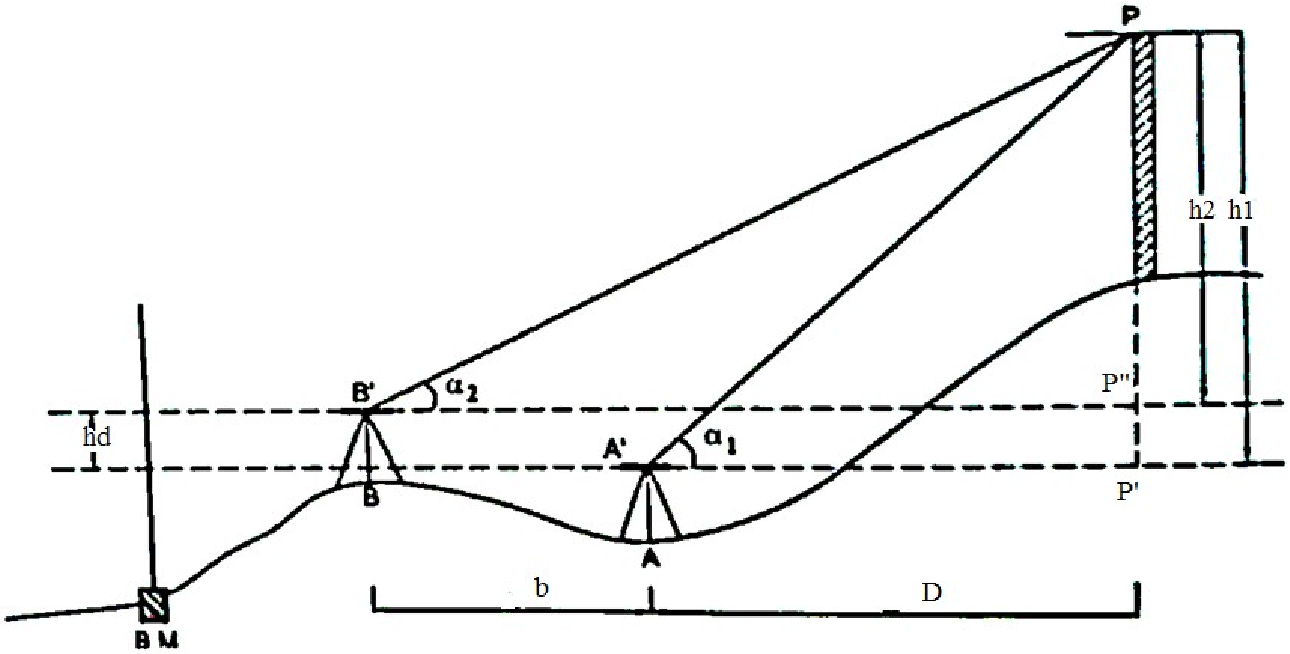

- From ΔPA'P'
- $\tan\alpha_1 = \frac{h_1}{D}$
- $h_1 = D \tan\alpha_1$ ………(1)

- From ΔPB'P"
- $\tan\alpha_2 = \frac{h_2}{b+D}$
- $h_2 = (b + D) \tan\alpha_2$ ……(2)

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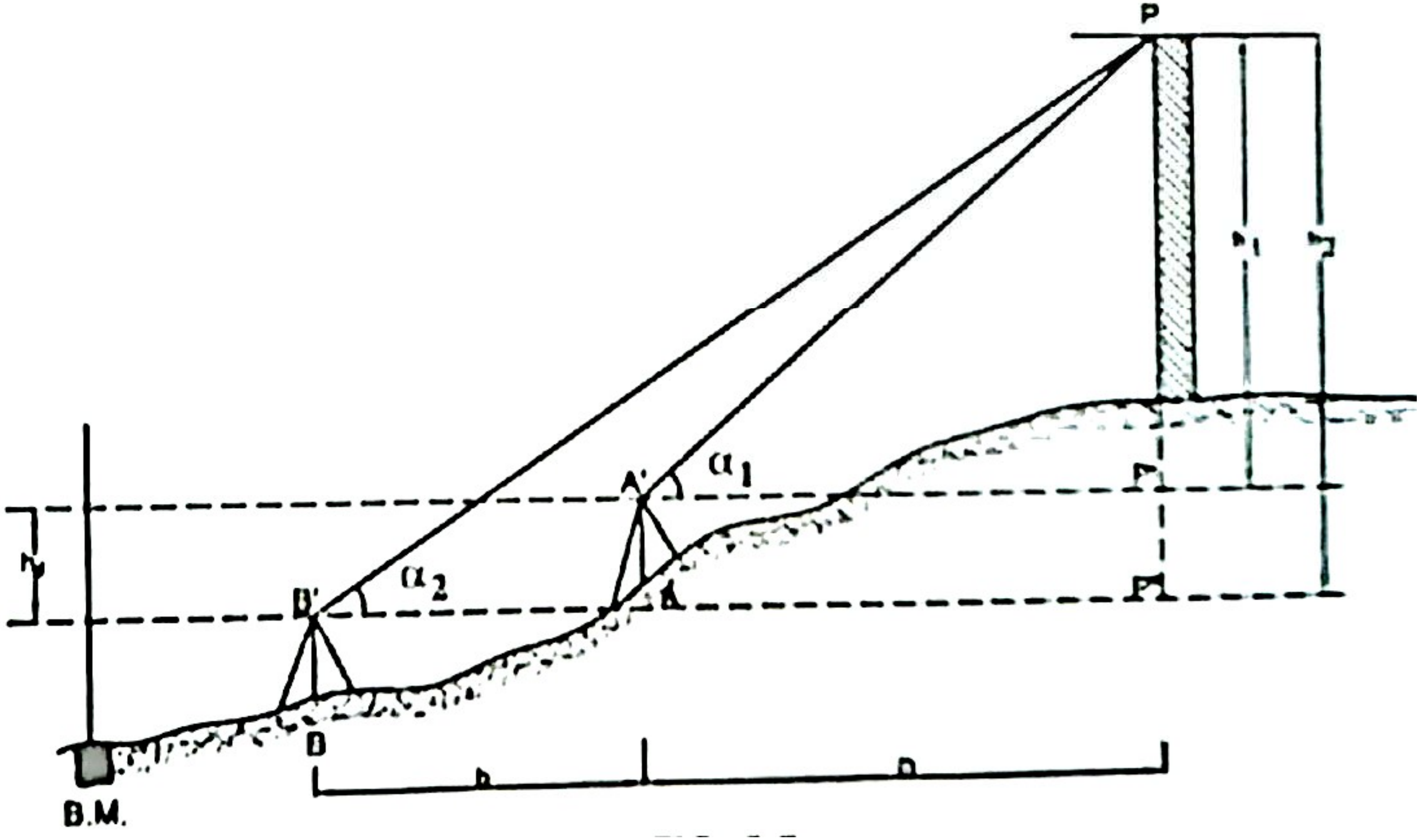
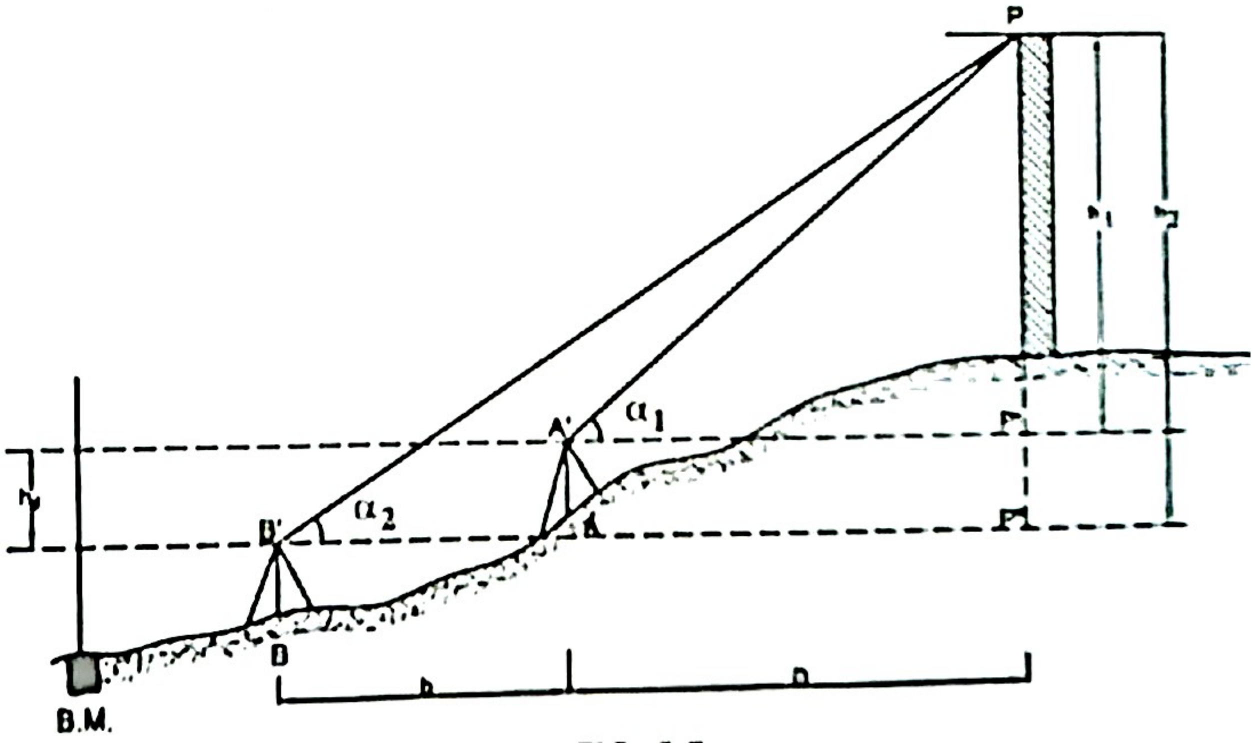
- Now hd = $h_1 - h_2$
- $h_d = (D \tan\alpha_1) - ((b + D) \tan\alpha_2)$
- $h_d = D \tan\alpha_1 - b \tan\alpha_2 - D \tan\alpha_2$
- $h_d = D (\tan\alpha_1 - \tan\alpha_2) - b \tan\alpha_2$
- $h_d + b \tan\alpha_2 = D (\tan\alpha_1 - \tan\alpha_2)$
- $D = \dfrac{h_d + b \tan\alpha_2}{(\tan\alpha_1 - \tan\alpha_2)}$ ....(3)
- Substitute the value of D in equation (1)
- $h_1 = \dfrac{(h_d + b \tan\alpha_2)\tan\alpha_1}{(\tan\alpha_1 - \tan\alpha_2)}$ ....(4)

2. *Height of the instrument axis near to the object is higher.*





- From $\Delta PA'P'$
- $\tan\alpha_1 = \frac{h_1}{D}$
- $h_1 = D \tan\alpha_1 \;......(1)$

- From $\Delta PB'P''$
- $\tan\alpha_2 = \frac{h_2}{b+D}$
- $h_2 = (b + D) \tan\alpha_2 \;....(2)$

- Now hd = $h_2 - h_1$
- $h_d = ((b + D) \tan\alpha_2) - (D \tan\alpha_1)$
- $h_d = b \tan\alpha_2 + D \tan\alpha_2 - D \tan\alpha_1$
- $h_d = D (\tan\alpha_2 - \tan\alpha_1) + b \tan\alpha_2$
- $h_d - b \tan\alpha_2 = D (\tan\alpha_2 - \tan\alpha_1)$
- $D = \dfrac{h_d - b \tan\alpha_2}{(\tan\alpha_2 - \tan\alpha_1)}$ ……………………………(3)
- Substitute the value of D in equation (1)
- $h_1 = \dfrac{(h_d - b \tan\alpha_2)\tan\alpha_1}{(\tan\alpha_2 - \tan\alpha_1)}$ …………………………(4)

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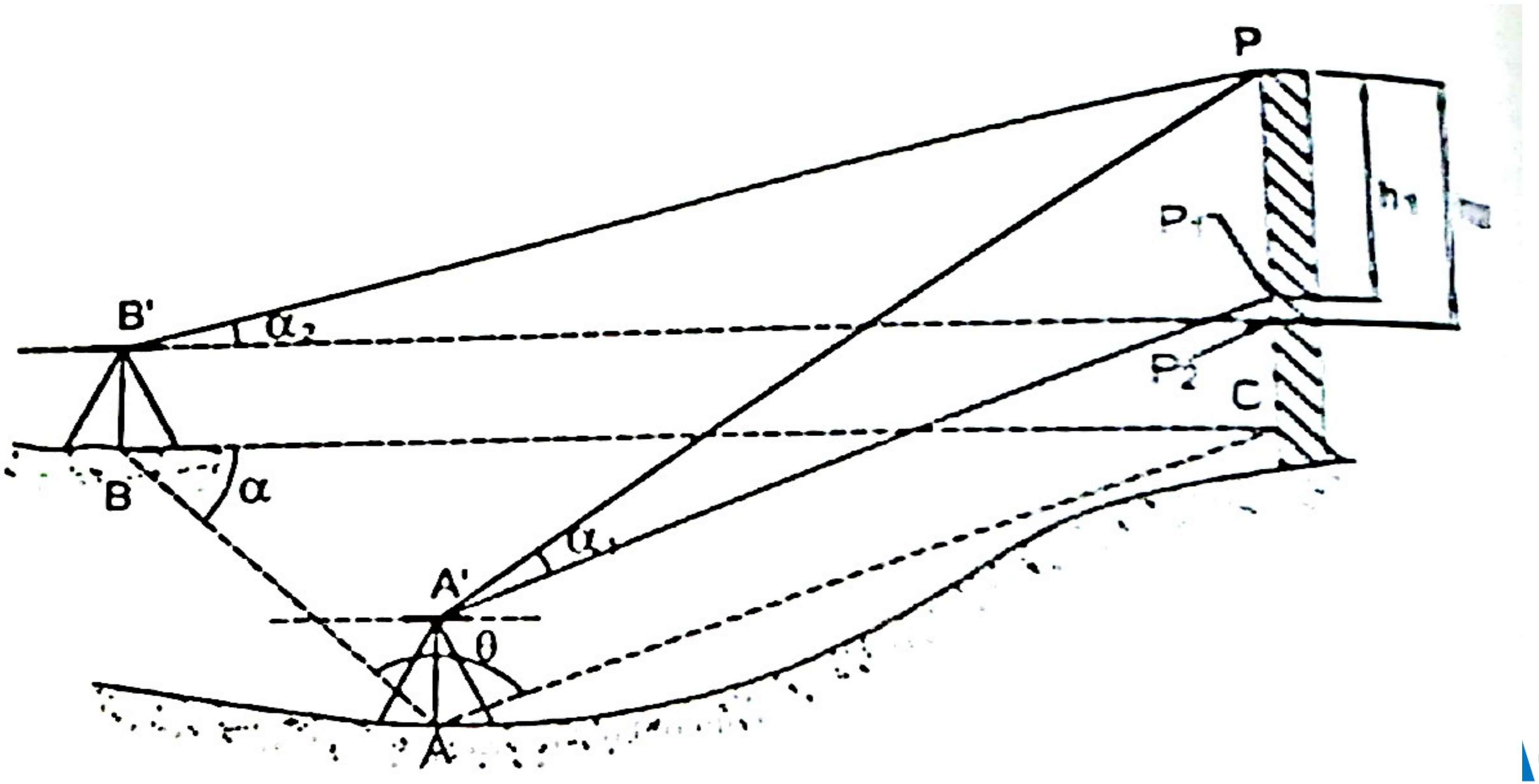
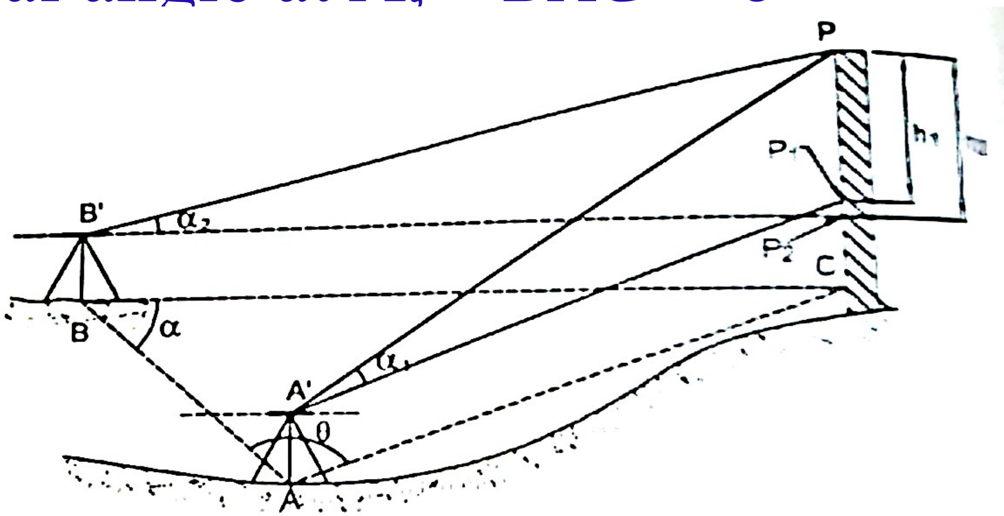
Case : 3 :- Base of the object inaccessible , instrument stations not in the same vertical plane as the elevated object.

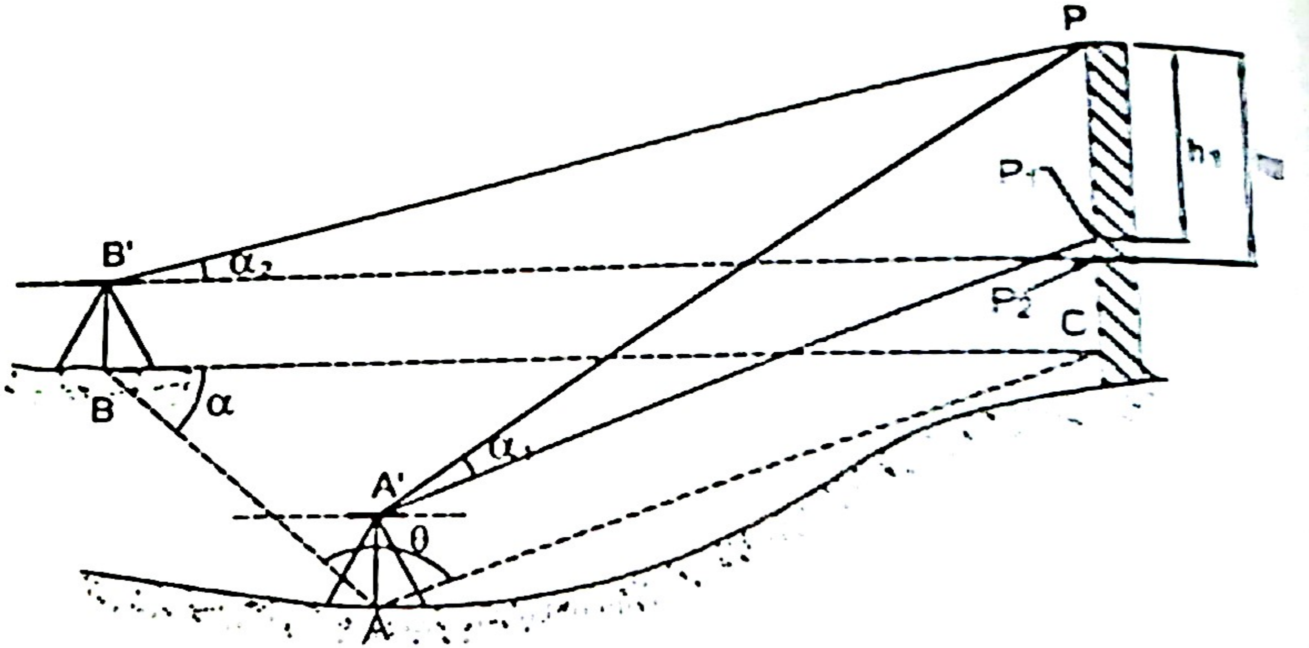
- Let A and B be the two instrument station not in tha same vertical plane as that of P.



- Select a two station A and B on leveled ground and measure b as horizontal distance between them.
- Set the instrument at A and level it.
- Set the vertical vernier 0^0.
- Bring the altitude bubble at the center and take a back sight hs on the staff and kept at B.M.
- Measure the angle of elevation α_1 to P.
- Measure the horizontal angle at A, $\angle BAC = \theta$



- Shift the instrument to B and measure the angle of elevation α_2 to P.
- Measure the horizontal angle at B as α.
- α_1 = angle of elevation from A to P
- α_2 = angle of elevation from A to P
- θ = Horizontal angle BAC at station A
- α = Horizontal angle CBA at station B
- $h_1 = PP_1$ = height of the object P from instrument axis of A.
- $h_2 = PP_2$ = height of the object P from instrument axis of B.



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- In Δ ABC, $\angle BAC = \theta$

$$\angle ABC = \alpha$$

$$\angle ACB = 180^0 - (\theta + \alpha)$$

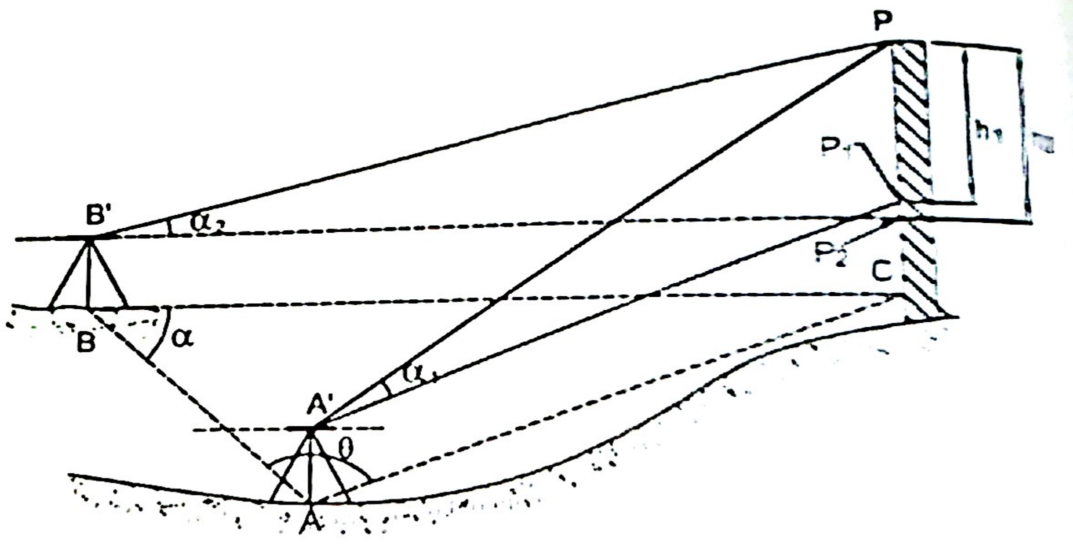
$$AB = b$$

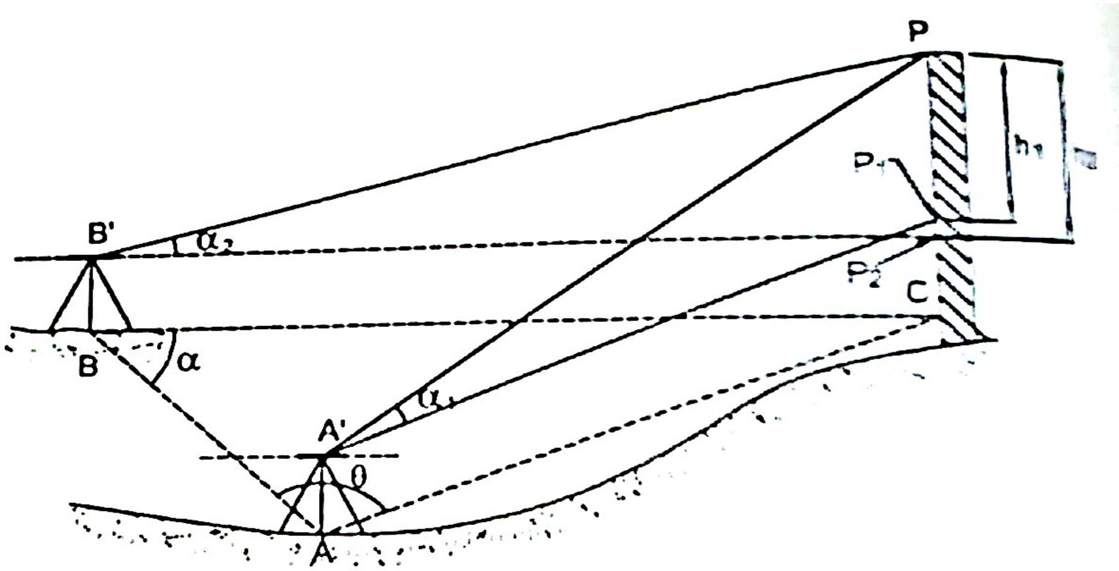
We know that, three angle and one side of the triangle ABC.

There for using a sin rule, we can calculate distance AC and BC as bellow

$$BC = \frac{b\ sin\theta}{\sin(180^0 - (\theta + \alpha))} \quad ..(1)$$

$$AC = \frac{b\ sin\alpha}{\sin(180^0 - (\theta + \alpha))} \quad ..(2)$$





- Now,
- $h_1 = AC \tan\alpha_1$
- $h_2 = BC \tan\alpha_2$
- Values of Ac and BC are obtained from equation (1) and (2) as above
- R.L of P = height of the instrument axis at A + h_1

or

- R.L of P = height of the instrument axis at B + h_2
- Height of the instrument axis at A = R.L of B.M + B.S
- Height of the instrument axis at B = R.L of B.M + B.S

Example : 1

- Calculate reduce level of the top of the tower from the following data.

Instrument station	Reading on B.M	Vertical angle
A	1.75m	15^0
B	2.10m	11^0

- R.L of B.M is 100.0m and observation are taken with the line of sight horizontal. AB = 50 m A, B and the top of lower are in the same vertical plane.

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Example : 2

- To obtain R.L of top of a ten storeyed building following observation were taken.

Instrument station	Reading of B.M	Vertical angle	R.L of B.M
A	2.625 m	19^0 48’	500 m
B	1.510 m	14^0 25’	500 m

- distance between A and B is 50 m A, B, B.M and the building are in same vertical plane.

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Example : 3

- In trigonometric levelling, calculate the R.L of the top of the tower from the following data.

Instrument station	Reading on B.M	Angle of Elevation of top of Tower	Remarks
A	1.55	150 24’	R.L of B.M = 100 m
B	1.53	150 42’	Distance AB = 30 m

- Horizontal angle at A, between B and tower = 75^{0} 24’
- Horizontal angle at B, between A and tower = 86^{0} 36’

Example : 4

- A theodolite was setup at a distance of 130 m from a tower. The angle of elevation to the top was 7^0 6' and the angle of depression to the bottom was 2^0 30'. If the R.L of Instrument axis is 100, find R.L of top and Bottom of tower.

Example : 6

- In trigonometric leveling following observations were taken for top of vertical cliff.

Instrument station	Reading on B.M	Vertical angle to top of cliff	Remark
A	1.40	15^0	R.L of B.M = 50 m
B	1.40	14^0	AB = 30

- If the top of the cliff and both instrument station are in same vertical plane, find the horizontal distance between cliff and A also find R.L of the top of cliff.

Example : 7 (March 2010)

- An instrument was setup at P and the angle of elevation of the top of an electric pole QR was $25^0 3'$. The horizontal distance between P and Q , the foot of the pole is 500 m. Determine the R.L of the top of the pole, if the staff reading held on B.M (R.L 100.00m) was 3.532 m with the telescope in horizontal plane.

Example : 8 (Dec 2010)

- A theodolite was setup at a distance of 150 m from tower. The angle of elevation to the top of the parapet was $10^{0}8'$ while the angle of depression to the foot of the wall was 3012’. The staff reading on B.M of R.L 50.217 with the telescope horizontal was 0.880. find the height of the tower and the R.L of the top of parapet.

Example : 9 (May 2011)

- Find out elevation of a hill top based on the following data set. Distance between O_1 and O_2 i

Instrument Station	Staff reading on B.M	Vertical angle hill top	R.L of B.M (m)
O1	1.545 m	$28^0 42'$	101.505 m
O2	1.545 m	$18^0 6'$	

Example : 10 (Dec 2011)

- To determine the height of the chimney, a theodolite was kept at two station I_1 and I_2 200 m apart. I_1 being nearer to chimney. The reading at the B.M of R.L 1020.375 m were 1.35 m from station I_1 & 2.15 from I_2. the vertical angle to the top of the chimney were $19^0 30'$ & $8^0 15'$ from station $I_1 I_2$ respectively. Find the horizontal distance & R.L of the top of the chimney.

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Example : 11 (May 2012)

- Determine the height of the pole above the ground on the basis of the following angles and elevation from two instruments station A and B in line with the pole angles of elevation from A to the top of bottom of pole = 29^0 and 20^0. angle of elevation of B to top of bottom pole = 36^0 and 27^0. horizontal distance AB = 35 m. the readings observed of staff at the B.M with the two instrument settings are 1.38 and 1.19 m respectively what is horizontal distance of the pole from A?

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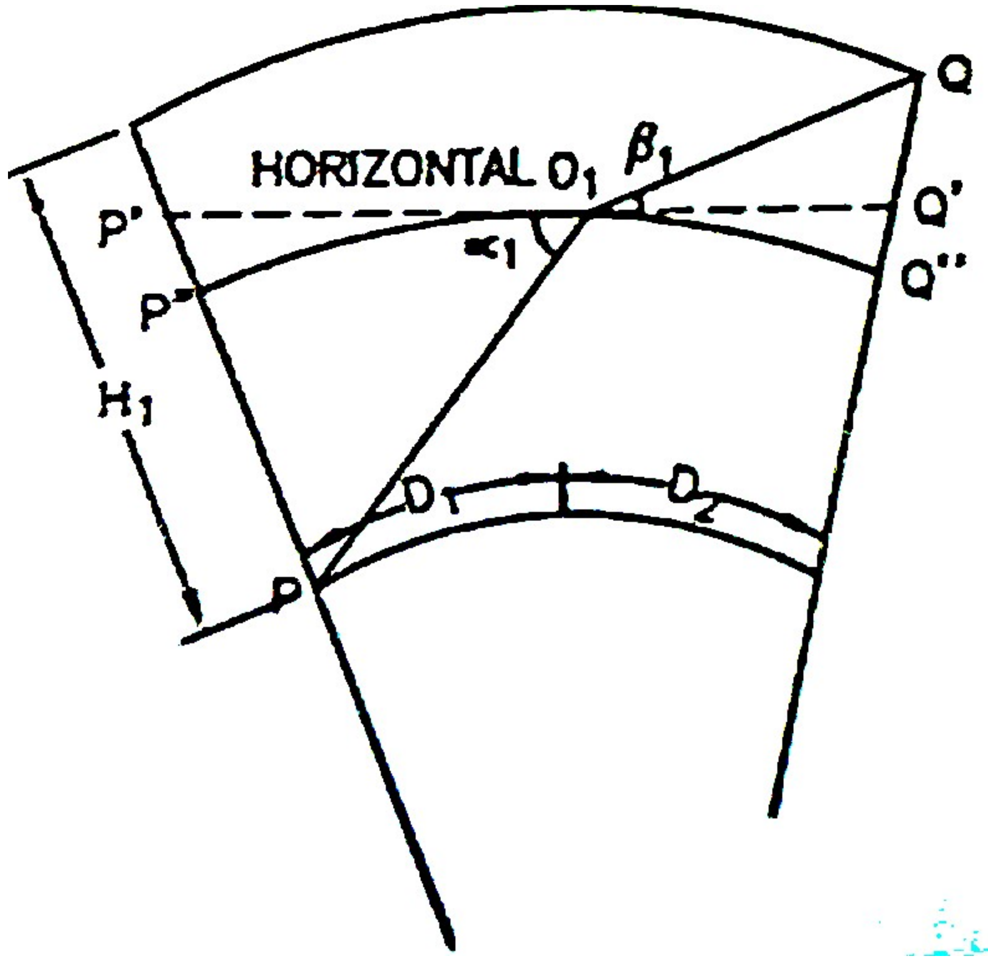
Indirect levelling on a rough terrain

- Indirect levelling can be used to determine the difference of elevation of two points which are quite apart.
- Select two point on ground. (P and Q)

- **Procedure :-**

1. Set up the instrument at convenient point O_1 mid way between P and Q.
2. Measure the vertical angle α_1 to the station P. Also measure the horizontal distance D_1 between O_1 and P.
3. Measure the vertical angle β_1 to the station Q. Also measure the horizontal distance D_2 between O_1 and Q.



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4. Determine the difference in elevation H1 between P and Q as explain bellow.

- Let us assume that,
- α_1 = angle of depression
- β_1 = angle of elevation
- H_1 = PP"+ QQ"

$$= (PP' - P'P'') + (QQ' + Q'Q'')$$

$$= (D_1 \tan \alpha_1 - C_1) + (C_2 + D_2 \tan \beta_1)$$

Where C_1 and C_2 are the correction due to curvature of earth and refraction.

As the distance D_1 and D_2 are nearly equal, the correction C_1 and C_2 are also approximately equal.

- $H_1 = D_1 \tan \alpha_1 + D_2 \tan \beta_1$

5. Now shift the instrument to the station O_2 midway between Q and R. measure the vertical angle α_2 and β_2 to the station Q and R and respective horizontal distance D_3 and D_4.

- Difference of elevation between Q and R is
- $H_2 = D_3 \tan \alpha_2 + D_4 \tan \beta_2$

6. Repeat above procedure at the station O_3

$$H_3 = D_5 \tan \alpha_3 + D_6 \tan \beta_3$$

7. Determine the difference in Elevation of P and S as.

$$H = H_1 + H_2 + H_3$$

$$\text{R.L of S} = \text{R.L of P} + H$$

- Indirect levelling is not as accurate as direct levelling.

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Indirect levelling on a steep slope

- If the ground is so steep so the method of indirect levelling can be used with advantages.
- The procedure for finding the difference of elevation between P and Q are as under.

- **Procedure :-**
- St up the instrument over convenient station O1 on the line PR.
- Make line of collimation roughly parallel to the slope of the ground, clamp the telescope.

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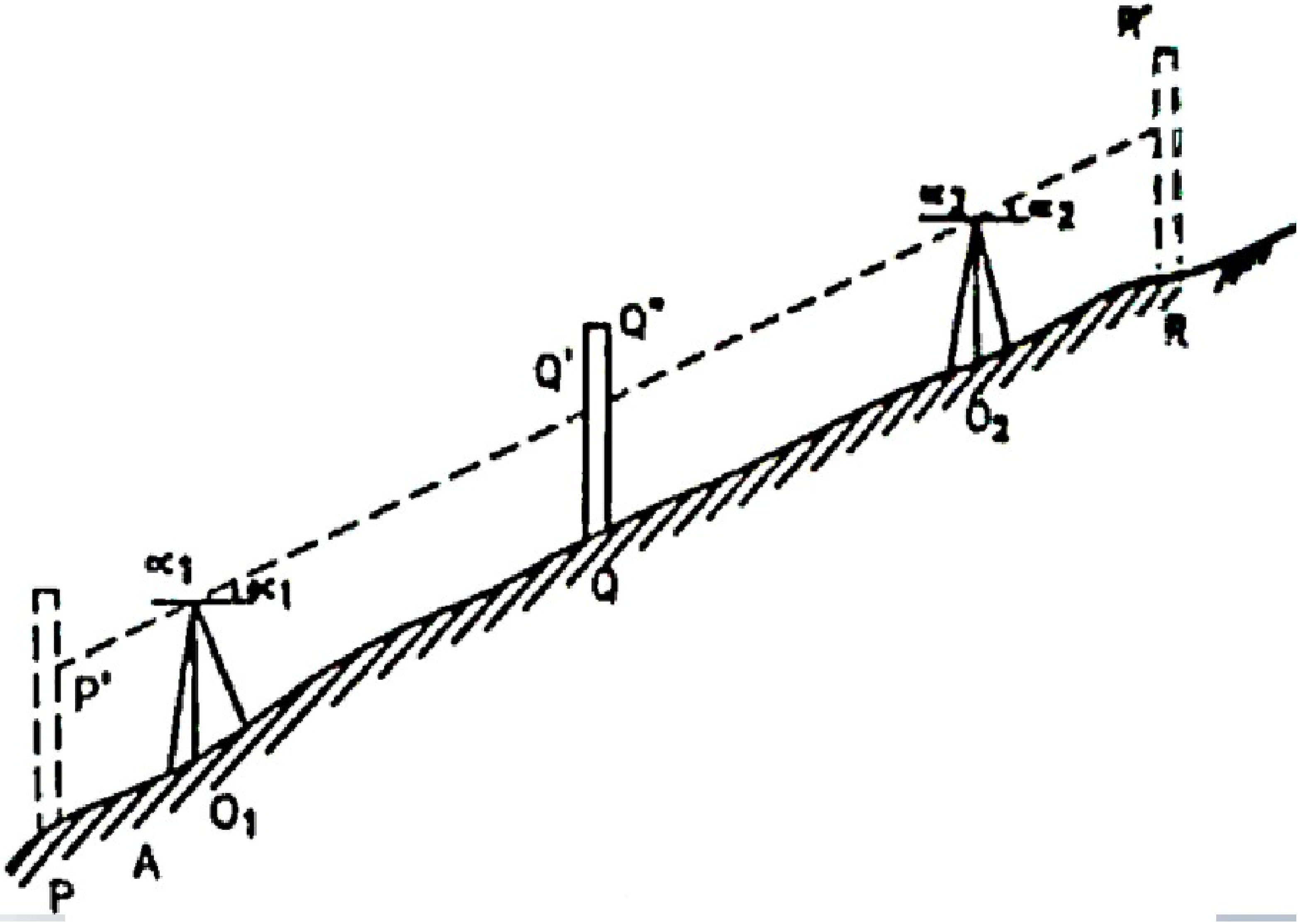
R"
Q"
Q'
P'
α_1
α_1
α_2
α_2
Q
R
O_2
O_1
A
P

- Take back sight PP’ on the staff held at P. Also measure the vertical angle α1 to P’. Determine the R.L of P’ as
- R.L of P’ = R.L of P + PP’
- Take a fore sight QQ’ on the staff held at the turning point Q, without changing vertical angle α1 , measure the solpe distance PQ between P and Q.
- R.L of Q = R.L of P’ + PQ sin α1 – QQ’
- Shift the instrument to the station O2 midway between Q and R. make the line of collimation roughly parallel to the slope of the ground, clamp the telescope.
- Take back sight QQ” on the staff held at the point R without changing the vertical angle α2. measure the sloping distance QR.
- R.L of R = R.L of Q” + QR sin α2 – PR’

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