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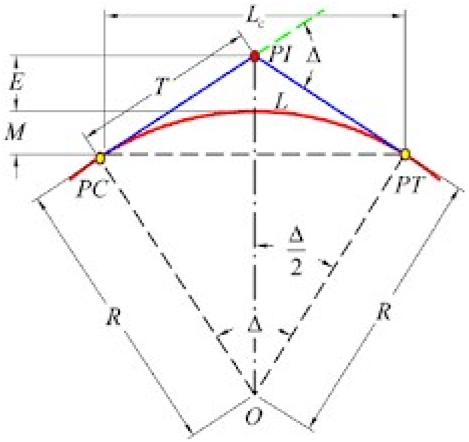
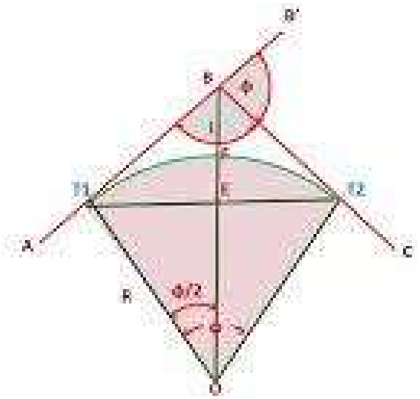
Module - 4
Curves





Subject:- Surveying Code:-3140601	Prepared by: Asst.Prof. Nutan Patel (Civil Department, ACET)	AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Introduction

- Generally used on Highway and Railway.
- Use for change the direction.
- Always tangential to the straight direction.
- The two line connected by a curve are called tangents.

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Types of Curves

- Curves
  - Horizontal Curves
    - Circular curve
    - Transition curve
  - Vertical curves

Types of Circular Curve

- There are three type of the circular curve.

Simple Curve

Compounded Curve

Reserve Curve

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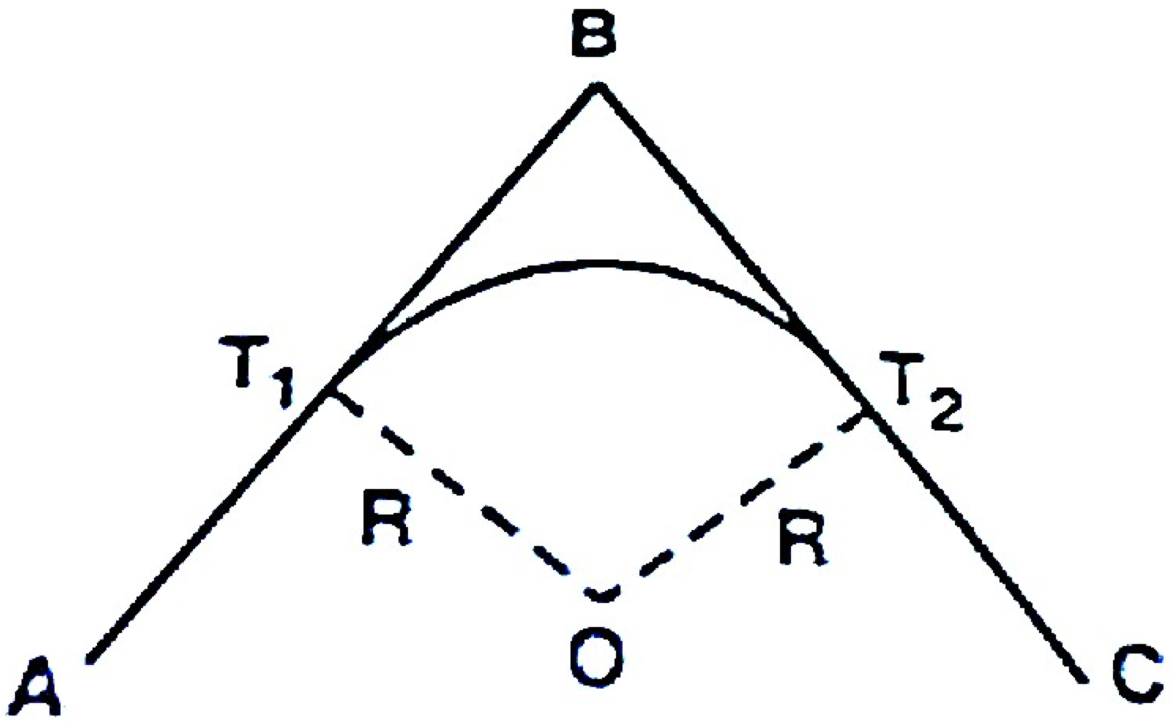
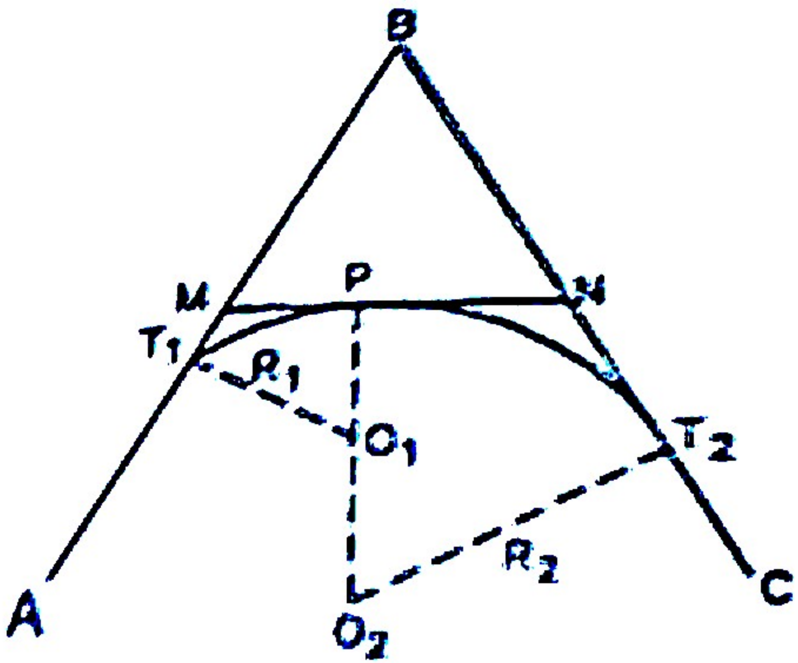
1. Simple Curve

- Consist of a single Arc.
- Tangential to both the straight line.



2. Compound Curve

- Two or more simple arc.
- In fig arc radius R_1 and centre O_1
- In fig arc radius R_2 and centre O_2



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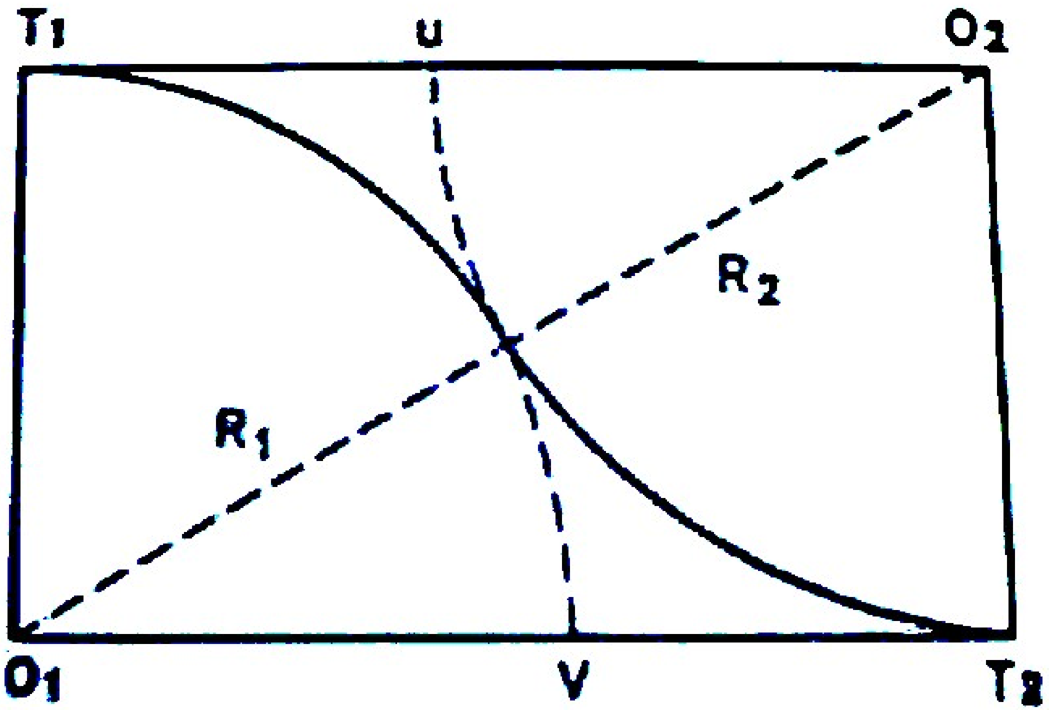
3. Reverse Curve

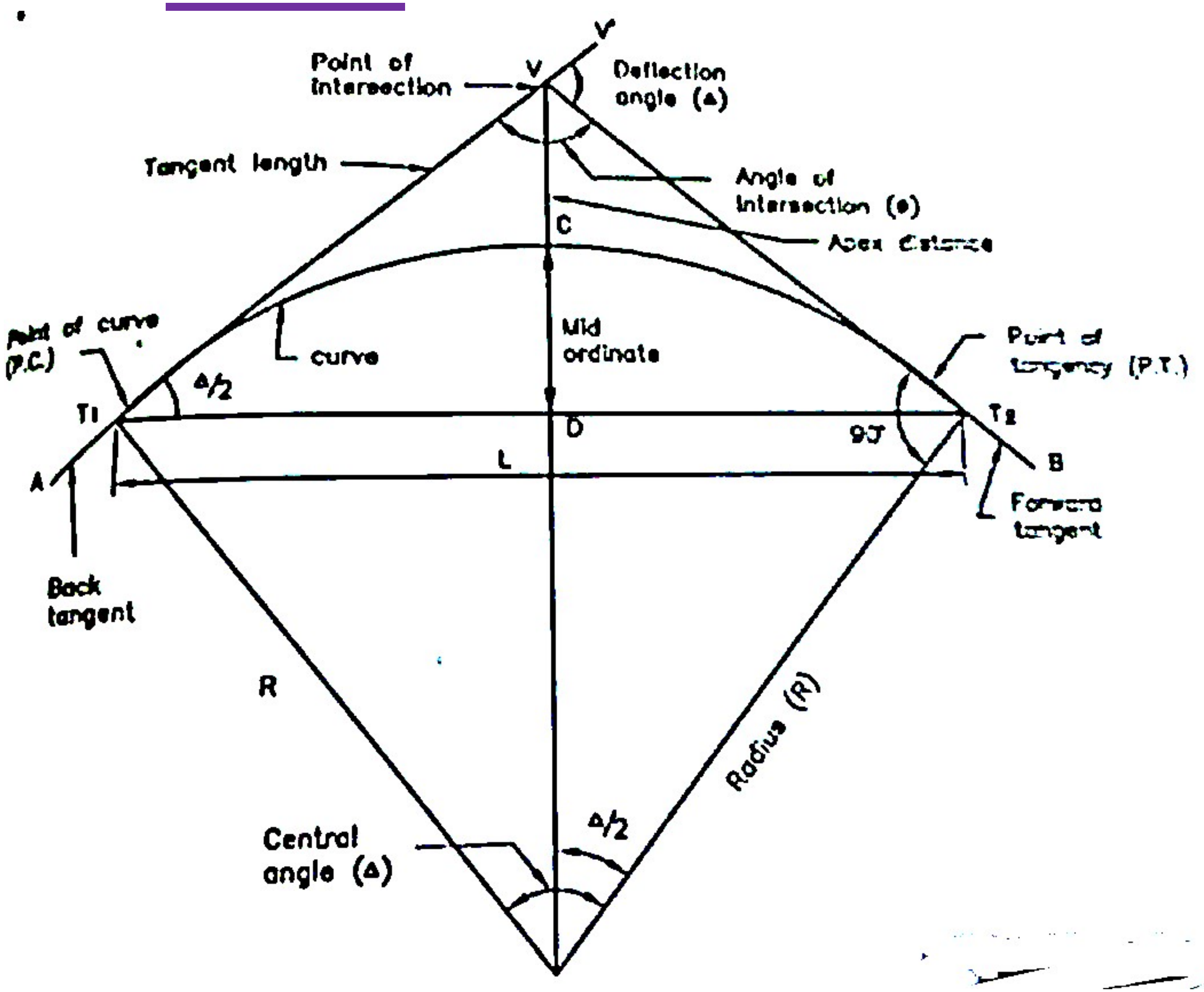
- Two circular arcs.
- Centre in opposite direction.
- Reverse curve are provided for low speeds roads and railway.



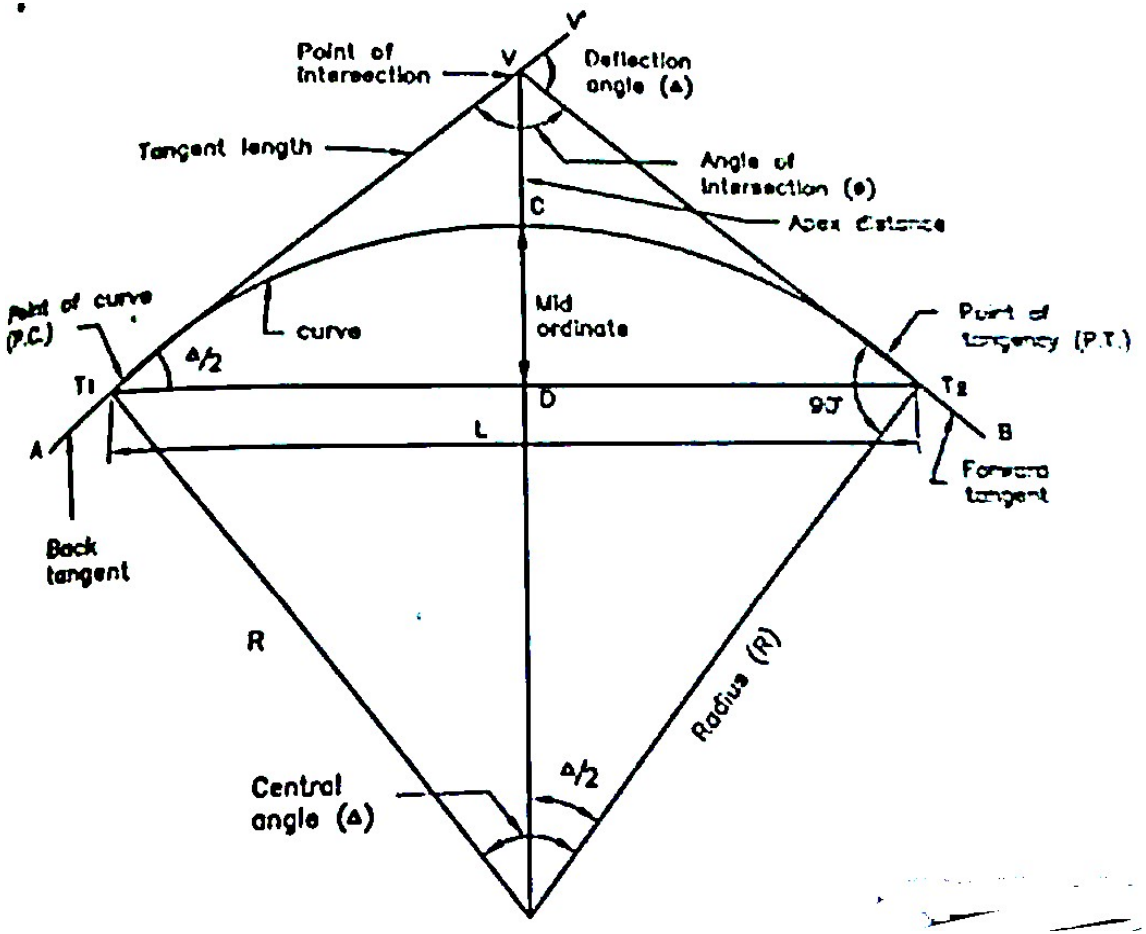
Definition and Notations for simple curve

- **Back tangent point :-**
- The tangent (AT_1) previous to the curve is called tha back tangent or first tangent point.

- **Forward tangent Point :-**
- The tangent (T_2B) following the curve is called the forward tangent point or second tangent point.



- **<u>Point of Intersection (P.I) :-</u>**
- If the tangents AT_1 and AT_2 are produced they will meet in a point, called the point of intersection (P.I).
- It is also called vertex (V)



- **<u>Point of Curve (P.C) :-</u>**
- It is the beginning point T_1 of a curve, at this point alignment is changes from a tangent to a curve.

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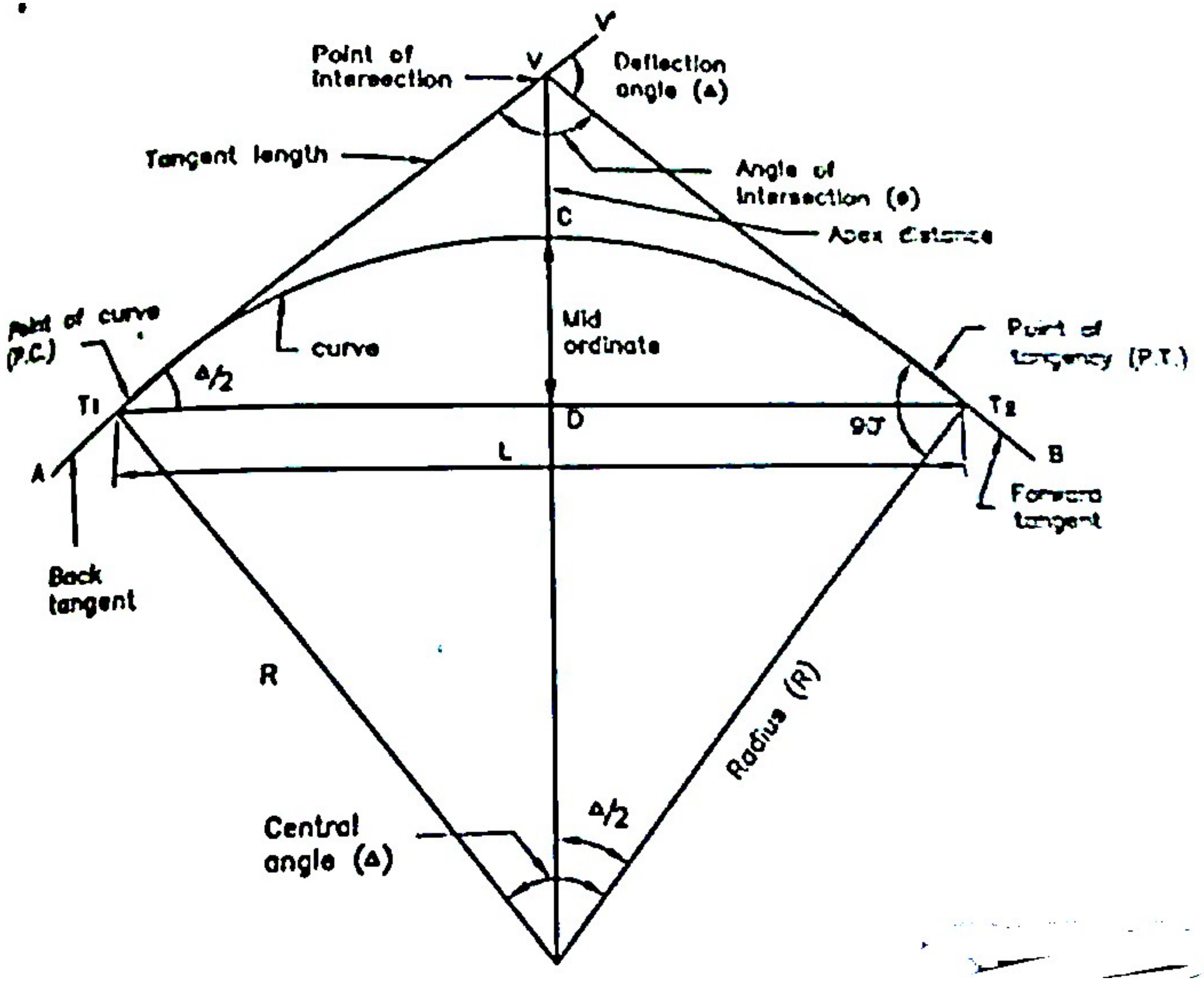
- **Point of tangency (P.T) :-**
- The end point of the curve (T_2) is called the point of tangency.



- **Intersection angle (Φ) :-**
- The angle AVB between tangent AV and tangent VB is called intersection angle.

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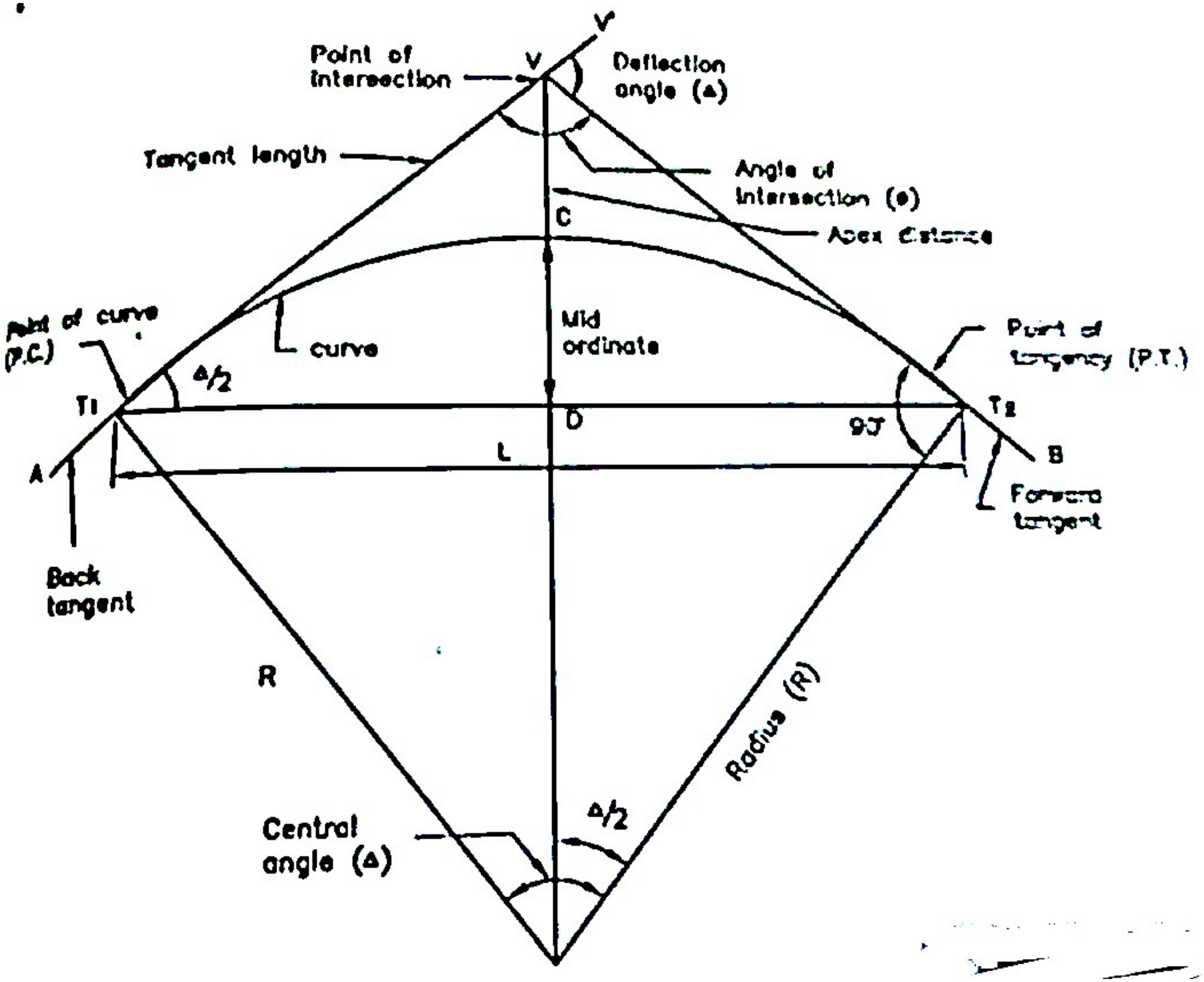
- **Deflection angle (Δ) :-**
- The angle at P.I between tangent AV produce and VB is called the deflection angle.



- **Tangent distance :-**
- It is the distance between the P.C to P.I, it is also distance between the P.I to P.T

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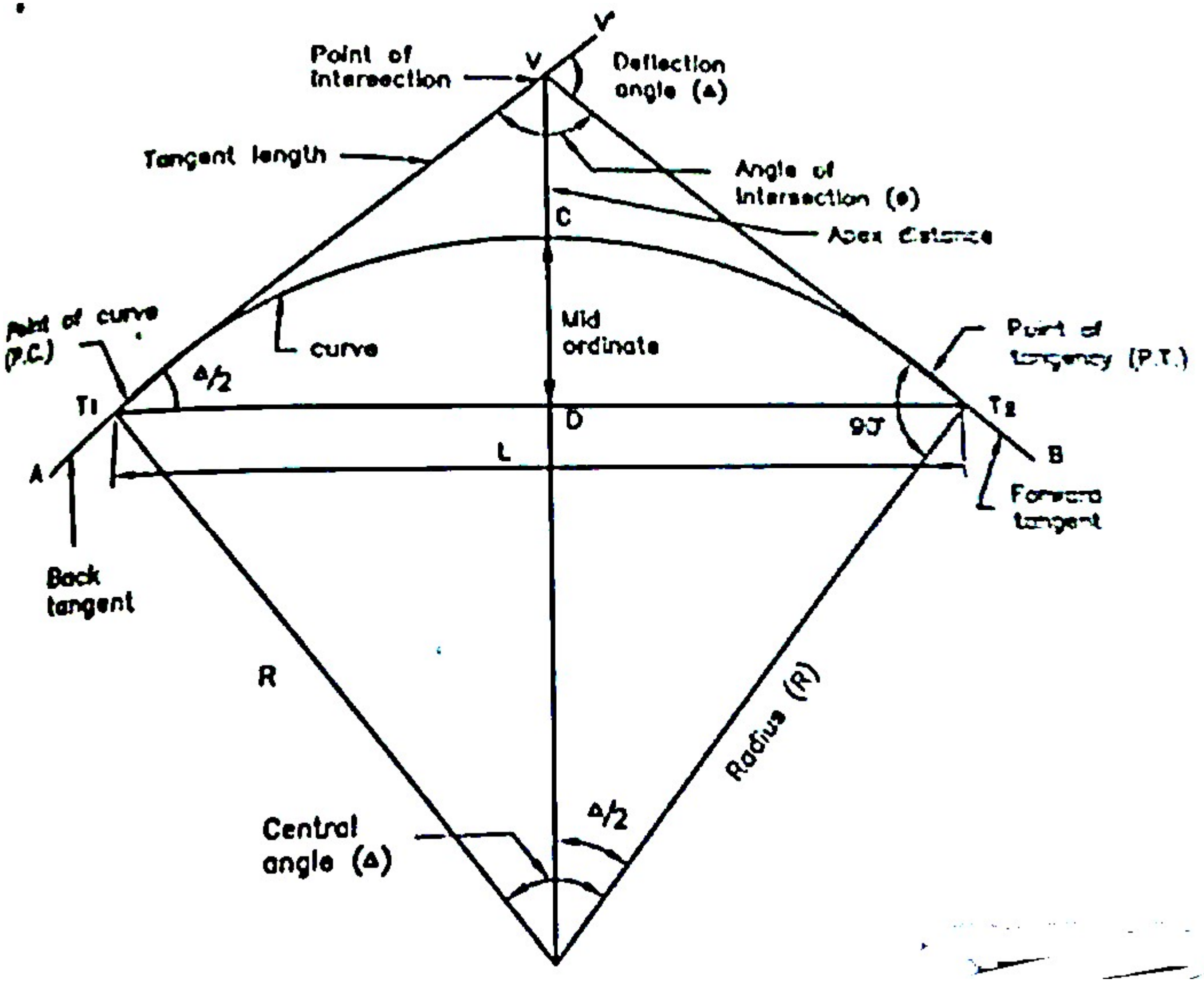
- **External distance (E) :-**
- It is the distance from the mid point of the curve to P.I.

- **Length of Curve (l) :-**
- It is the total length of curve from P.C to P.T.



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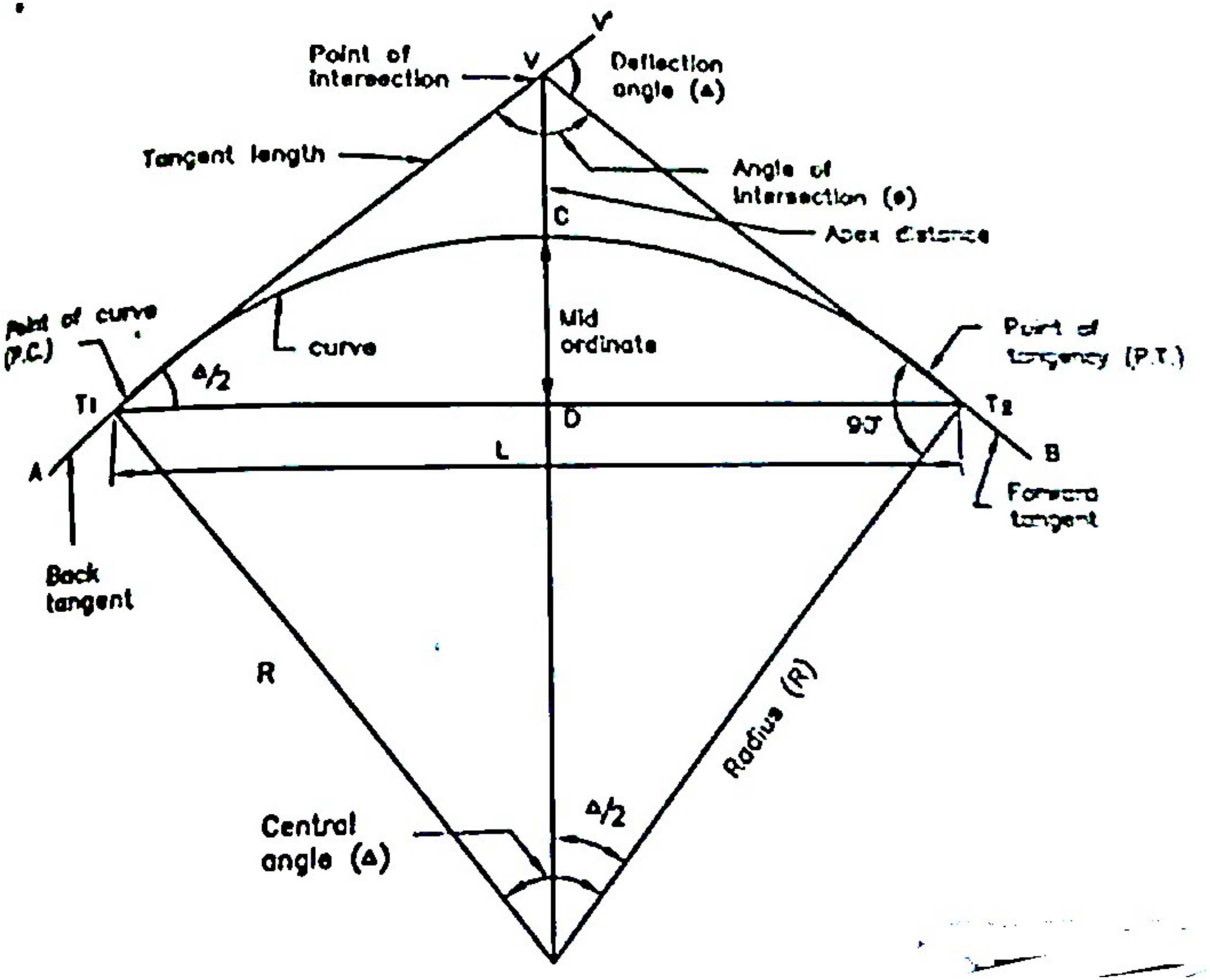
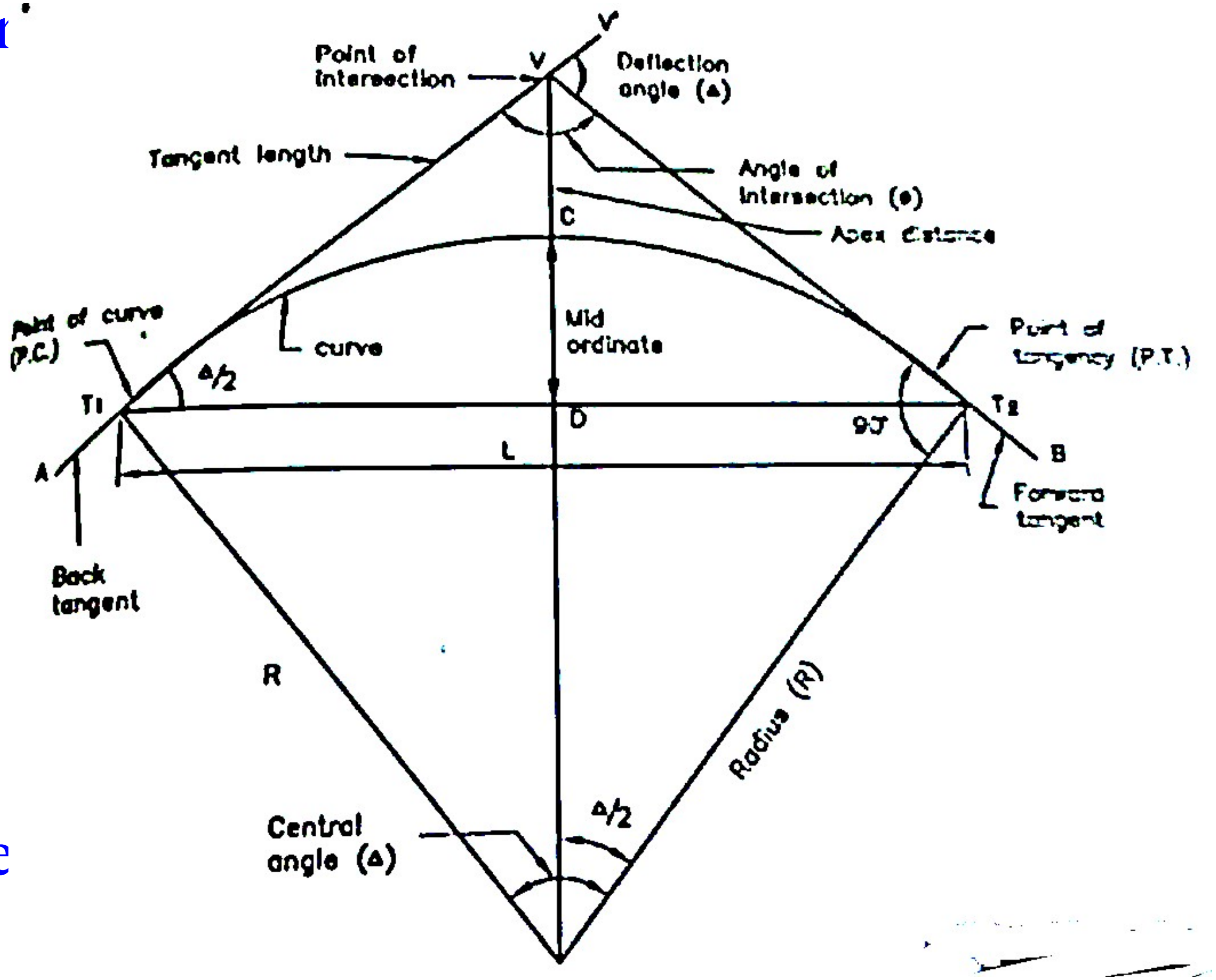
- **Long Chord :-**
- It is chord joining P.C to P.T T1, T2 is a long chord.

- **Normal chord :-**
- A chord between two successive regular station on a curve is called normal chord.



- **Sub Chord :-**
- The chord shorter than normal (Shorter than 20m) is called Sub chord.

- **Versed sine :-**
- The distance between mid point of long chord (D) and the apex point C is called versed sine.



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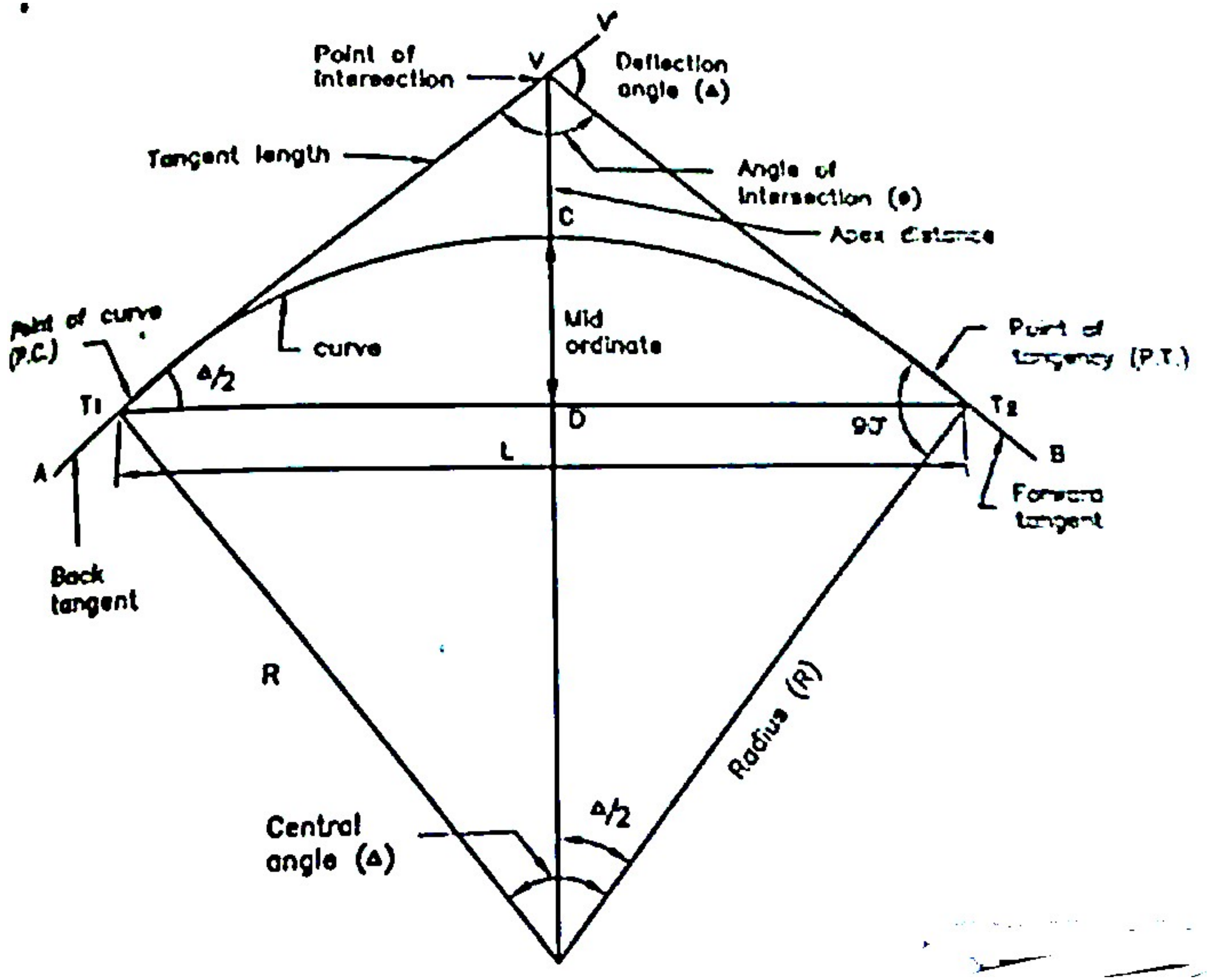
- **Right hand curve :-**
- If the curve deflect to the right of the direction of the progress of survey, it is called right hand curve.

- **Left hand curve :-**
- If the curve deflect to the left of the direction of the progress of survey, it is called left hand curve.



Designation of Curve

- The sharpness of Curve is designated by two ways :-

1) By radius (R)
2) By degree of curvature (D)

1) **By radius (R) :-**

- In this method the curve is known by the length of its radius (R).
- Ex :-
- 200m curve means the curve having radius 200m.
- 6 chain curve means the curve having radius 6 chain.

2) By Degree of Curvature :-

- In this method curve is designated by degree.
- The degree of curvature can be divided in to two ways.

1. Chord definition :-

- The angle subtended at the centre of curve by a chord of 20m is called degree of curvature.
- Ex :- If the angle subtended at the centre of the curve by a chord of 20m is 5^0 the curve is called 5^0 degree curve.

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2. Arc definition :-

- The angle subtended at the centre of the curve by an arc of 20m length is called degree of curve.
- Used in America, Canada, India.

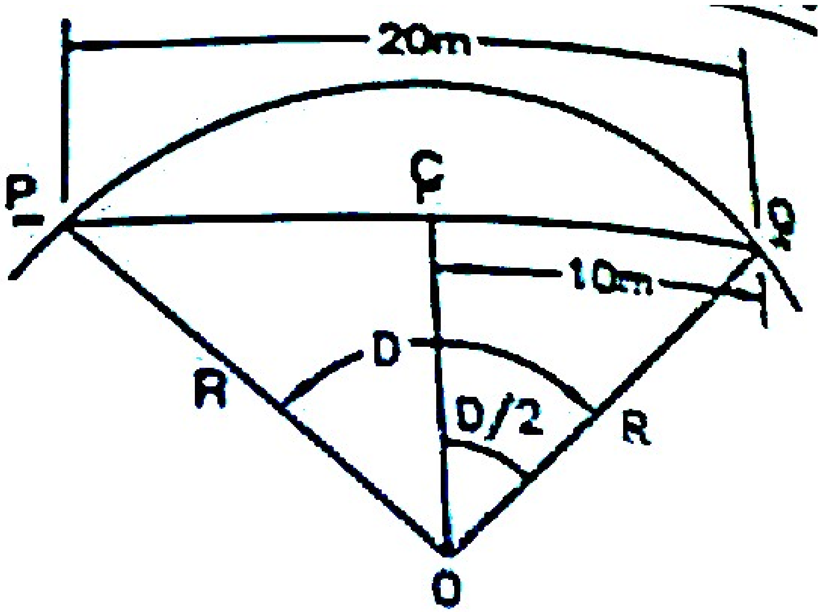
Relation Between Radius and Degree of Curve

By Chord Definition

By Arc Definition

By Chord Definition

- The angle subtended at the centre of curve by a chord of 20m is called degree of curve.



- R = Radius of curve
- D = Degree of Curve
- PQ = 20 m = Length of Chord

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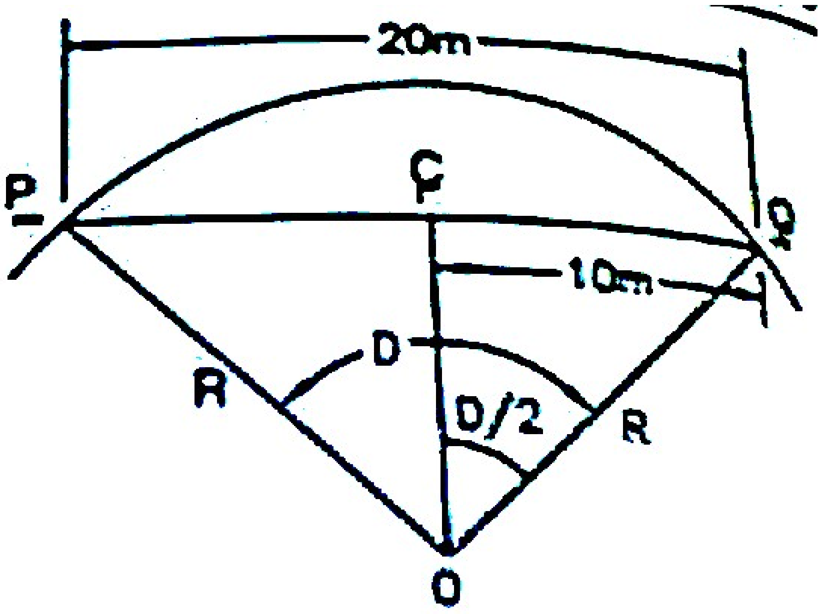
- From triangle PCO
- $\text{Sin}\ \frac{D}{2} = \frac{10}{R}$
- $R = \frac{10}{\text{Sin}\ \frac{D}{2}}$
- When D is small, $\text{Sin}\ \frac{D}{2}$ may be taken equall to $\frac{D}{2}$



- $\text{Sin}\ \frac{D}{2} = \frac{D}{2}$
- $R = \frac{10}{\frac{D}{2}\ X\ \frac{\pi}{180}}$
- $R = \frac{10\ X\ 360}{\pi\ D} = \frac{1146}{D}$

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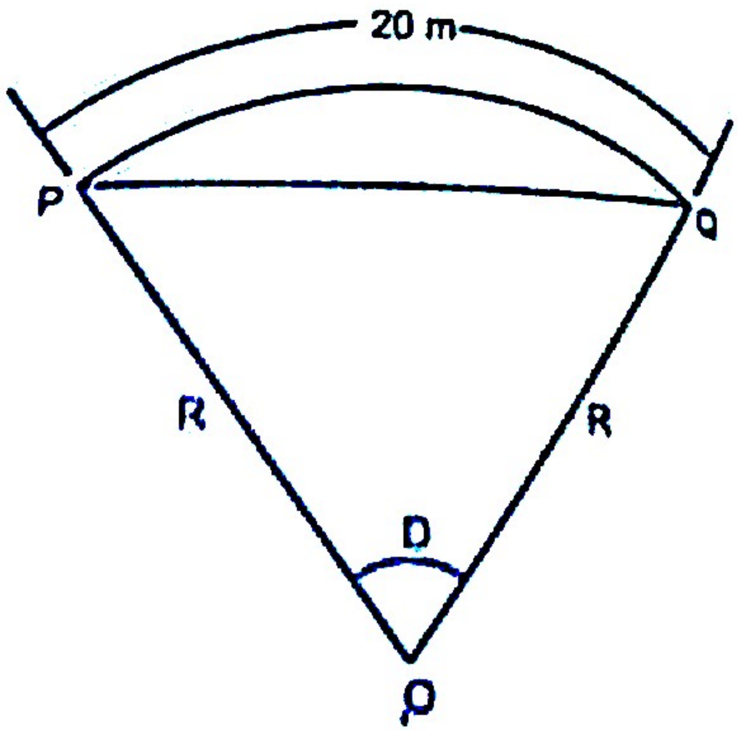
By Arc Definition

- The angle subtended at the centre of curve by an arc of 20m length is called degree of curve.
- $\frac{2\,\pi\,R}{360} = \frac{20}{D}$
- $R = \frac{20\ X\ 360}{2\,\pi\,D}$
- $R = \frac{1145.92}{D}$
- $\boxed{R = \frac{1146}{D}}$



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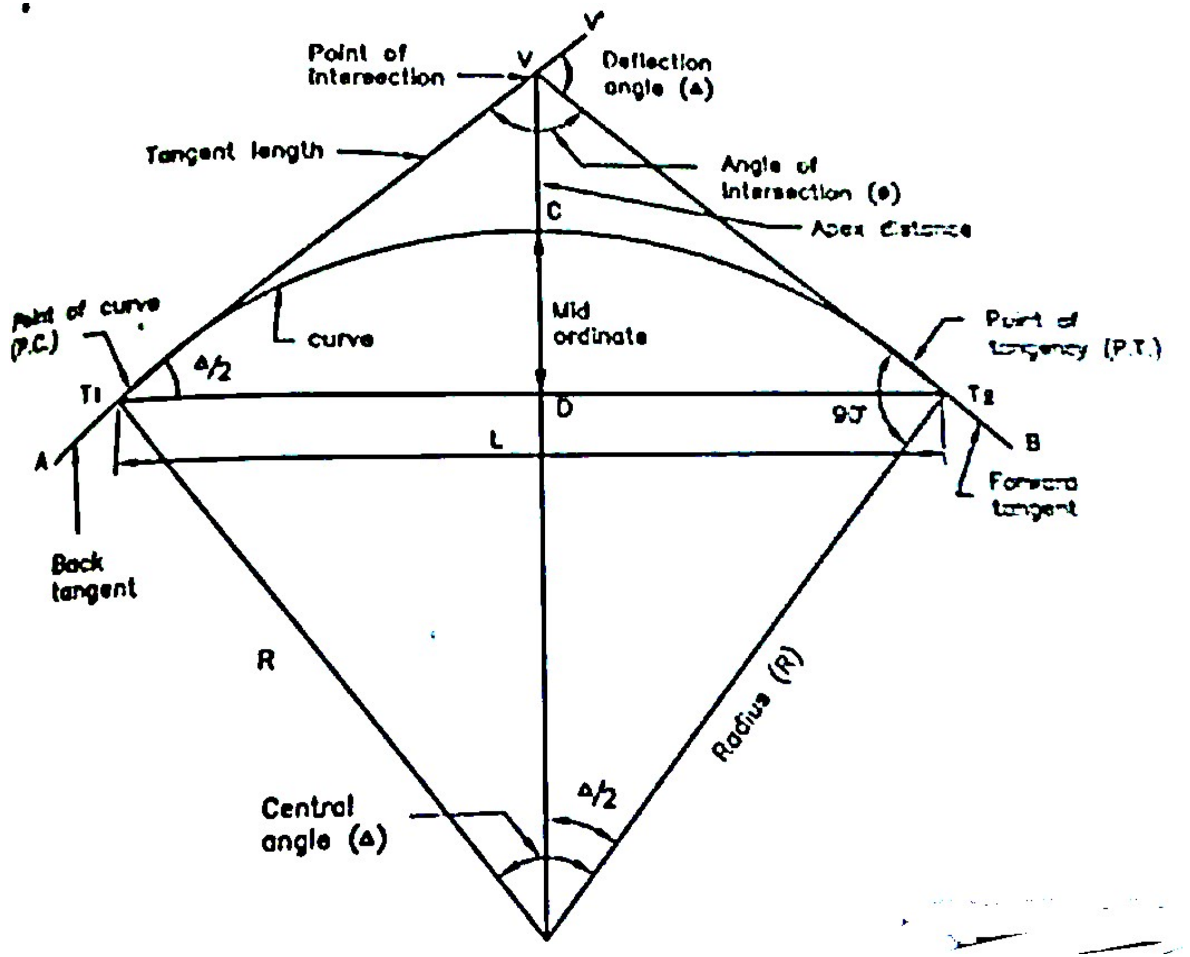
Elements of Simple Circular Curve



Length of Curve (l)

❖ **If curve designated by radius**

- l = length of arc $T_1\ C\ T_2$
- $l = R\ \Delta$ (Where Δ is in radian)
- $l = \dfrac{R\ \Delta\ \pi}{180}$ (Where Δ is in Degree)

❖ **If curve designated by degree**

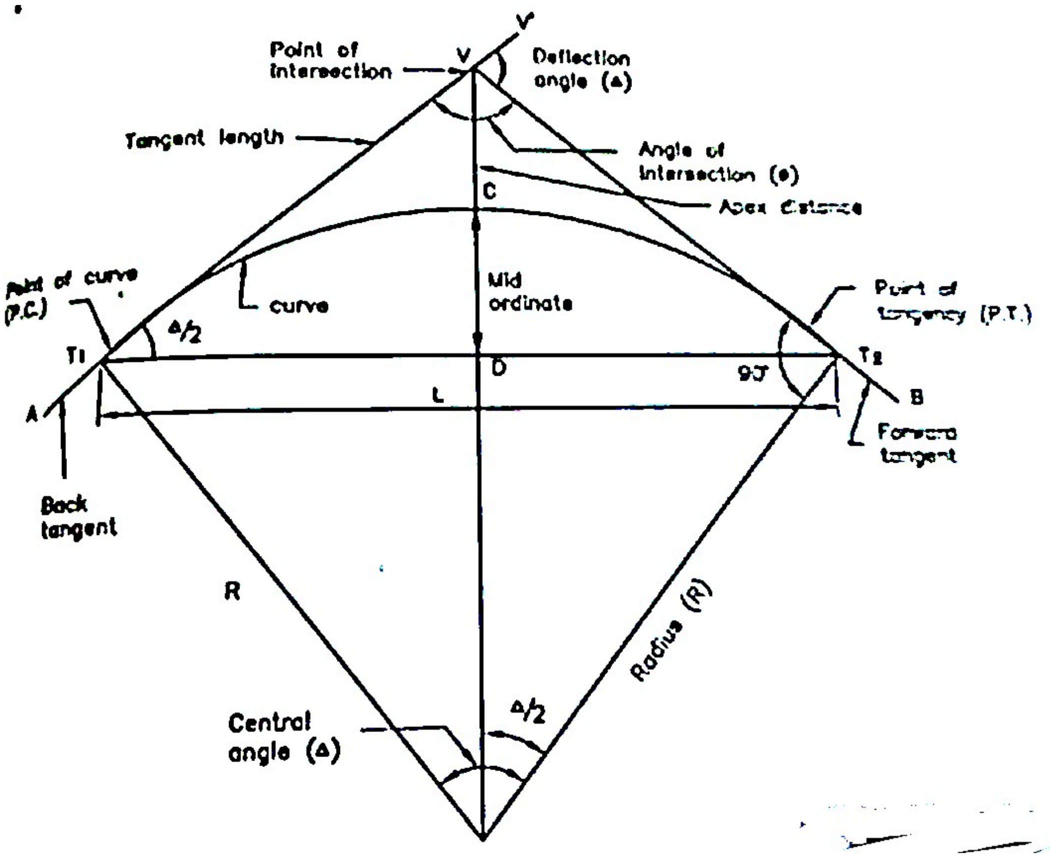
- Length of arc = 20 m
- Length of curve = $l = \dfrac{20\ \Delta}{D}$ m

  (D = degree of curve for 20 m arc)

Tangent Length (T)

- VT_1 and VT_2 are the tangent lengths
- $T = VT_1 = VT_2 =$ tangent length
- From $\Delta\, V\, T_1\, O$
- $\tan\frac{\Delta}{2} = \frac{VT_1}{OT_1} = \frac{T}{R}$

  ($\angle VT_1O$ and $\angle VT_2O$ are right angle)
- $T = R\tan\frac{\Delta}{2}$



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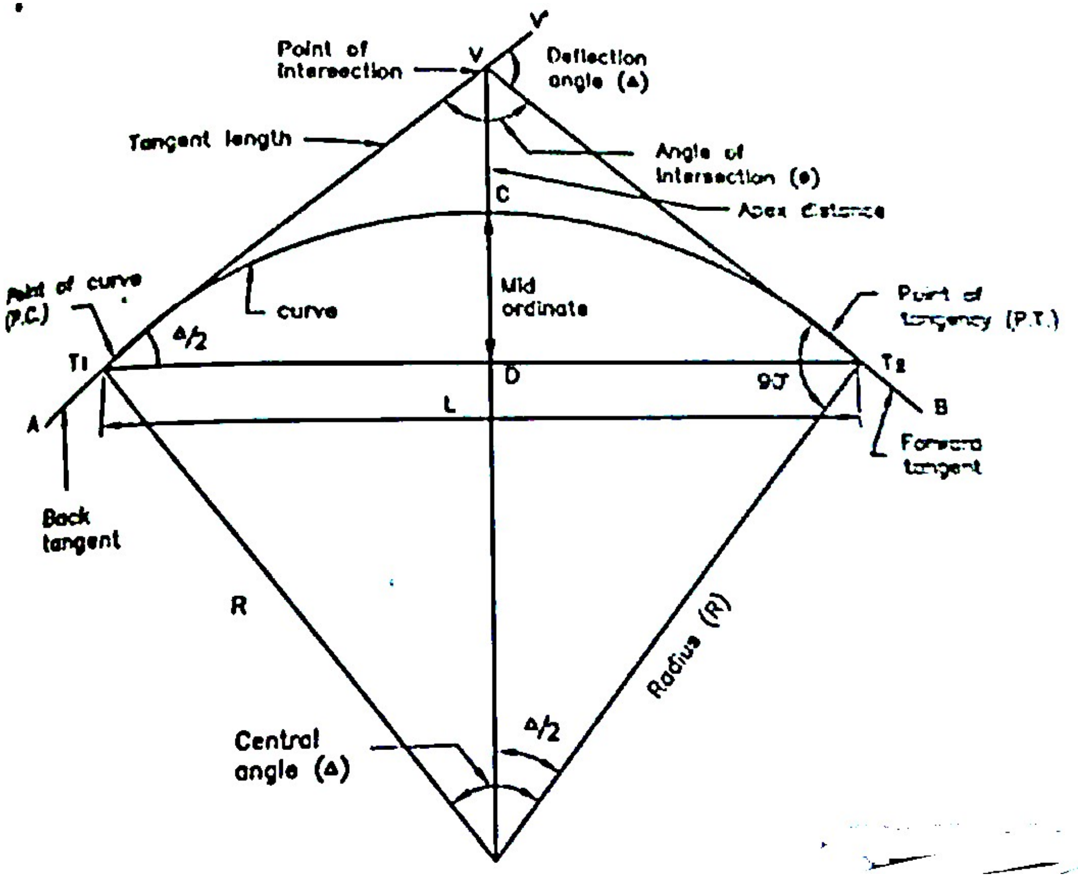
Length of Chord (L)

- In fig $T_1 T_2$ is long chord.
- Length of long chord $= L = T_1 T_2 = 2T_1 D$
- From triangle T_1 DO
- $\text{Sin}\,\frac{\Delta}{2} = \frac{T_1 D}{T_1 O} = \frac{T_1 D}{R}$
- $R\,\text{Sin}\,\frac{\Delta}{2} = T_1 D$
- $L = 2T_1 D$
- $L = 2R\ \sin\frac{\Delta}{2}$



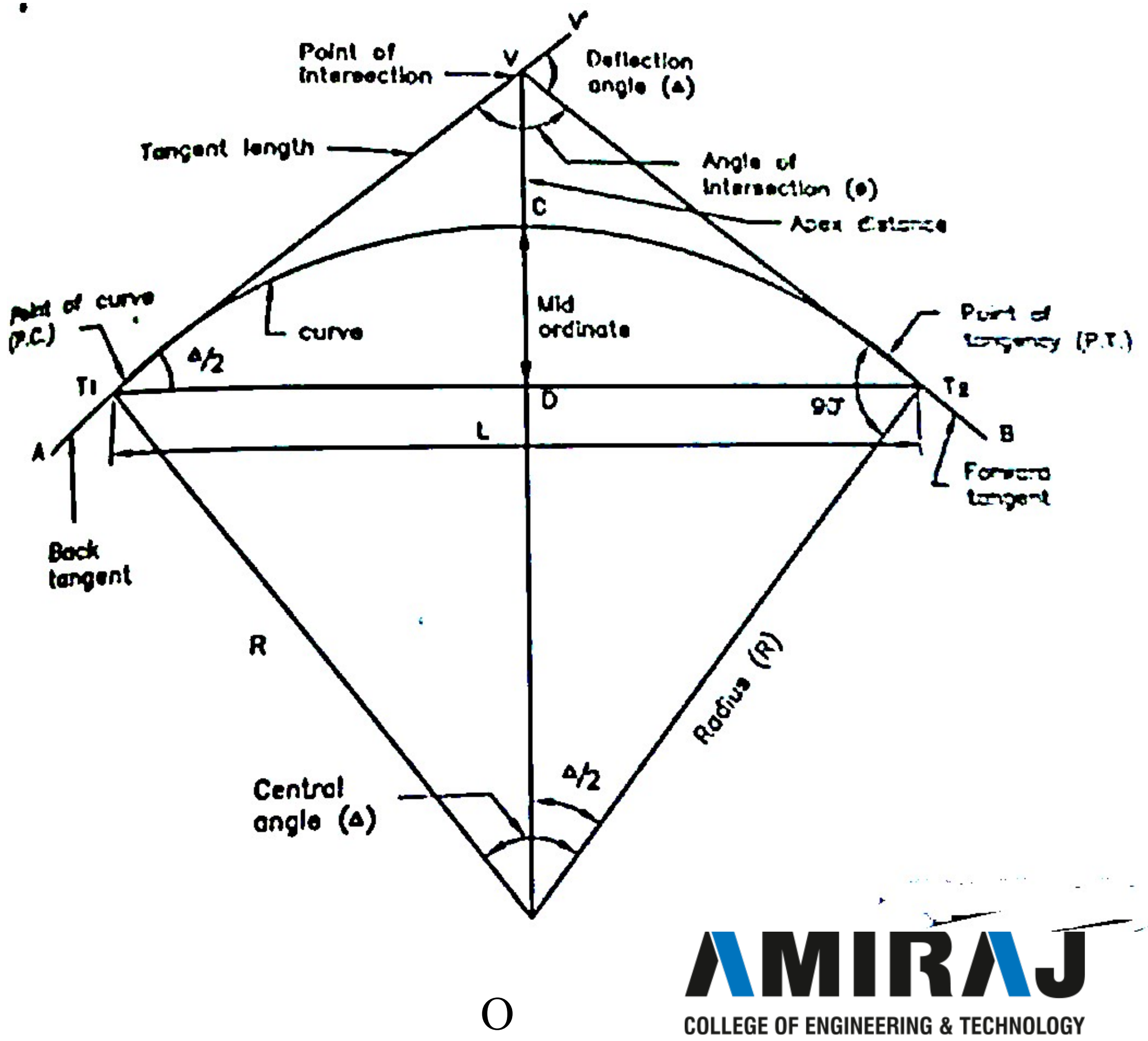
O

External Distance (E)

- In the fig, VC is the external distance.
- External Distance = E = VC = OV – OC
- From triangle VT_1O
- $\text{Cos}\,\dfrac{\Delta}{2} = \dfrac{OT_1}{OV} = \dfrac{R}{OV}$
- $OV = \dfrac{R}{\text{Cos}\,\frac{\Delta}{2}} = R\ \text{Sec}\,\dfrac{\Delta}{2}$
- E = OV – OC

$$= R\ \text{Sec}\,\frac{\Delta}{2} - R$$

$$= R\left(\text{Sec}\,\frac{\Delta}{2} - 1\right)$$

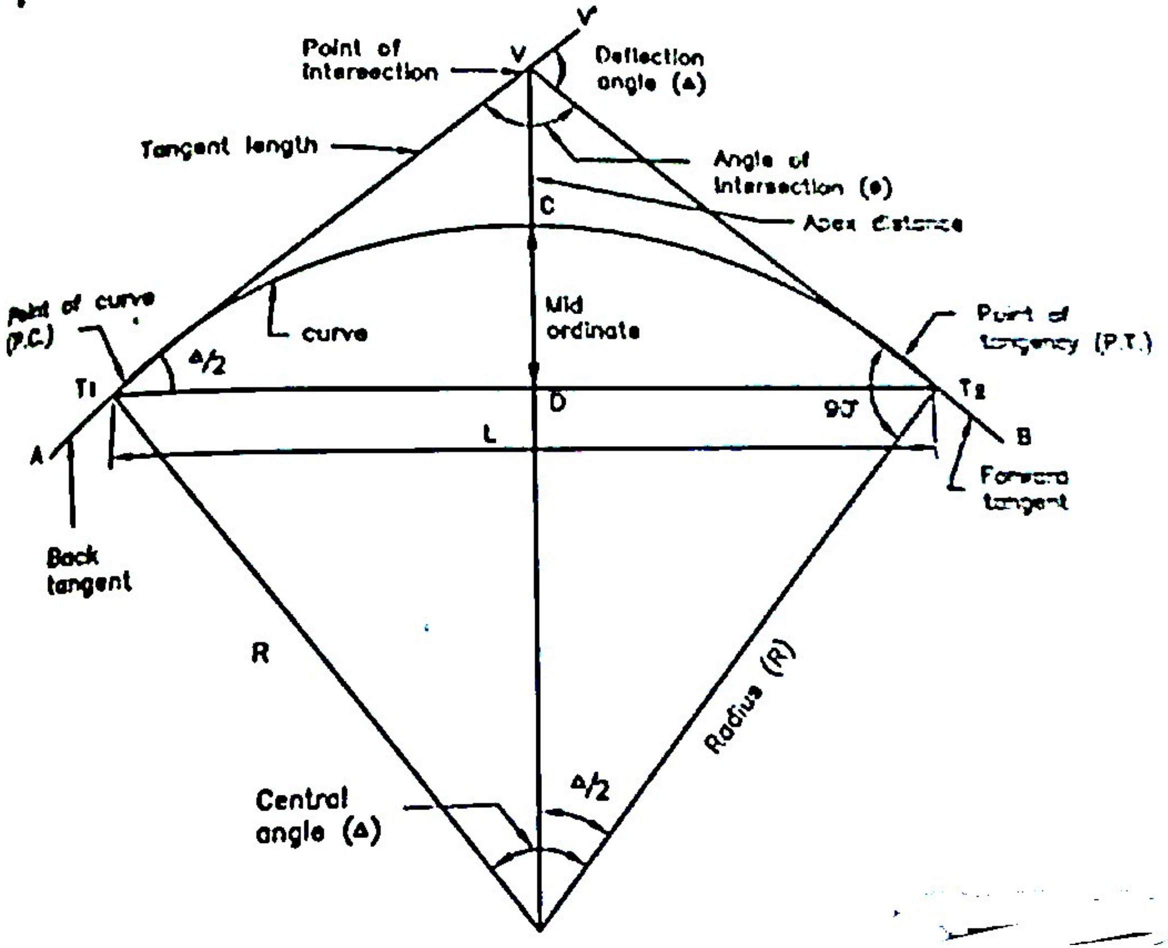


Mid Ordinate (M)

- In the fig, CD is the mid ordinate.
- It is also called versed sine.
- Mid ordinate = M
- M = CD = OC – OD
- From ΔT_1DO
- $\text{Cos}\,\frac{\Delta}{2} = \frac{OD}{OT_1} = \frac{OD}{R}$
- $OD = R\,\text{Cos}\,\frac{\Delta}{2}$
- M = OC – OD

$$= R - R\,\text{Cos}\,\frac{\Delta}{2}$$

$$= R\,(1 - \text{Cos}\,\frac{\Delta}{2})$$



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Setting Out of Simple Circular Curve

- First of all, tangent point should be located on the ground very accurately.

❖ Location of Tangent Point :-

✓ First of all surveyor study the working plan.

✓ Knowing offsets to certain points on both tangents and marked on ground.

✓ Both the tangent AV and BV, intersect at a point V, known as point of intersection.

✓ Set the theodolite at V and measure the angle AVB = $\emptyset$

✓ Deflection angle = $\Delta = 180 - \emptyset$

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- Calculate the tangent length $= T = R \tan \frac{\Delta}{2}$
- Now select point T_1 on line AV at a distance T from V.
- Similarly select point T_2 on line BV at a distance T from V.

Chainage of tangent Point :-

- The distance of any point from the beginning of the chain line is called chainage of that point.
- Point A is the starting point of the chain line. Chainage of point V, B, D are measure from the point A.
- Chainage of T1 = chainage of V – T (tangent Length)
- Chainage T2 = chainage of T1 + length of curve (l)
- $l = \frac{R \, \Delta \, \pi}{180}$

❖ Normal Chord and Sub Chord :-

- ✓ On the alignment of the curve, at a certain distance interval pegs are driven in to the ground.
- ✓ The distance between the two pegs is normally kept equal to 20 m.
- ✓ The distance is known as peg interval.
- ✓ If the peg are driven at 20m interval, the peg station are called main peg stations.
- ✓ The chord joining the tangent point T_1 and the first main peg station is called first sub chord.
- ✓ All the chord joining adjacent peg station are called full chord or normal chord.
- ✓ The length of normal chord is generally taken equal to 20m.
- ✓ The chord joining last main peg station and the tangent point T_2 is called last sub chord.

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Method of setting out of Simple circular curve

- Angular Method
- Linear method

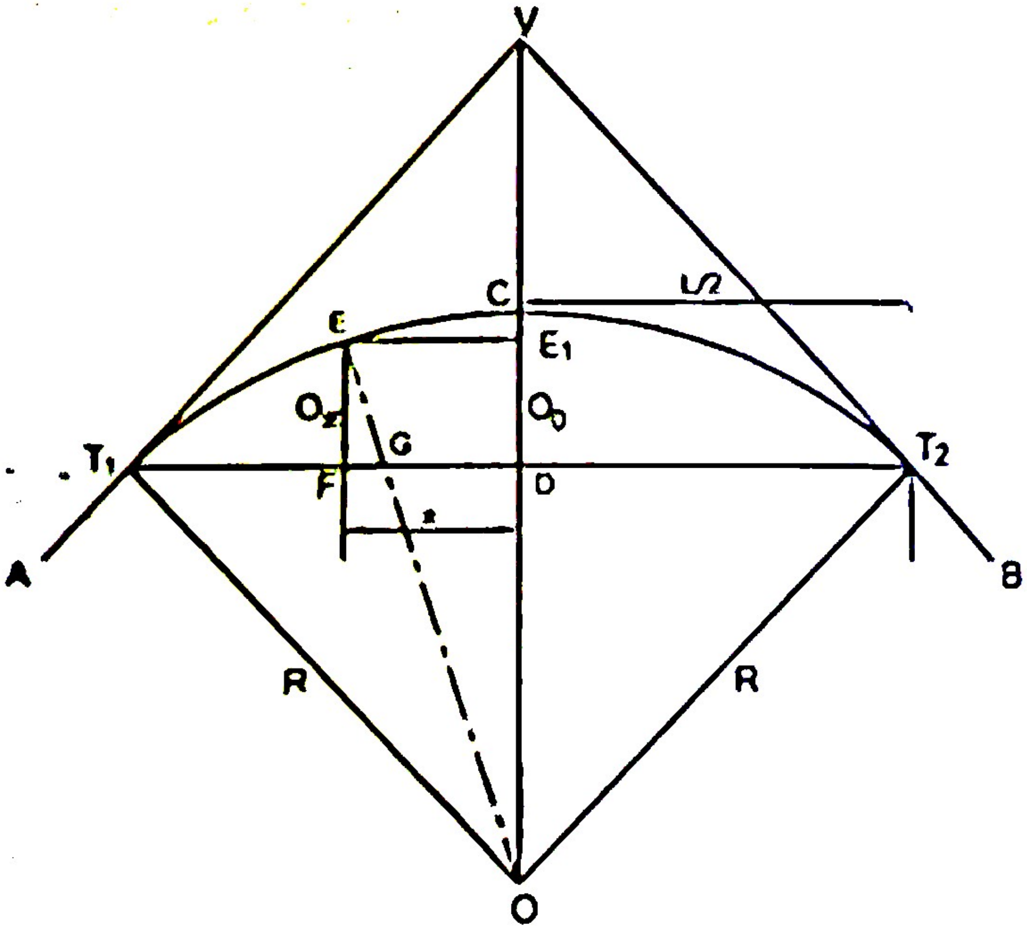
Linear Method

- Only chain or tap are required.
- Angle measurement instrument are not used.
- Method are used where high degree of accuracy is not required.
- Method is used where curve is very short.

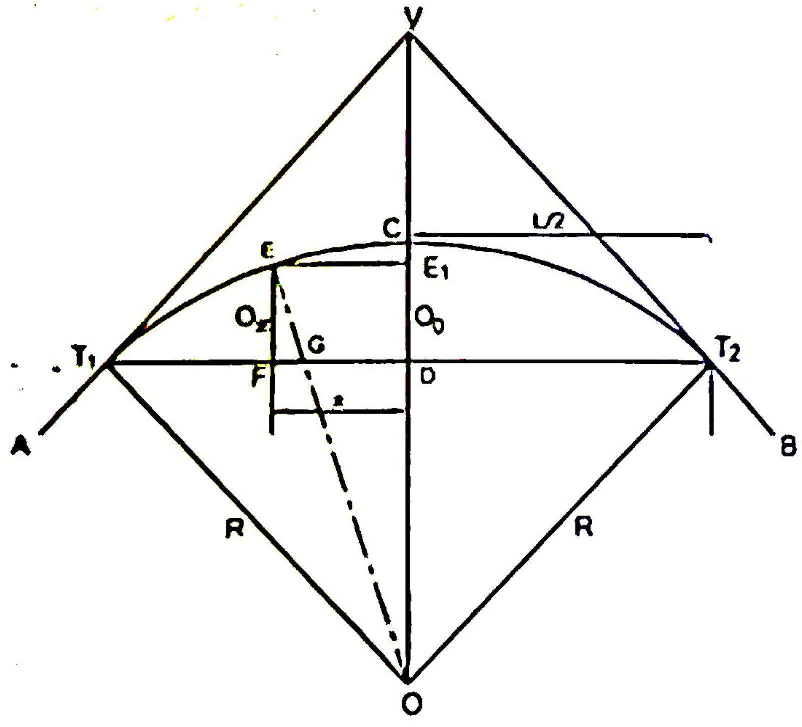
Linear methods are

i. By offset or ordinate from the long chord.
ii. By successive bisection of arcs or chords.
iii. By offsets from the tangents
iv. By offsets from chords produce

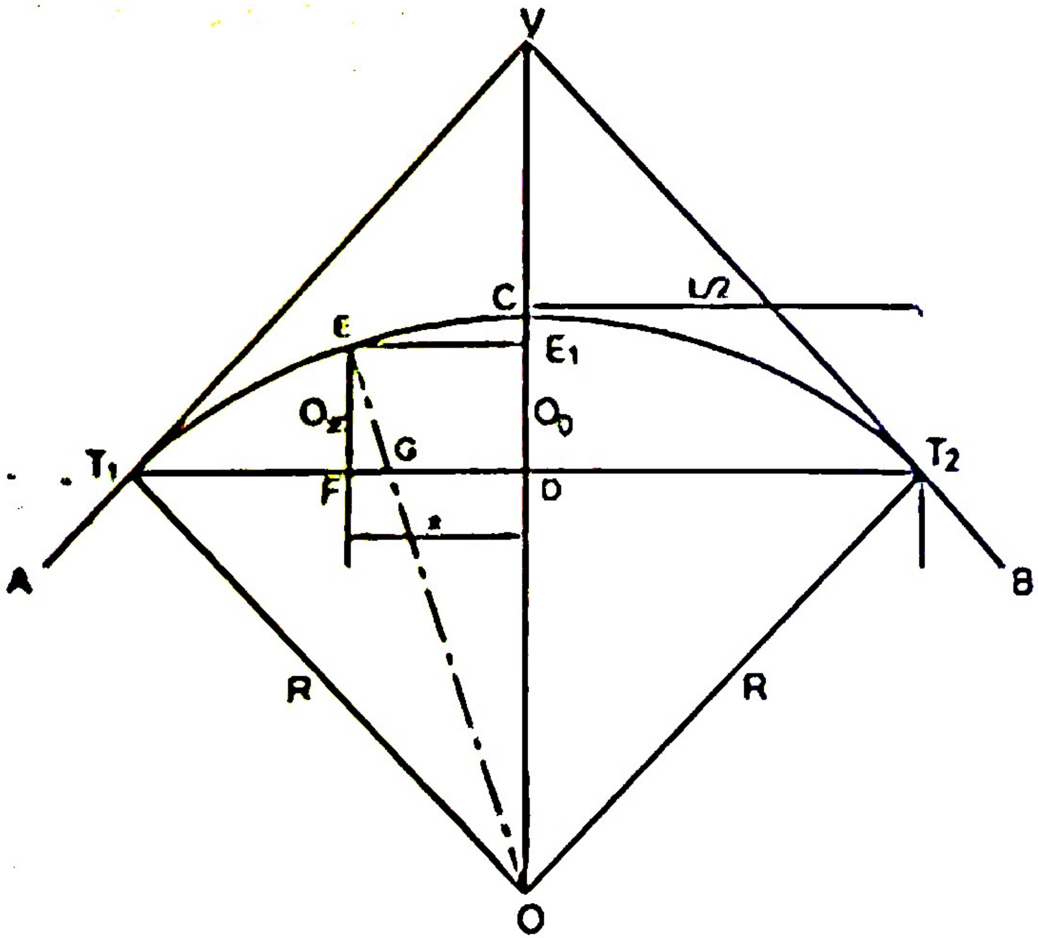
1. By offset or ordinate from the long chord.



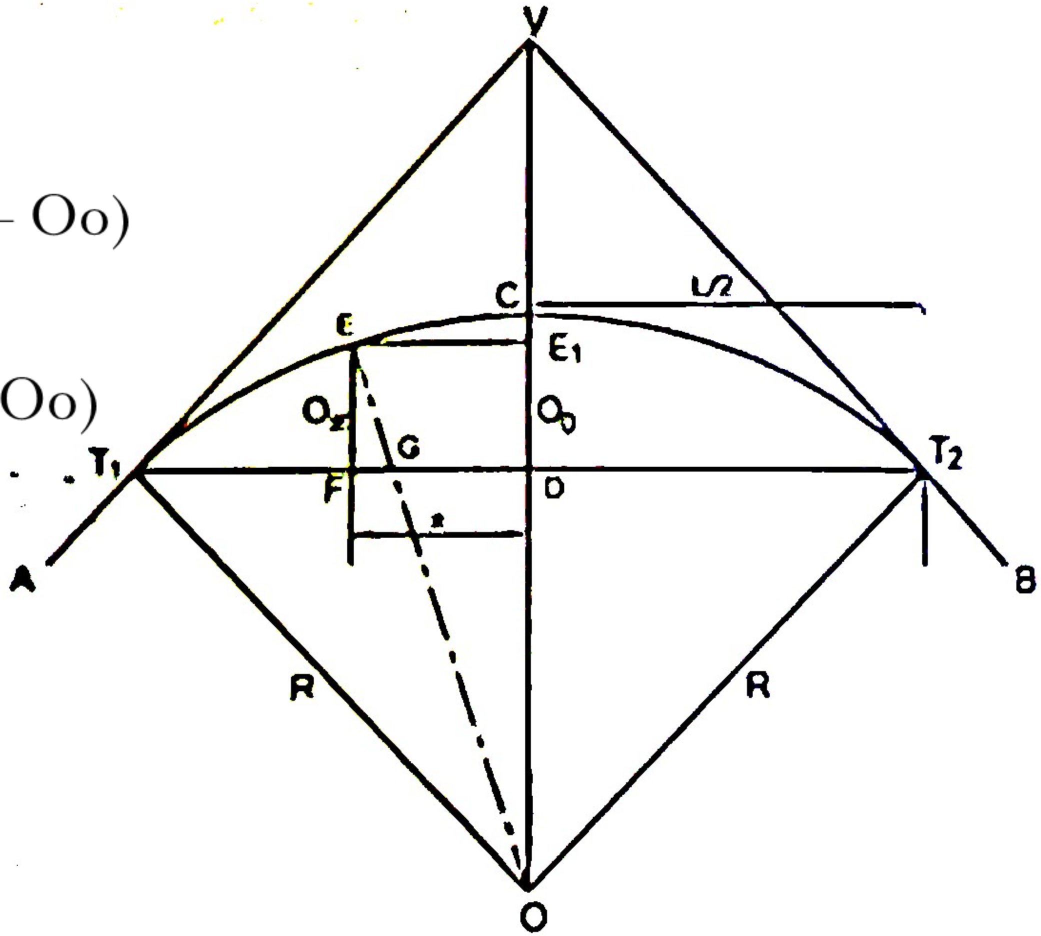
- R = Radius of curve
- O_o = Mid-Ordinate
- O_x = Ordinate at distance x from the mid point of the chord.
- T_1 and T_2 = Tangent Points
- L = Length of Long chord



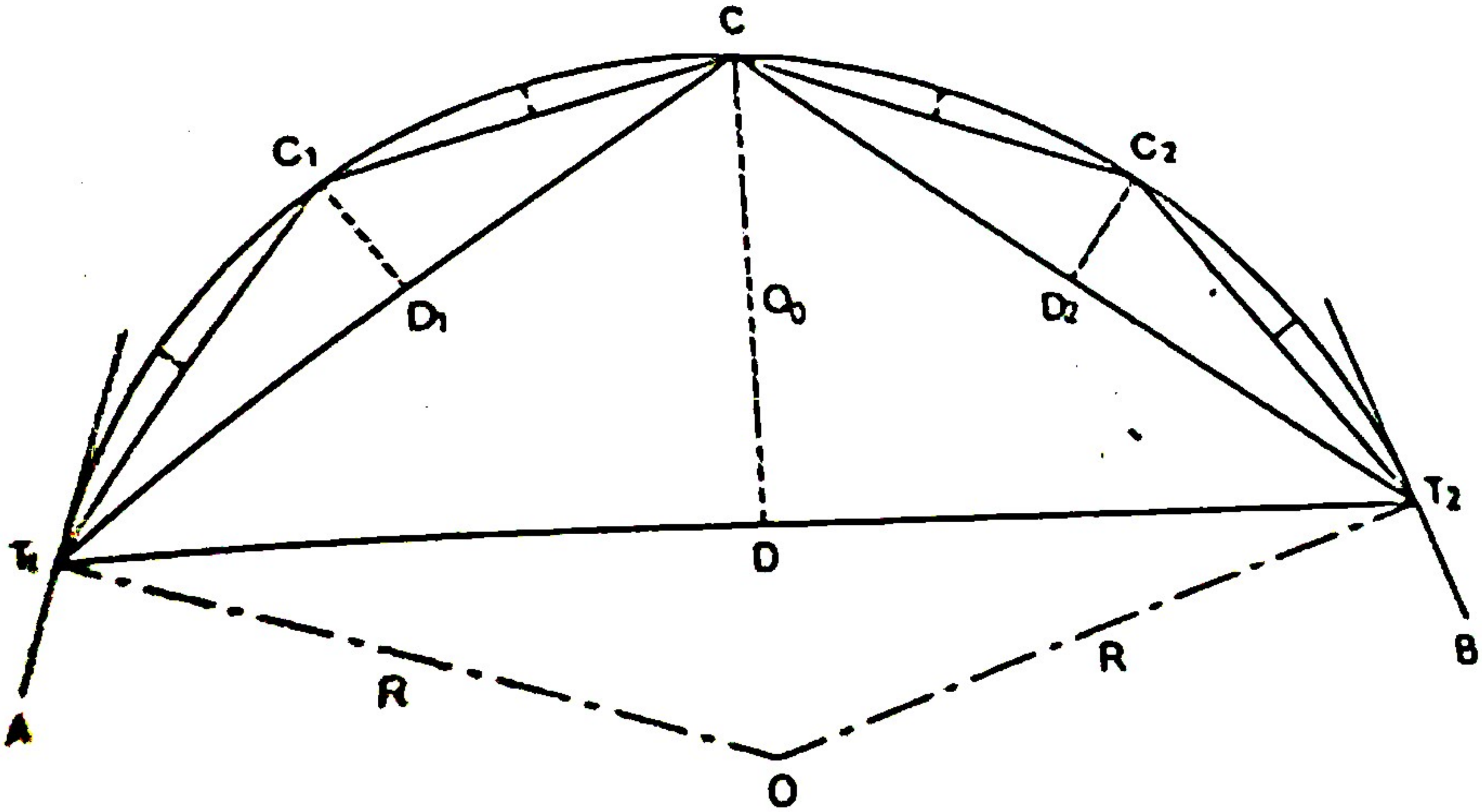
- To obtain equation for O_o :-
- From triangle OT_1D,
- $(OT_1)^2 = (DT_1)^2 + (OD)^2$
- $R^2 = \left(\frac{L}{2}\right)^2 + (OD)^2$
- $OD = \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$
- $O_o = R - OD$
- $O_o = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$



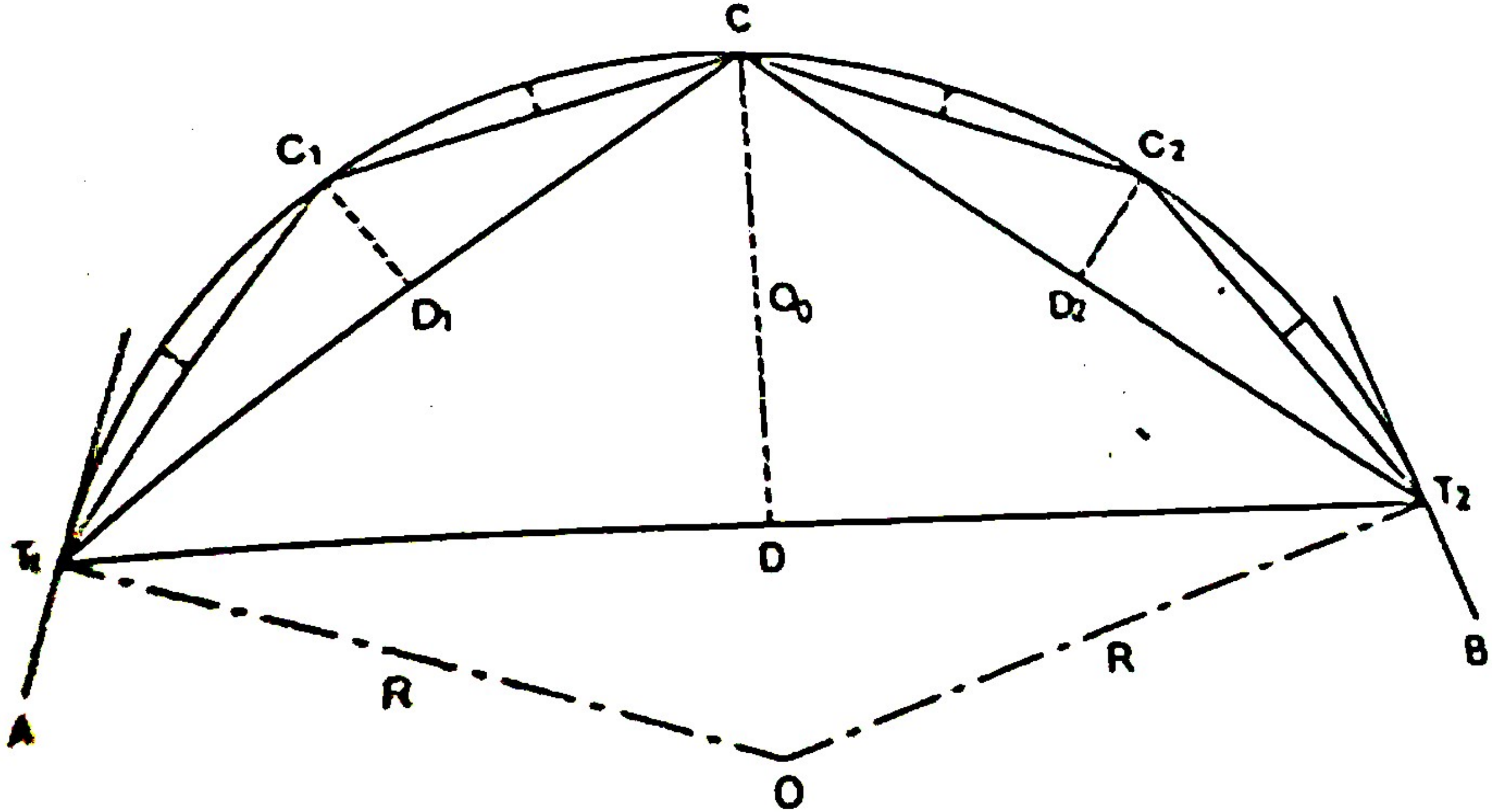
- In order to calculate ordinate Ox to any point E, draw the line EE_1, parallel to the long chord $T_1 T_2$. joint EO to cut long chord in G.
- $Ox = EF = E_1D$
- $\quad = E_1O - DO$
- $\quad = \sqrt{(EO)^2 - (EE_1)^2} - (R - Oo)$
- $Ox = \sqrt{(R)^2 - (X)^2} - (R - Oo)$



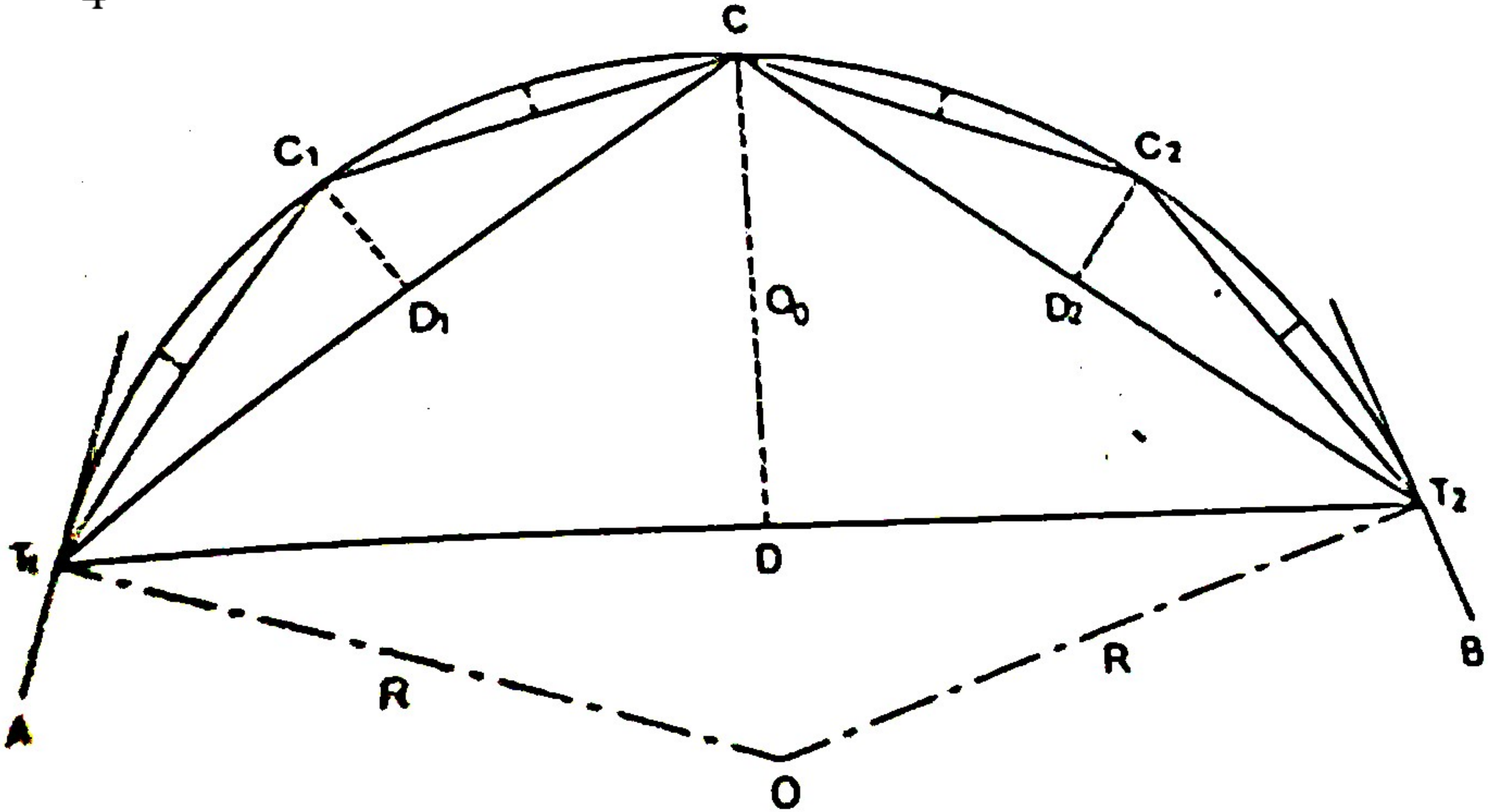
2. By successive bisection of arcs or chords.



- Joint point T_1 and T_2 and bisect long chord at D.
- Erect perpendicular DC at D equall to mid ordinate (M)
- Mid ordinate = M = CD = $R(1 - \cos\frac{\Delta}{2})$



- $O_o = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$
- Joint T_1C and T_2C and bisect them at D_1 and D_2 respectively.
- At D1 and D2 set out perpendicular offsets $C_1D_1 = C_2D_2 = (1 - \text{Cos}\frac{\Delta}{4})$ and obtain C_1 and C_2 on the curve.



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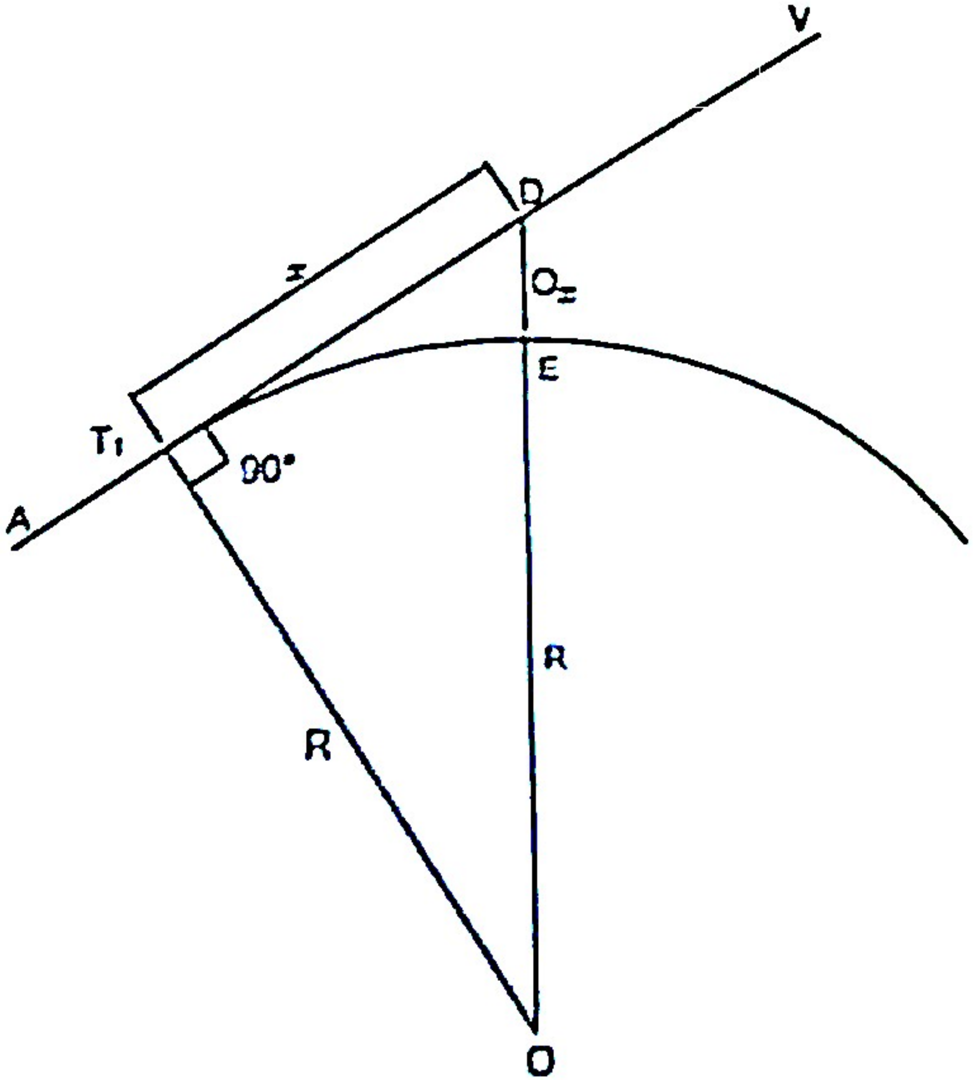
3. By offsets from the tangents

- The off set from the tangent point can be divided in to two types :-

1) Radial offset
2) Perpendicular offsets.

1. Radial offset

- Ox = Radial offset DE at any distance x from T_1 along the tangent.
- $T_1D = x$
- From ΔT_1DO
- $(OD)^2 = (OT_1)^2 + (DT_1)^2$
- $(R + Ox)^2 = (R)^2 + (X)^2$
- $(R + Ox) = \sqrt{(R)^2 + (X)^2}$
- $Ox = \sqrt{(R)^2 + (X)^2} - R$



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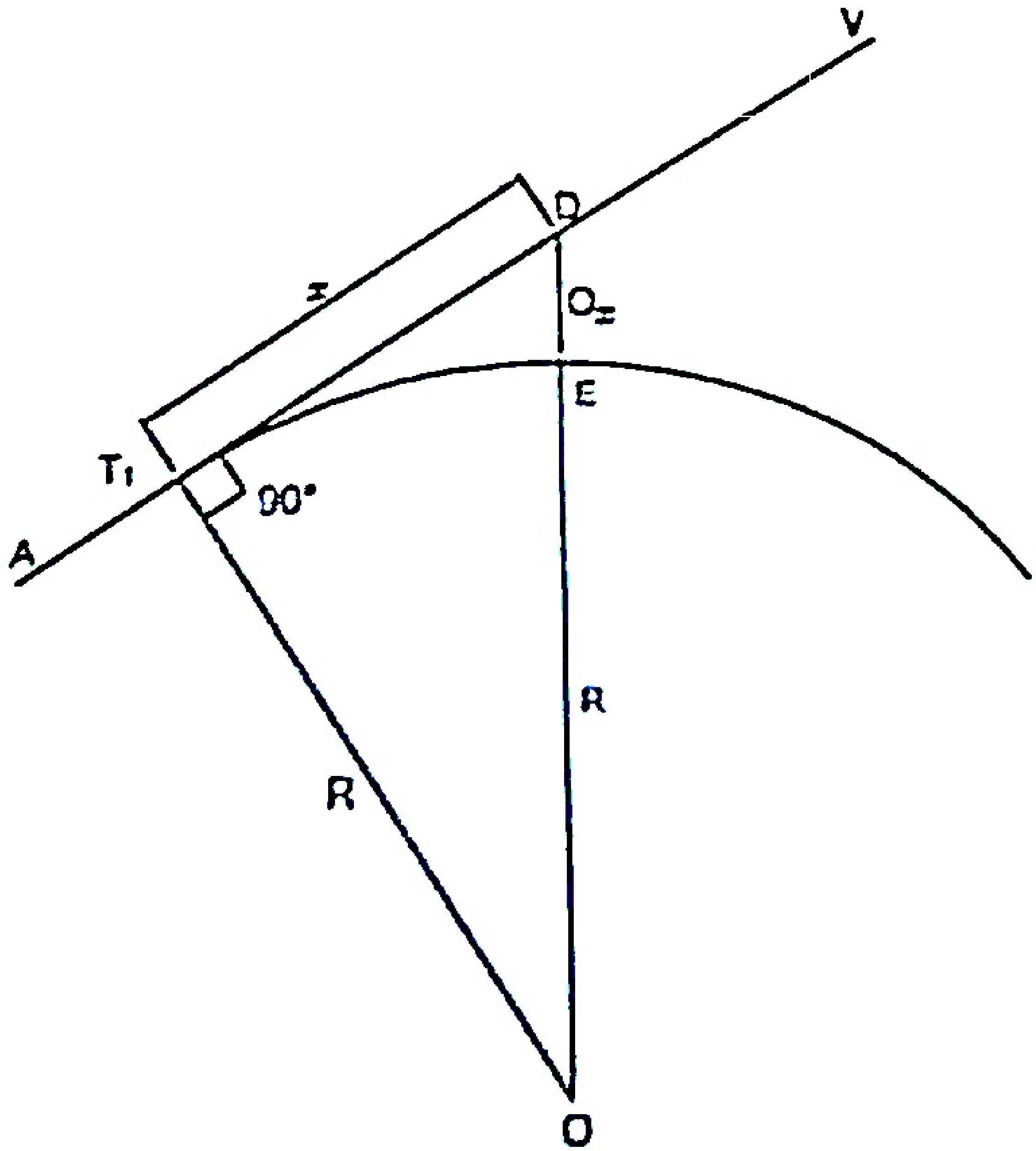
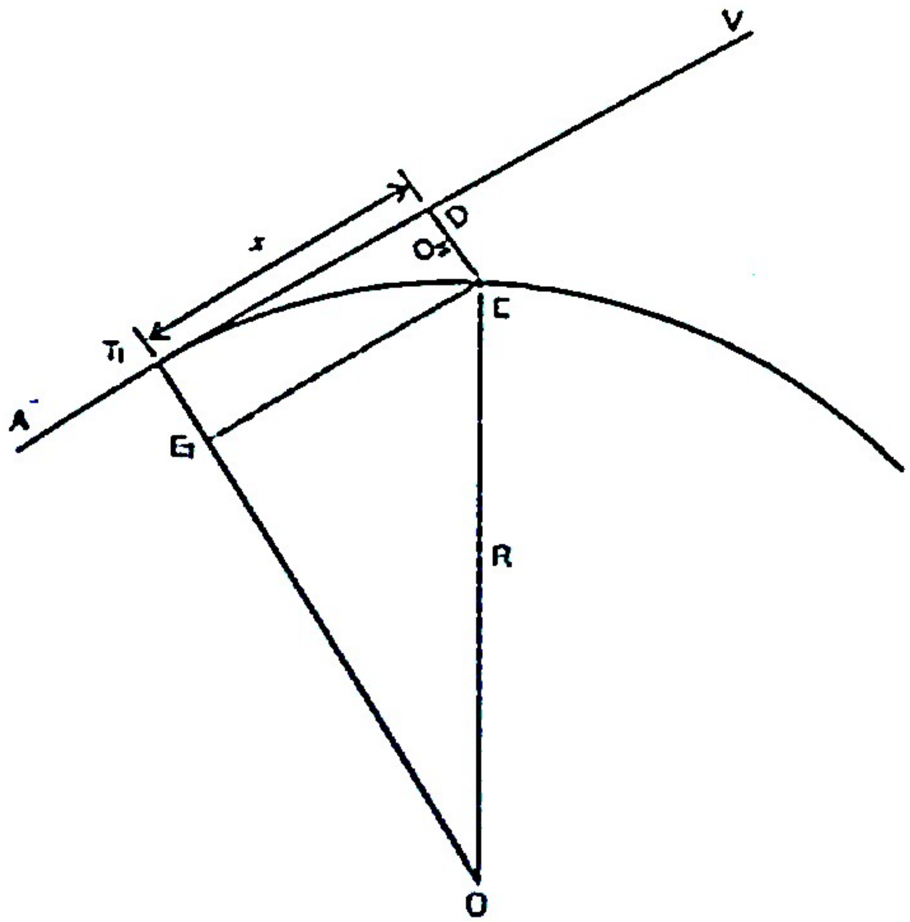


V
D
x
O_x
E
T_1
90°
A
R
R
O

2. Perpendicular offset

- Ox = Offset perpendicular to the tangent
- DE = Ox
- T_1D = x, measured along tangen
- From ΔEE_1O, we have
- $(OE)^2 = (OE_1)^2 + (EE_1)^2$
- $(OE_1)^2 = (OE)^2 - (EE_1)^2$
- $(OT_1 - T_1E_1)^2 = (OE)^2 - (EE_1)^2$
- $(R - Ox)^2 = R^2 - x^2$
- $R - Ox = \sqrt{R^2 - x^2}$
- $Ox = R - \sqrt{R^2 - x^2}$



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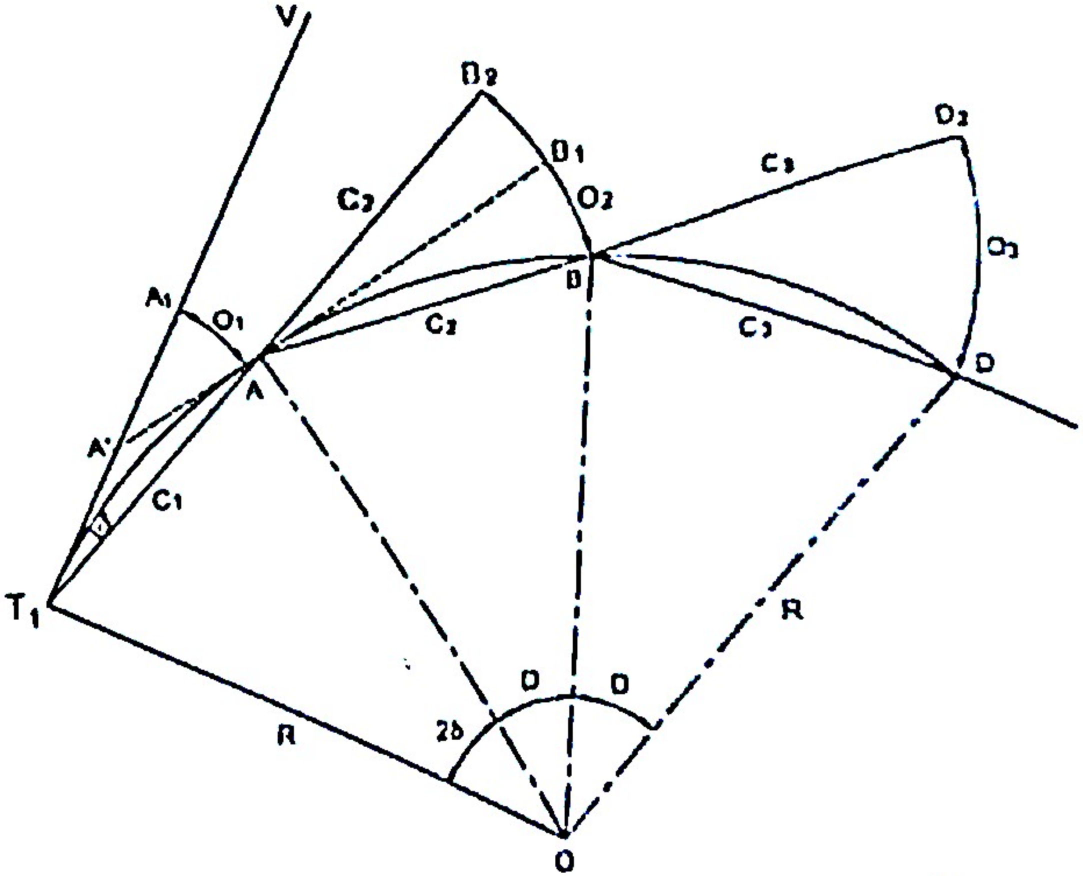
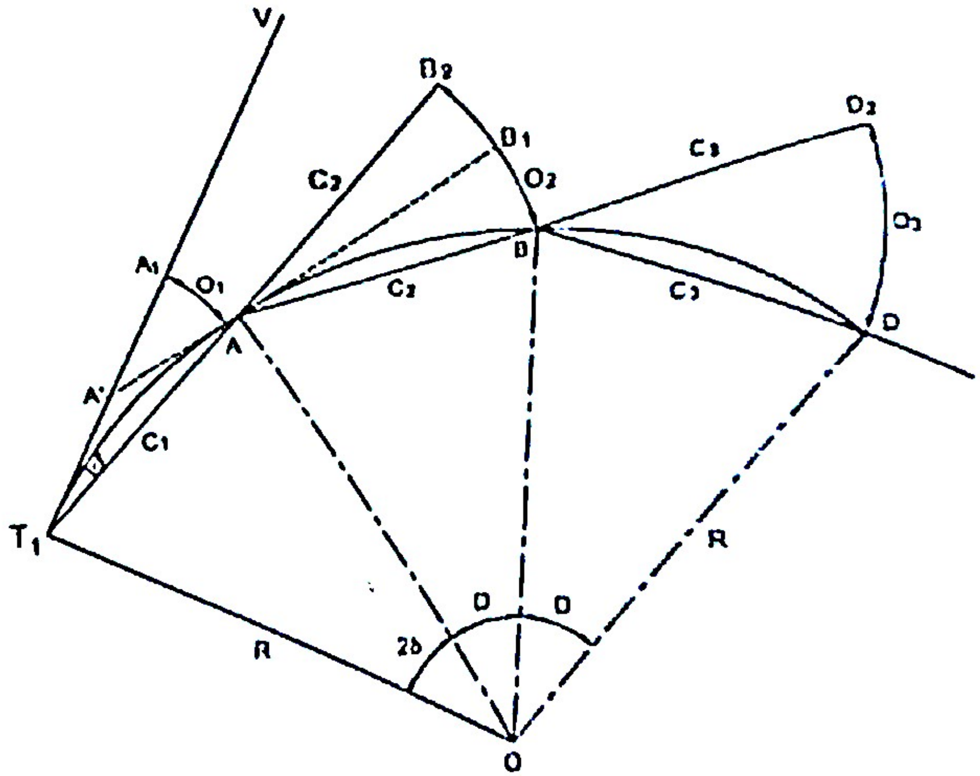
4. By offset from chord produce

- This method is used for long curve.
- This method is used when the theodolite is not available.



- $T_1A_1 = T_1A$ = initial sub-chord $= C_1$
- $AB = C_2$ = Normal chord
- $BD = C_3$ = Normal chord
- T_1V = back tangent
- $\angle A_1T_1A = \boldsymbol{\delta}$ = deflection angle of the first chord
- $A_1A = O_1$ = first offset
- $B_2B = O_2$ = second offset
- $D_2D = O_3$ = third offset
- Now arc $A_1A = O_1 = T_1\,A\,\boldsymbol{\delta}$ ……………………………….(1)
- T_1V is the tangent to the circle at T_1
- $\angle T_1OA = 2\,\angle A_1T_1A = 2\boldsymbol{\delta}$
- $T_1A = R\,2\boldsymbol{\delta}$
- $\boldsymbol{\delta} = \frac{T_1A}{2R}$ ………………(2)



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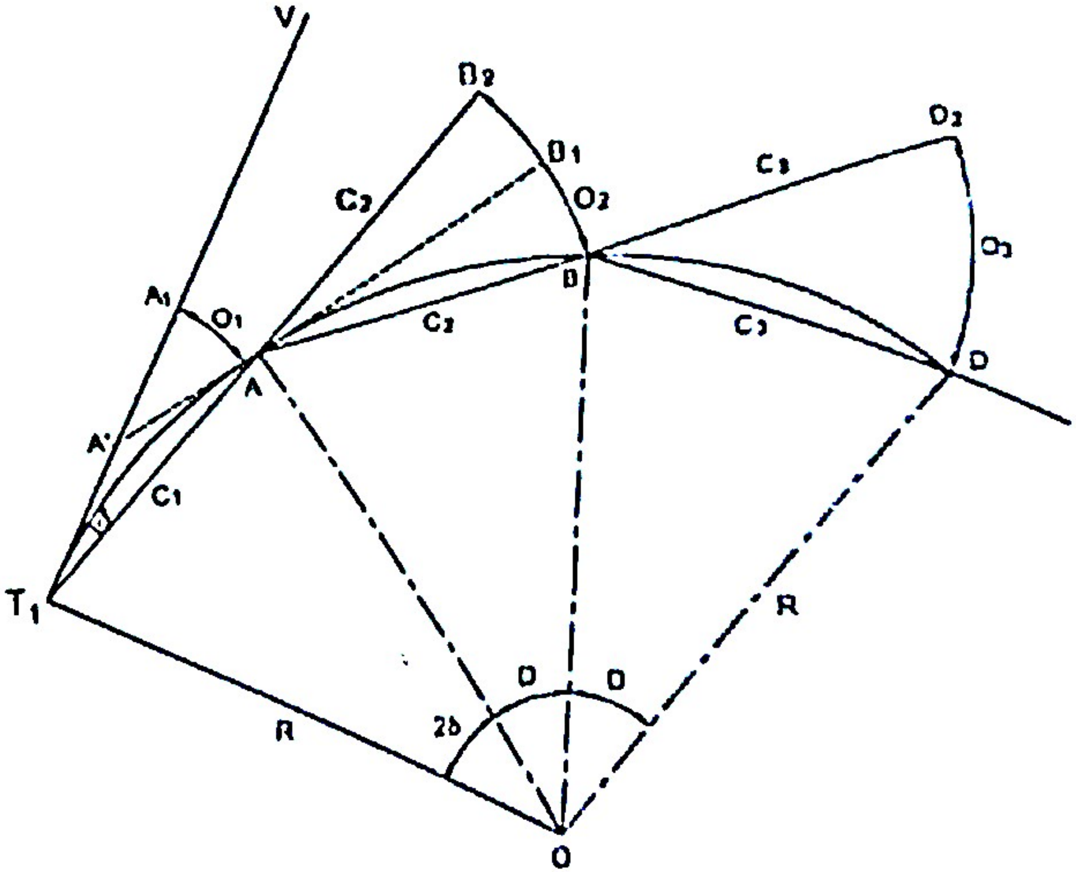
- Substituting the value of $\boldsymbol{\delta}$ in eq. (1), we get
- $O_1 = T_1\,A\,\boldsymbol{\delta}$

$$= T_1\,A\left(\frac{T_1 A}{2R}\right)$$

$$= \frac{(T_1 A)^2}{2R}$$

$$O_1 = \frac{{C_1}^2}{2R}$$



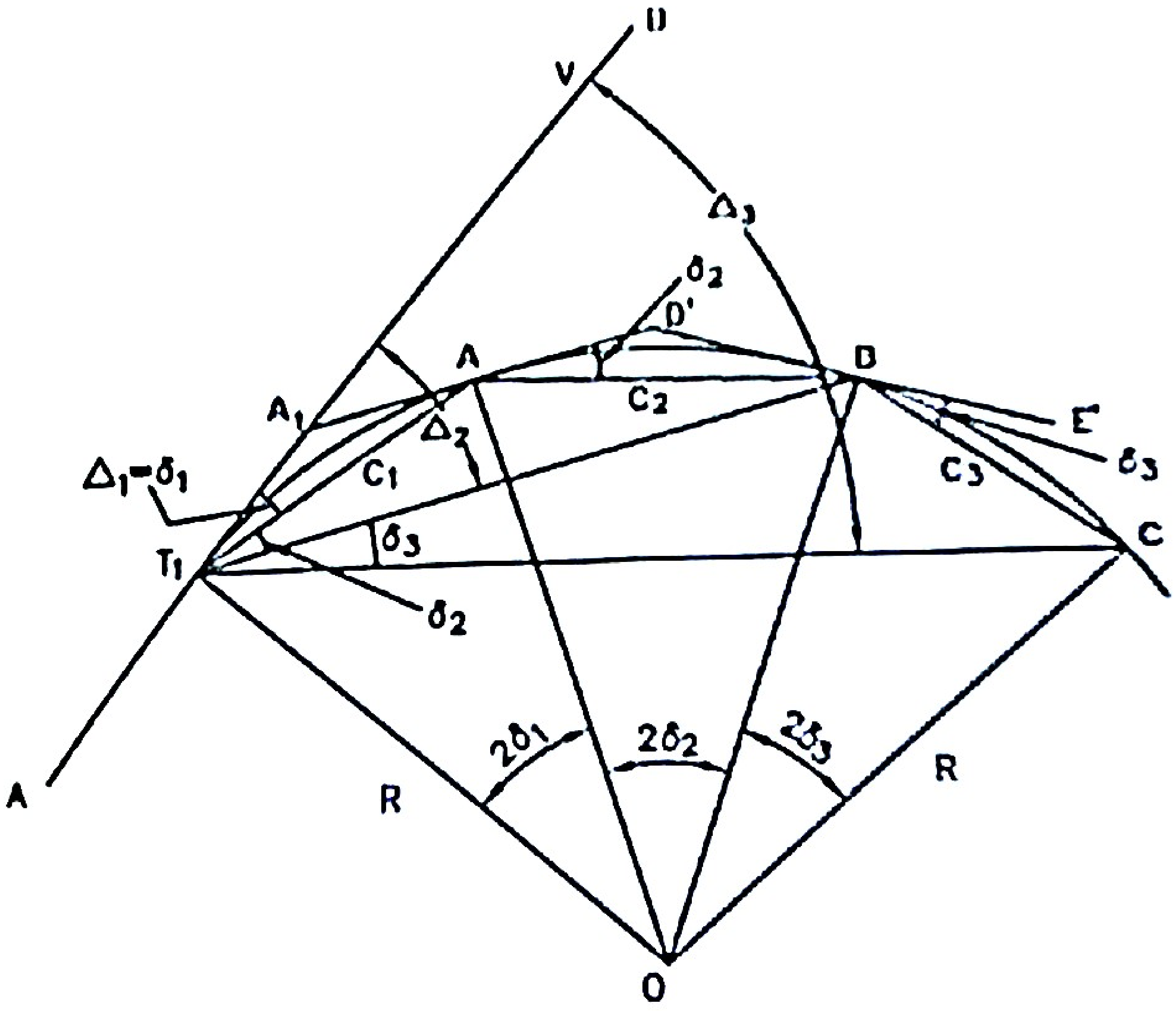
Angular method

- Theodolite are used.
- Some time chain and tap is also used.
- This method is used when the length of the curve is very large.
- More accurate method.
- There are main three method.

1) Rankine method of tangential angles.
2) Two theodolite method.
3) Tachometric method.

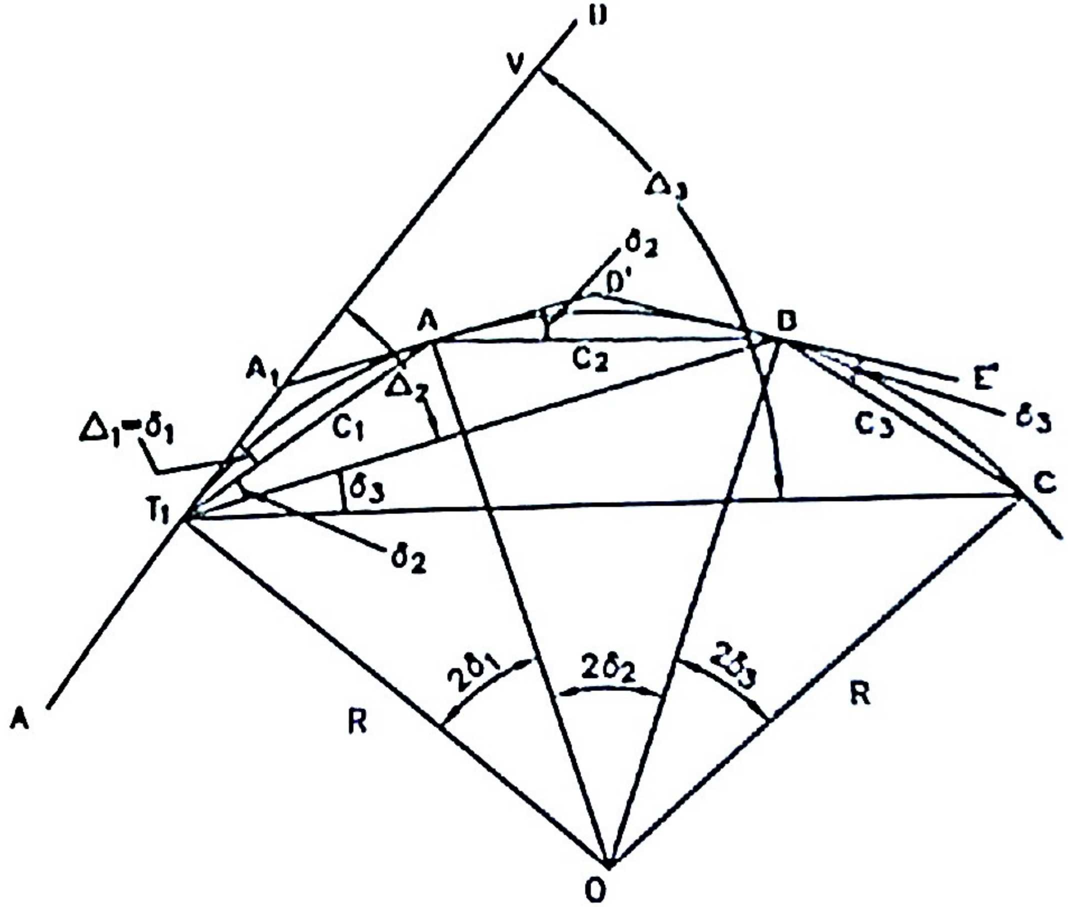
1. Rankine method of tangential angles

- Also called one theodolite method.
- Most frequently used.
- Useful for setting out of work like railway, Highway, express way with more accuracy.



V
D
Δ_3
δ_2
D'
A
B
A_1
C_2
E'
Δ_2
$\Delta_1 = \delta_1$
C_1
C_3
δ_3
C
T_1
δ_3
δ_2
$2\delta_1$
$2\delta_2$
$2\delta_3$
A
R
R
O

- C_1, C_2, C_3 = length of chords T1A, AB, BC
- $\boldsymbol{\delta}_1$, $\boldsymbol{\delta}_2$, $\boldsymbol{\delta}_3$ = Angle which each of the successive chords T_1A, AB, BC makes with the respective tangents to the curve at T_1, A, B
- Δ_1, Δ_2, Δ_3 = deflection angle



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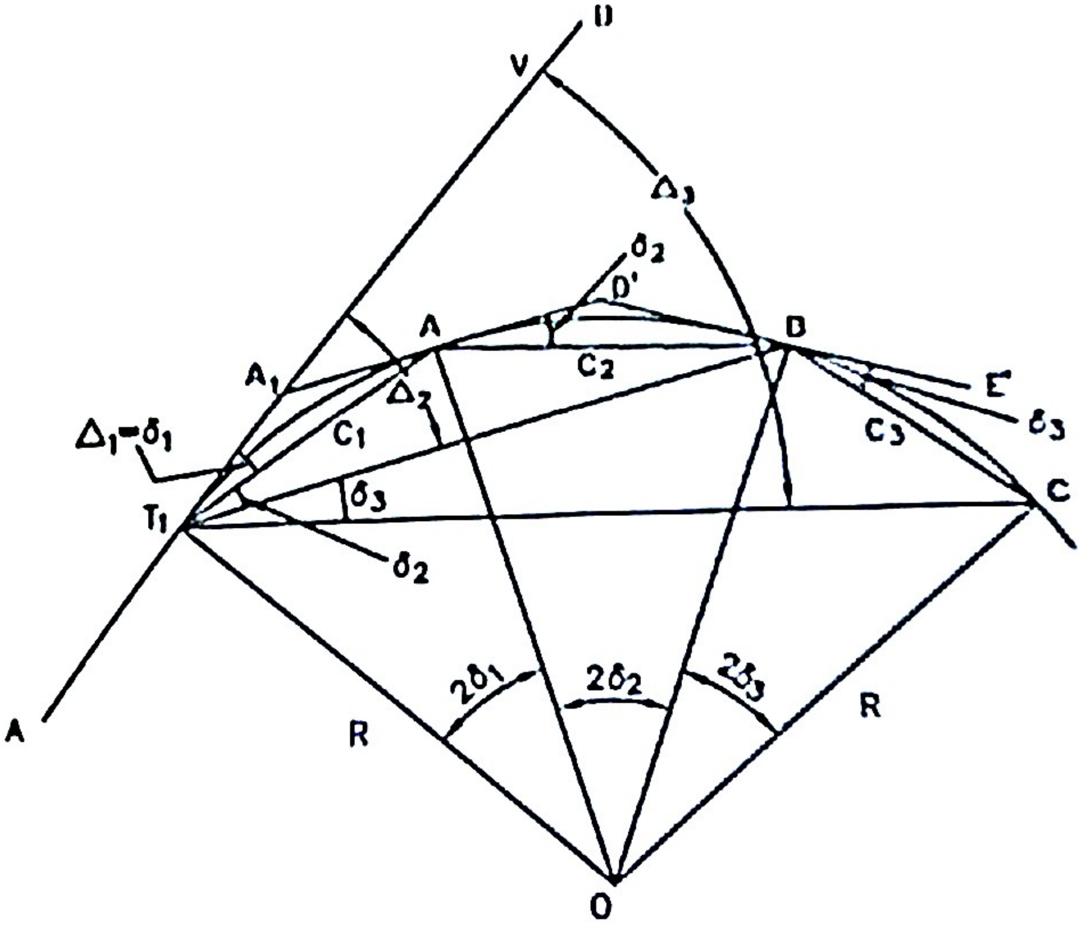
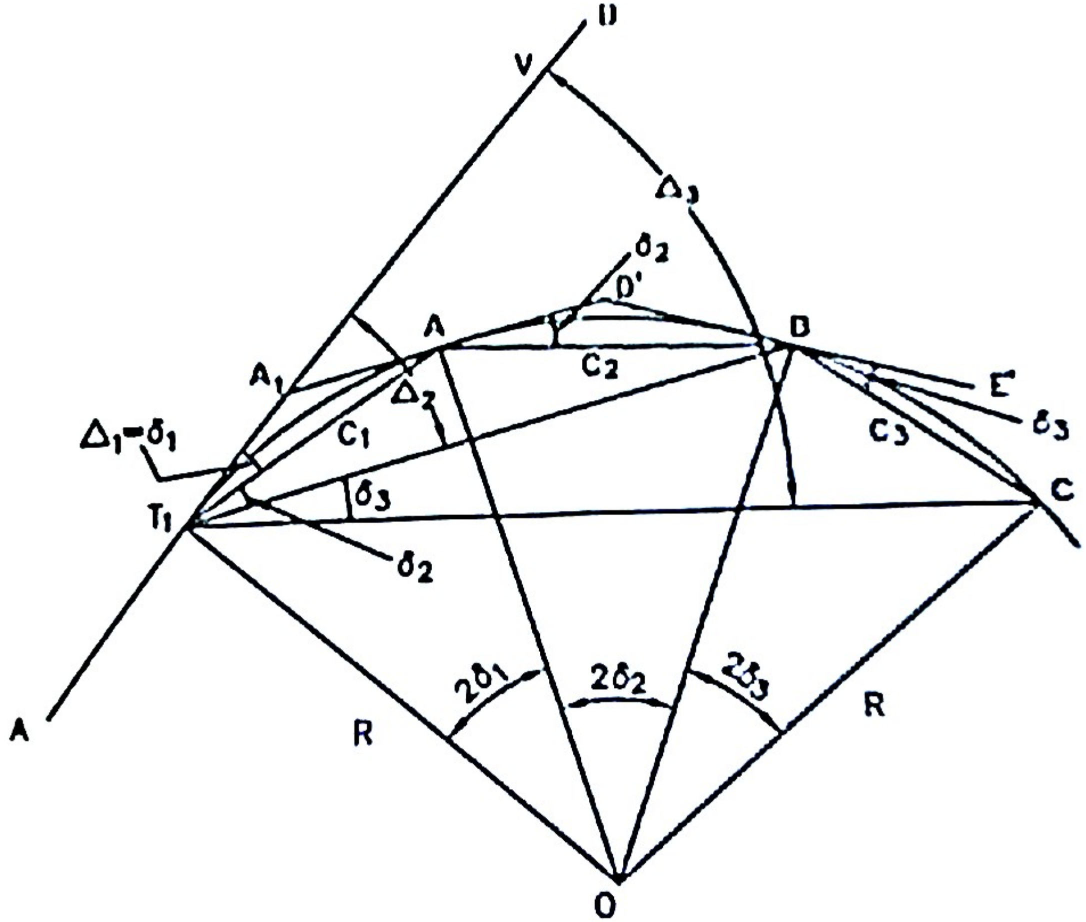
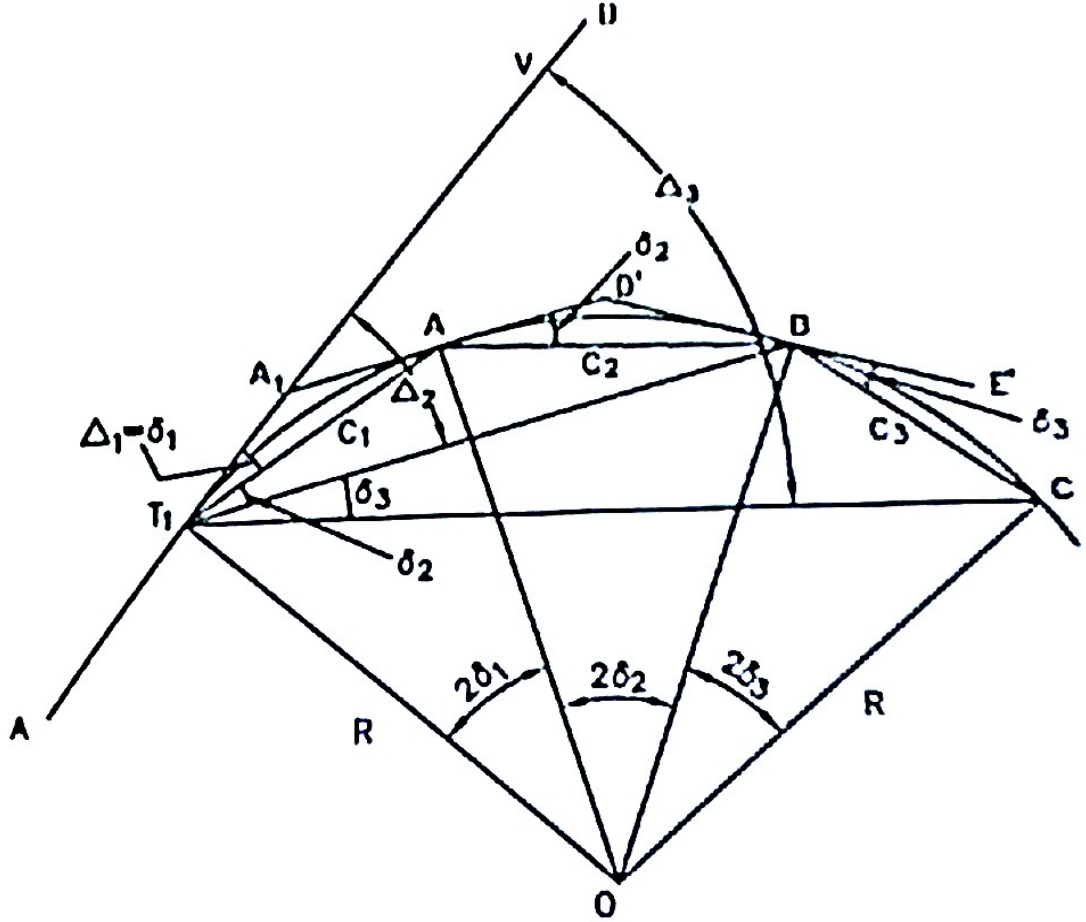
- From the property of the circle
- $\angle VT_1A = \frac{1}{2} \angle T_1OA$
- $\angle T_1OA = 2 \angle VT_1A = 2 \boldsymbol{\delta}_1$
- Now;
- $\frac{\angle T_1OA}{C_1} = \frac{180^0}{\Pi R} = 2 \boldsymbol{\delta}_1$
- $\boldsymbol{\delta}_1 = \frac{C_1 \, 90^0}{\Pi R}$ (Deg)
- $= \frac{C_1 \, 90 \text{ x } 60}{\Pi R}$
- $= 1778.9 \frac{C_1}{R}$ (minute)



- Similarly;
- $\boldsymbol{\delta}_2 = 1778.9\,\frac{C_2}{R}$
- $\boldsymbol{\delta}_3 = 1778.9\,\frac{C_3}{R}$
- $\boldsymbol{\delta} = 1778.9\,\frac{C}{R}$



- For the first chord T_1A, deflection angle = its tangential angle.
- $\Delta_1 = \boldsymbol{\delta}_1$
- Let the deflection angle for point B is Δ_2
- $\boldsymbol{\delta}_2 =$ tangential angle for chord AB.
- $\angle AOB = 2\,\boldsymbol{\delta}_2$



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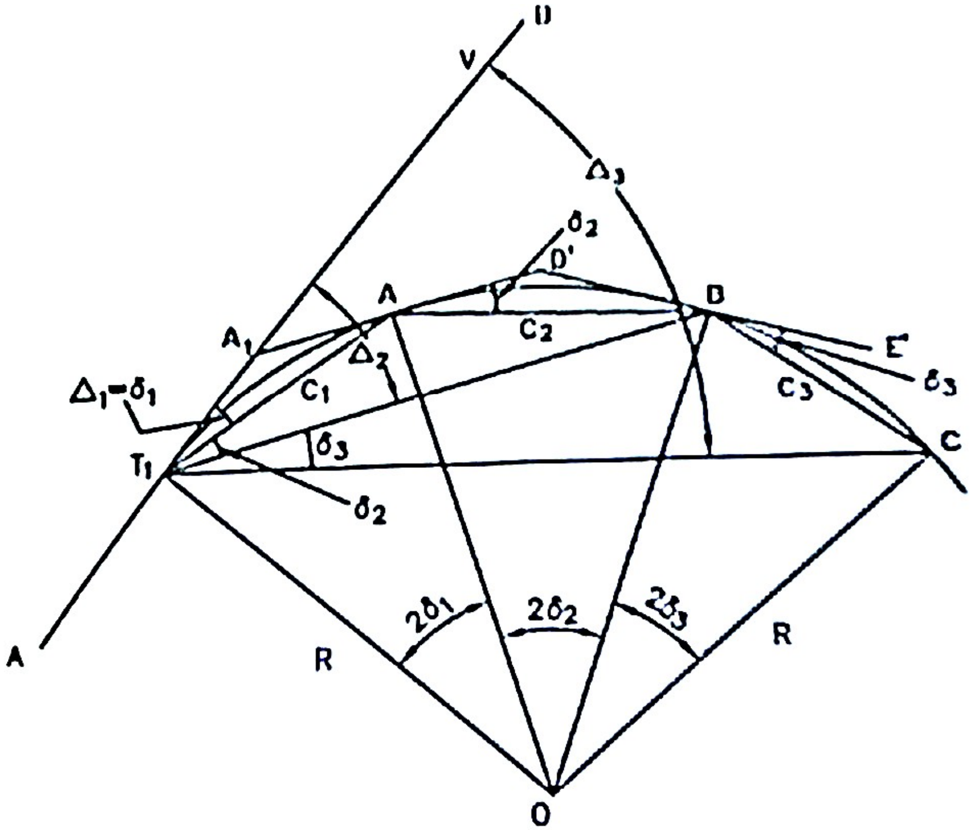
- $\angle AT_1B$ = half of the angle subtended by AB at the centre = $\boldsymbol{\delta}_2$
- $AT_1B = \boldsymbol{\delta}_2$
- now,
- $\Delta_2 = \angle VT_1B$
- $= \angle A_1T_1A + \angle AT_1B$
- $\Delta_2 = \boldsymbol{\delta}_1 + \boldsymbol{\delta}_2$
- $\Delta_2 = \Delta_1 + \boldsymbol{\delta}_2$
- Similarly,
- $\Delta_3 = \Delta_2 + \boldsymbol{\delta}_3$
- $\Delta_4 = \Delta_3 + \boldsymbol{\delta}_4$



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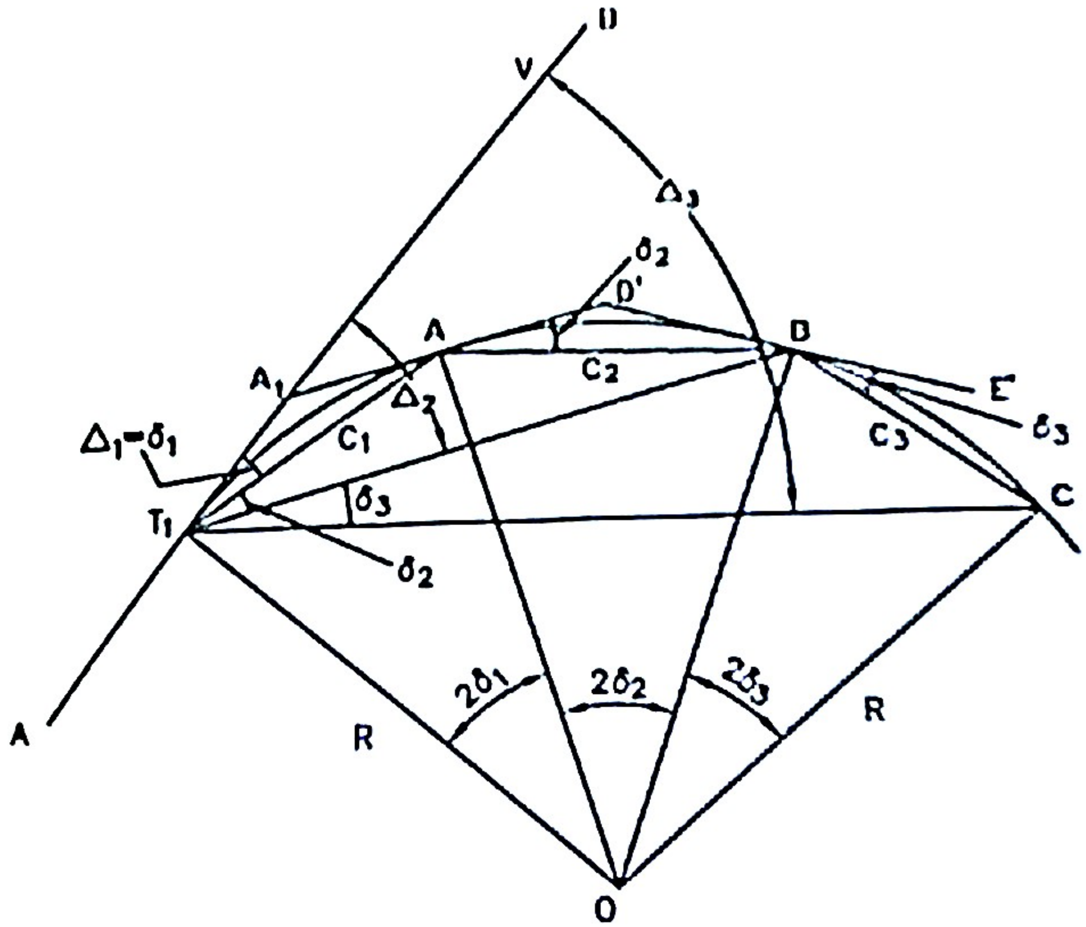
- Set out the tangent point T_1 and T_2 on ground.
- Set out the theodolite on the point of the curve T_1.
- With both the plates clamped to the zero, direct the theodolite to bisect the intersection (V).



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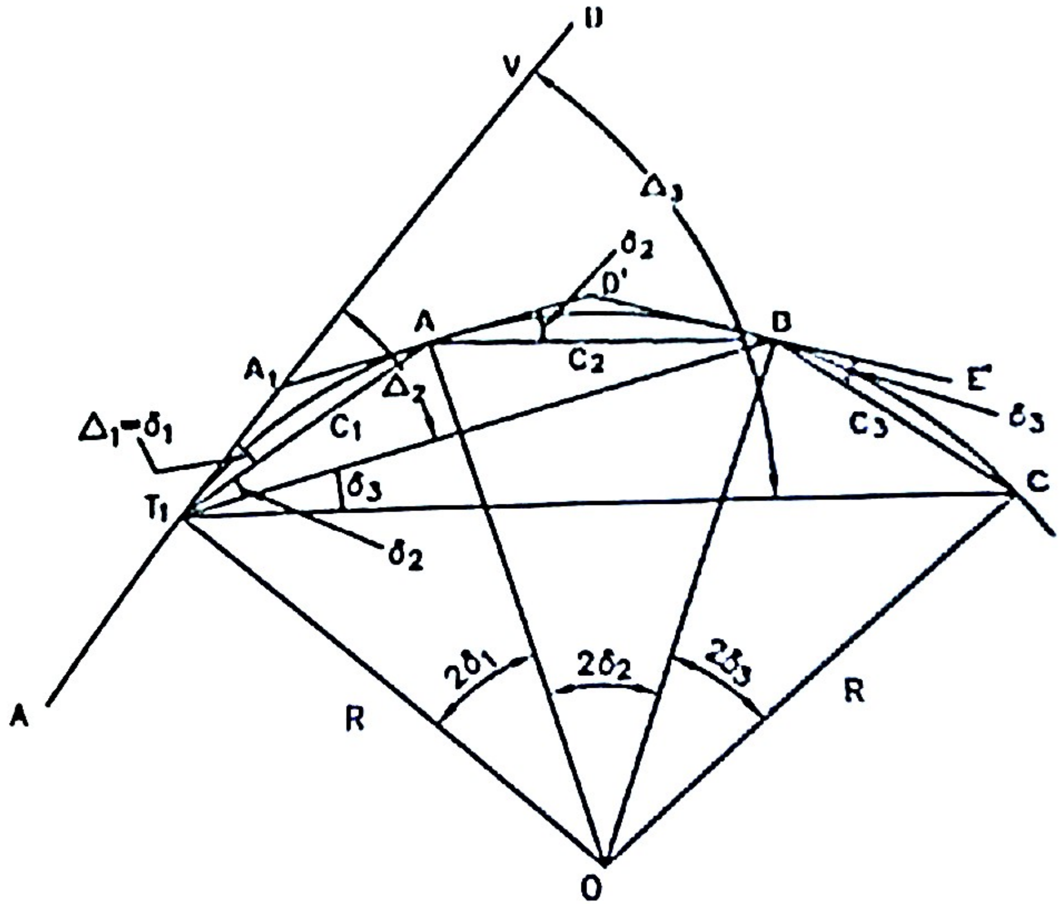
- Release the upper clamp and screw and set the angle Δ_1 on the vernier.
- The line of sight direct the chord T_1A.
- With the zero end of the tap pointed at T_1 and an arrow held at a distance $T_1A = C_1$, thus the point is fixed on ground.



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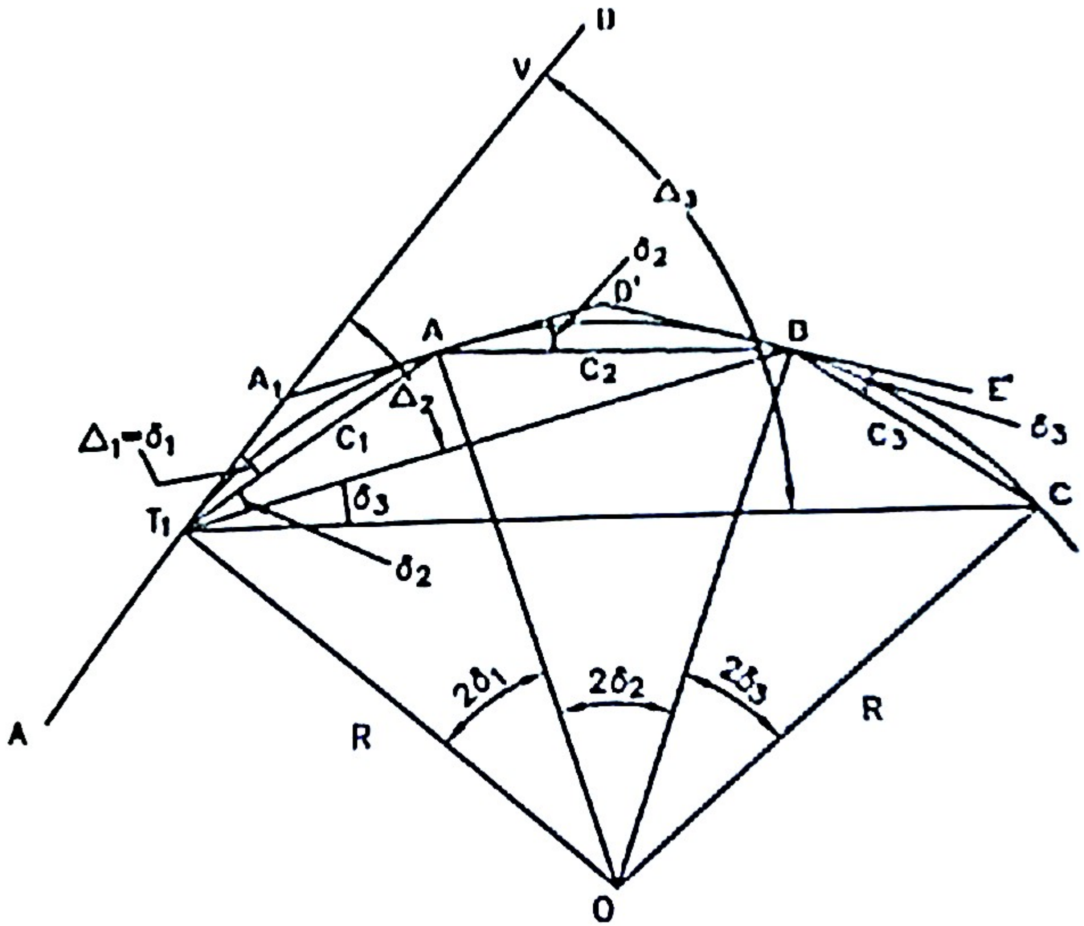
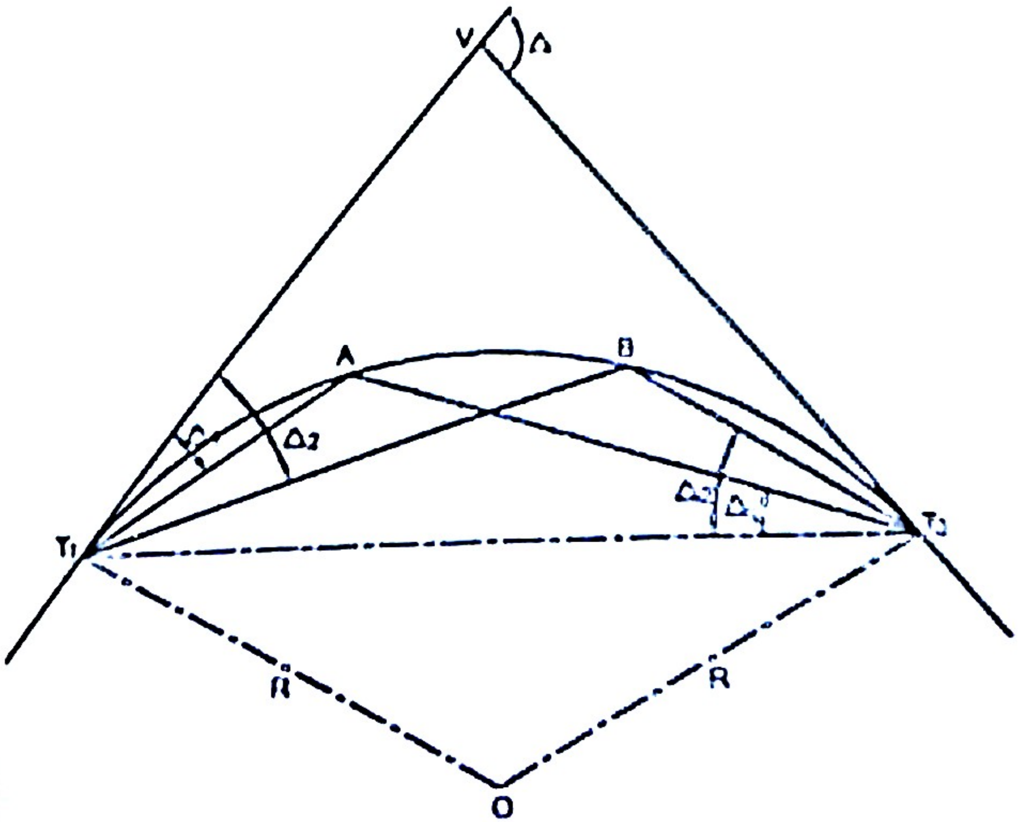
- Now release the upper plate and set the second deflection angle Δ_2 on the vernier so the line of sight is direct along T_1B.
- With the zero end of the tap is pointed at A, and an arrow held at a distance AB = C_2, thus fix the second point.
- Repeat the procedure until the last point T_2.
- Joint T_1, A, B, ……………., T_2 to obtain the required curve.



2. Two theodolite method

- Two theodolite is used.
- In this method chain or tap is not required.
- $\angle VT_1A = \Delta_1$ = Deflection angle for A.
- But $\angle AT_2T_1$ is the angle subtended by long chord T_1A in the opposite segment.
- $\angle AT_2T_1 = \angle VT_1A = \Delta_1$
- Similarly
- $\angle T_1T_2B = \angle VT_1B = \Delta_2$



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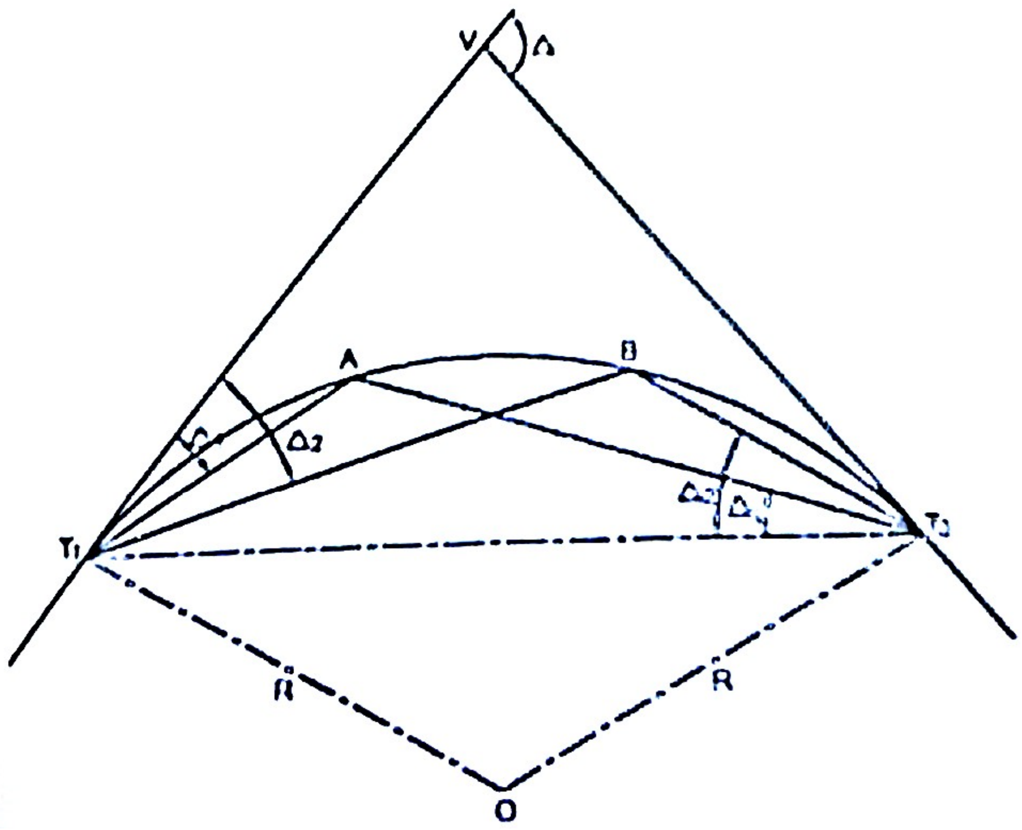
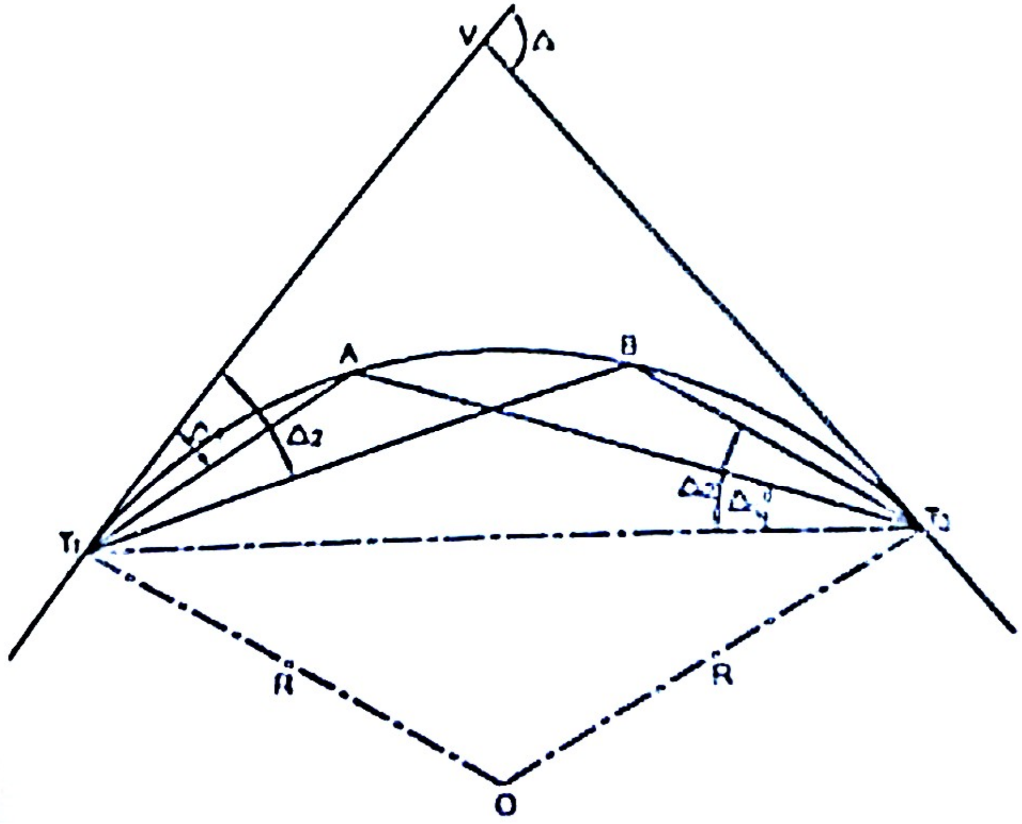
Procedure :-

- Set up the theodolite at T_1 and T_2 point.
- Clamp both the plate transit zero degree.
- With zero reading, direct the line of sight of the transit at T_1 towards V.



- Similarly direct the line of sight of the other transit at T_2 toward T_1.
- Verier A of the both the theodolite will shows the zero reading.
- Transit the deflection angle for the first point A equal to Δ_1.



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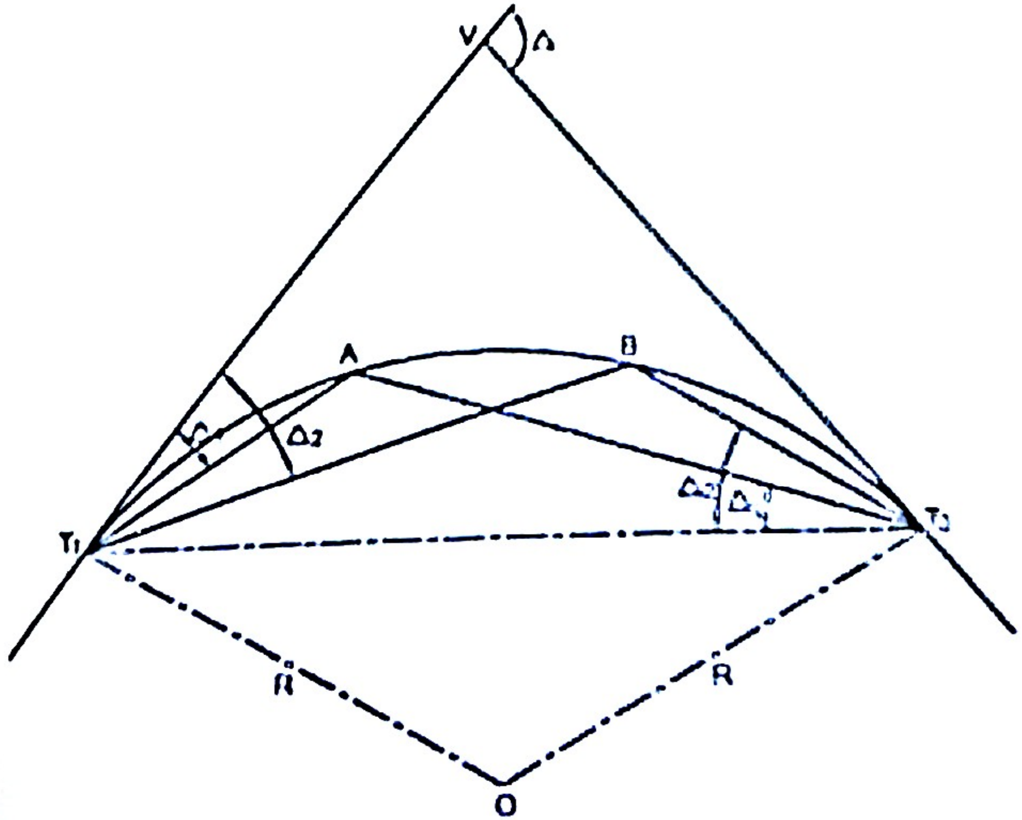
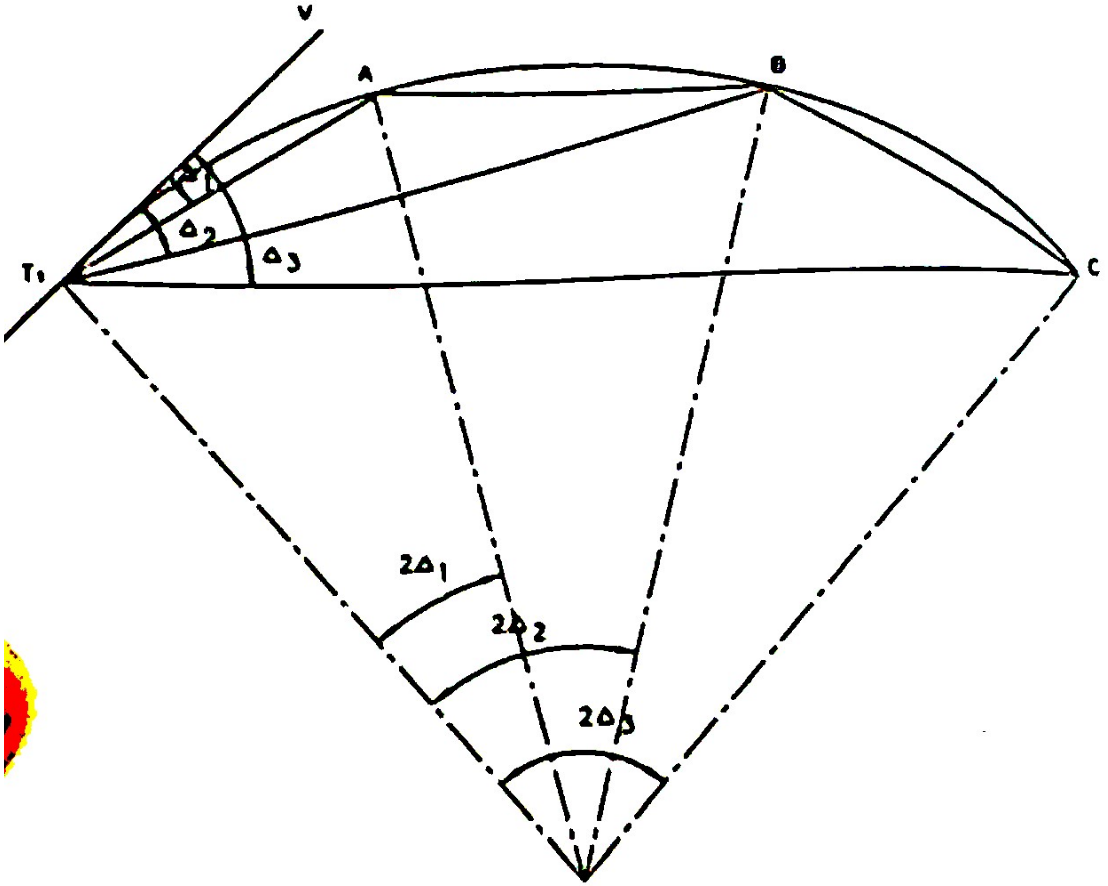
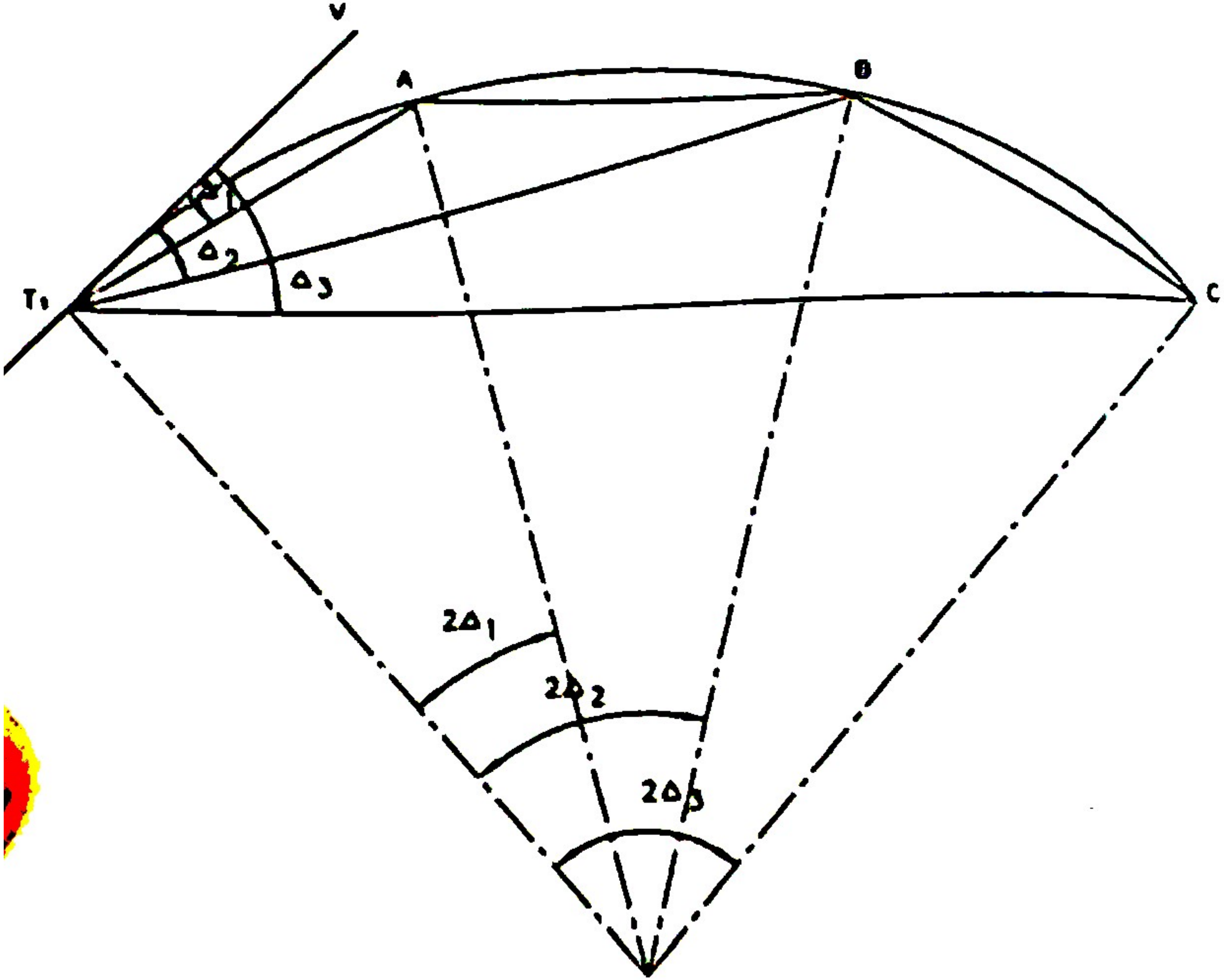
- Second theodolite set the same angle and turn the telescope to A at angle Δ_1.
- We get point A.
- Repeat the same procedure and get number of the point.



3. Tachometric Method

- Linear and angular measurement are used in this method.
- Less accurate than the rankine's method.
- Here T_1A, T_1B , T_1C are the different chords and joining of this chord point A, B, C we get the curve.
- $T_1A = L_1 = 2R \text{ Sin } \Delta_1$
- $T_1B = L_2 = 2R \text{ Sin } \Delta_2$
- $T_1C = L_3 = 2R \text{ Sin } \Delta_3$
- $T_1T_2 = Ln = 2R \sin \Delta_n$
- $= 2R \sin \dfrac{\Delta}{2} = L$
- = Length of Long Chord





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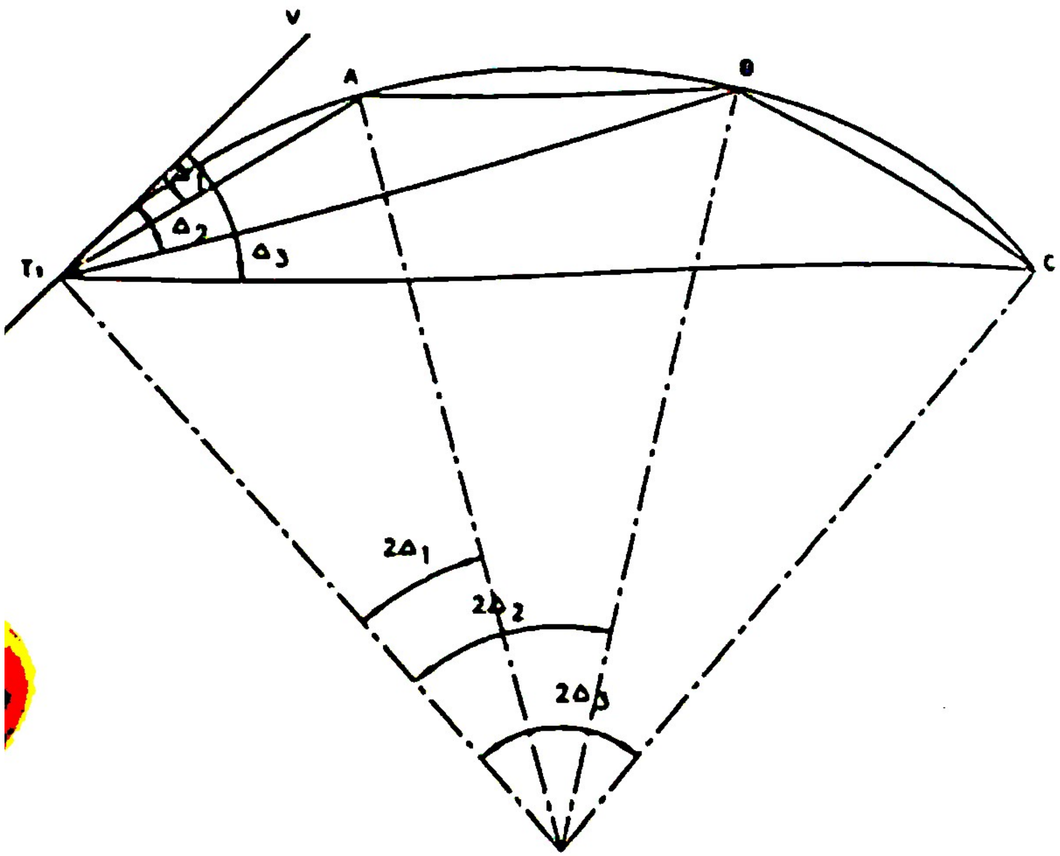
Procedure

- Set the tachometer at T_1 and sight the point of intersection (V) at that time set the reading is zero.
- Set the deflection angle Δ_1 on the vernier, thus the directing the line of sight along T_1A.



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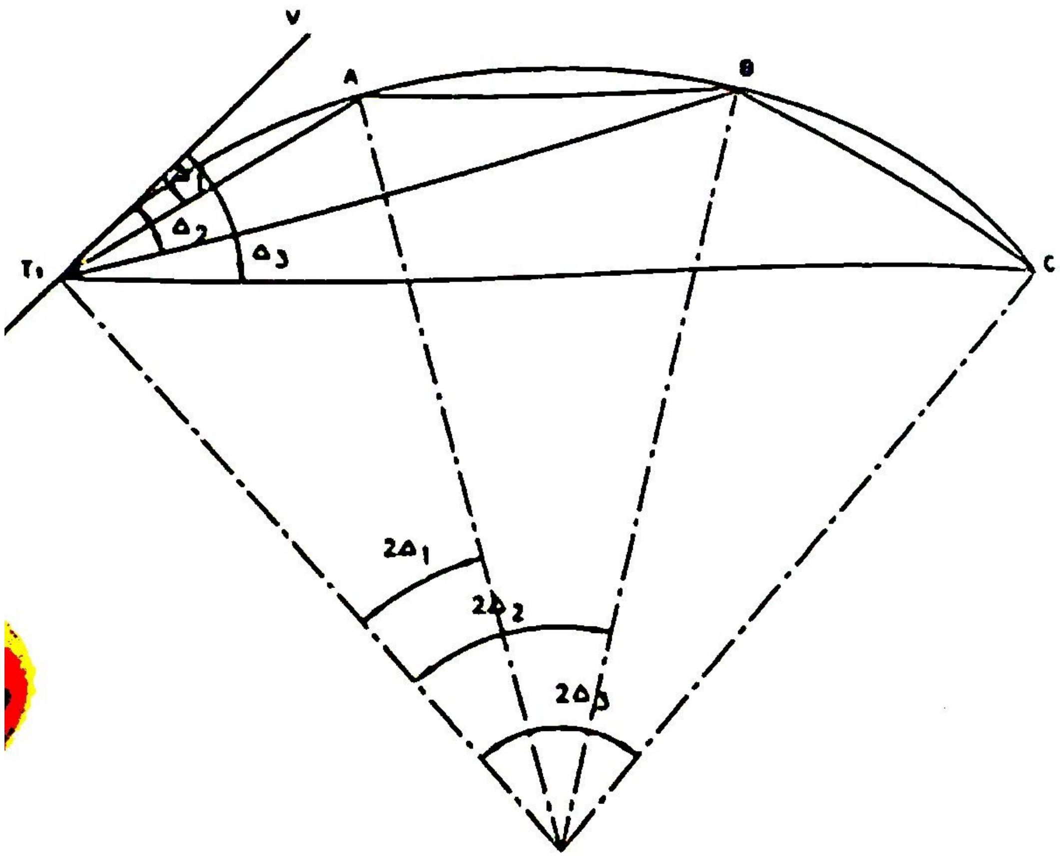
Procedure

- Direct the staff man to move in the direction T_1A till the calculated staff intercept S_1 is obtained. The staff held vertical. Thus the first point A is fixed.
- Set the deflection angle Δ_2 on the vernier, thus the directing the line of sight along T_1B.
- Repeat the same procedure.



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Transition Curve

- When a vehicle moves on a curve, there are two force acting.

1) Weight of the vehicle (W)
2) Centrifugal force (P)

- Both the force passing through the C.G of the vehicle.
- Weight of the vehicle is act as vertical.
- Resultant of the two forces should be normal to the road surface

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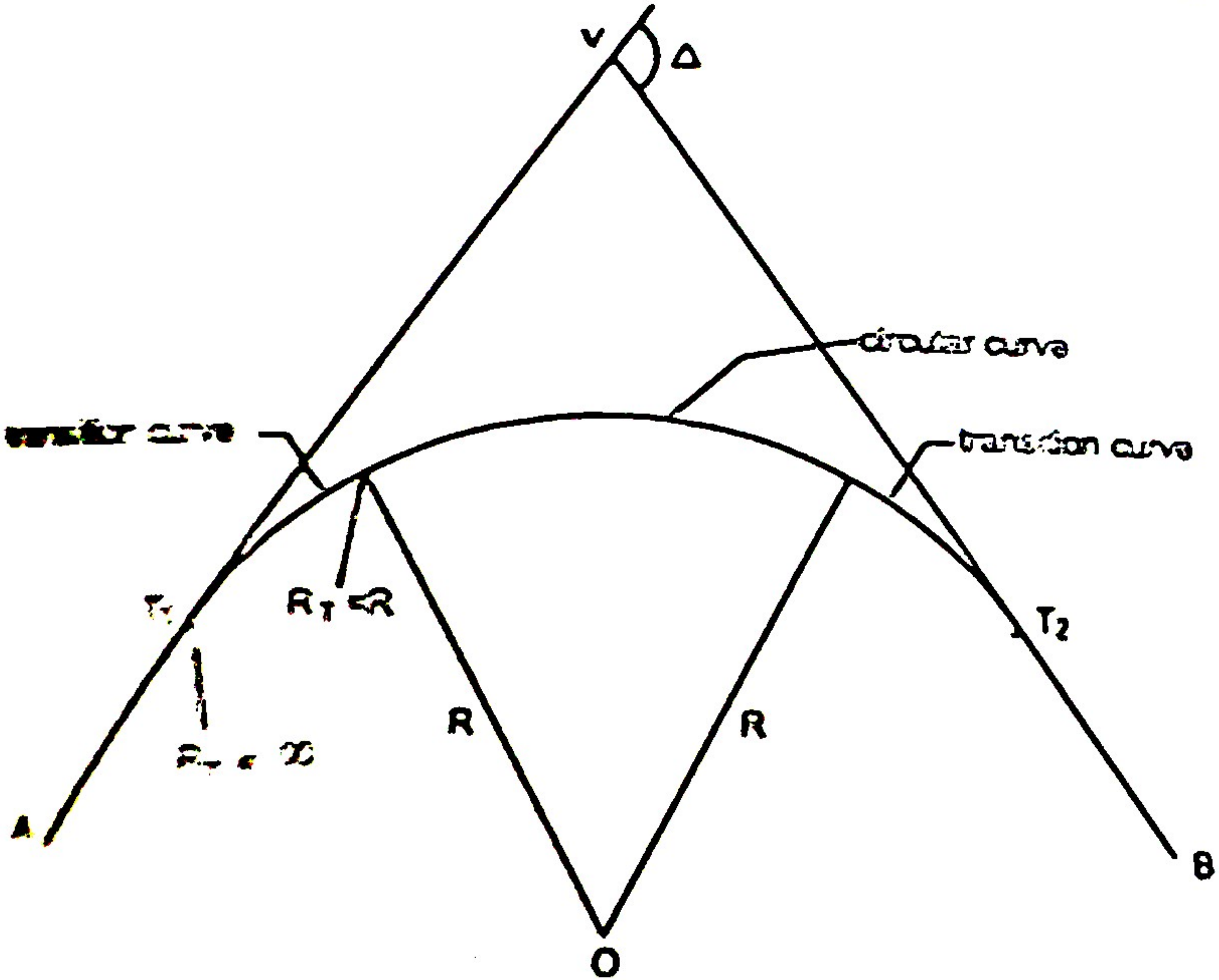
- The centrifugal force is given by
- $P = \frac{Wv^2}{gR}$
- Where,
- P = Centrifugal force
- W = weight of the vehicle Kg or N
- v = Speed of vehicle, m/sec
- g = Acceleration due to gravity m/sec^2 (9.81 m/sec^2)
- R = Radius of Curve

- $P = \frac{Wv^2}{gR}$
- Centrifugal force is inversely proportional to the radius of the curve.
- Radius decrease so the centrifugal force is increase.
- When the vehicle enters from straight road to the curve, its radius is change from infinite to R.
- Resulting in sudden increase the centrifugal force on the vehicle.
- If this value is exceed so vehicle may overturn.



V
Δ
circular curve
transition curve
$R_T = R$
T_2
R
R
A
B
O

Requirement of the transition curve

- It should be tangential to the straight line.
- It should meet the circular curve tangentially.
- Its curve should be zero at the origin of the straight curve.
- Its curvature at the junction with the circular curve should be same as that of the circular curve.
- Full super elevation is attained at the junction with circular curve.

Purpose of providing transition curve

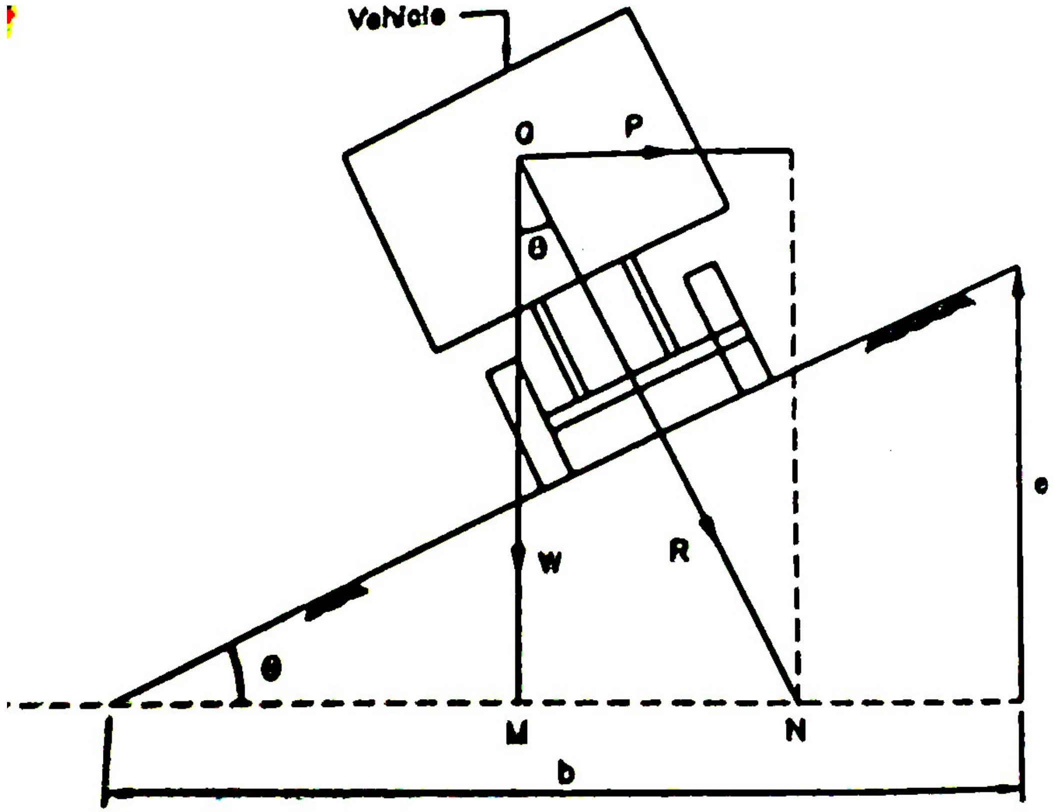
- Increase the curvature gradually.
- Provide medium for super elevation.
- Provide extra widening on the circular curve gradually.

Advantages

- Reduce the discomfort.
- Reduce the chances of overturning of the vehicles.
- Allows higher speed at curve.
- Wear on running gears is reduce.

Super Elevation or Cant

- Super elevation is defined as the rising of the outer edge of a road respect to its inner edge.



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- The centrifugal force P is given by,
- $P = \frac{Wv^2}{gR}$
- Where,
- P = Centrifugal force in Kg or N
- W = weight of the vehicle Kg or N
- v = Speed of vehicle, m/sec
- g = Acceleration due to gravity m/sec^2 ($9.81\ m/sec^2$)
- R = Radius of Curve

- $\frac{P}{W} = \frac{v^2}{gR}$

- $\tan\theta = \frac{e}{b}$

$= \frac{MN}{MG}$

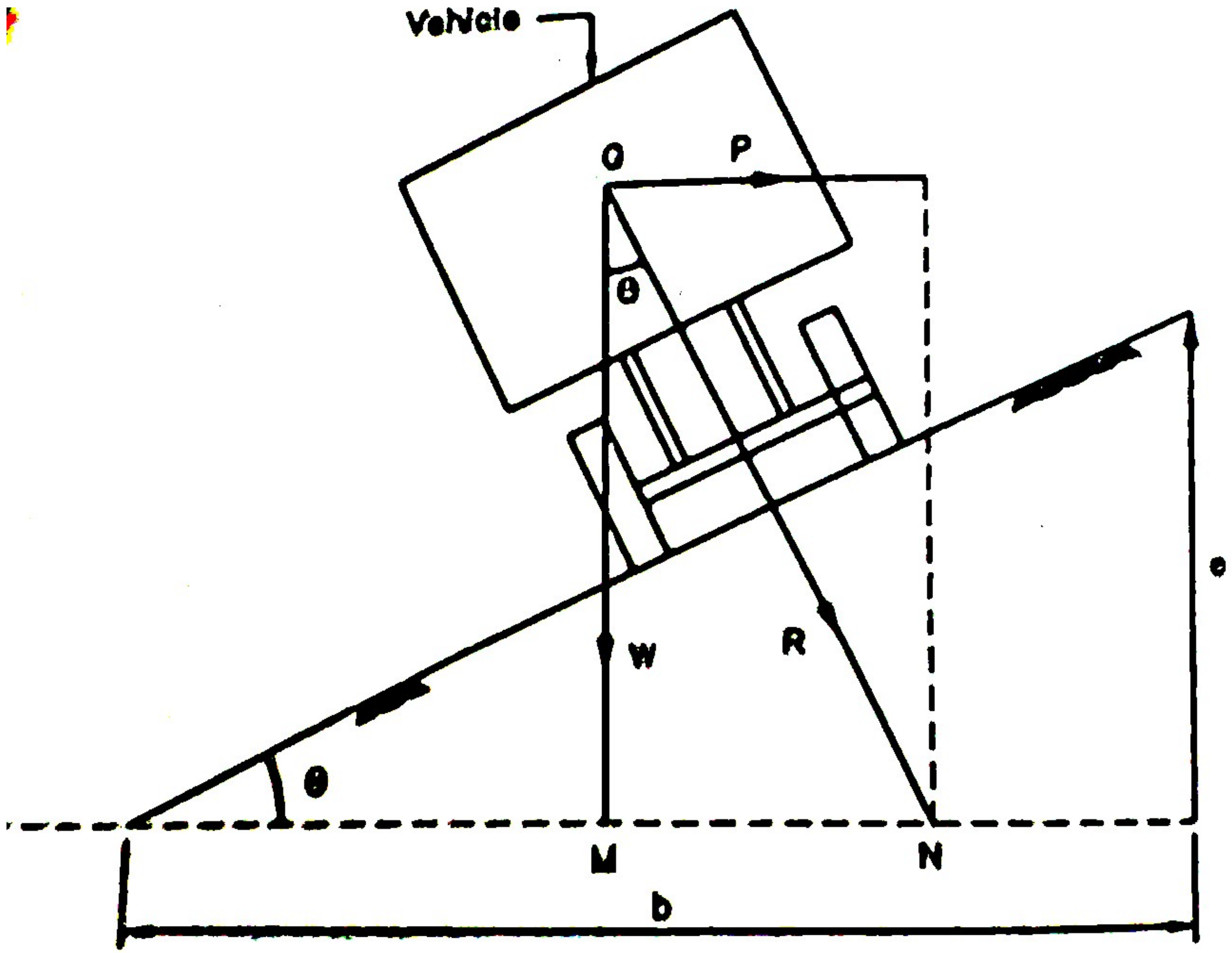
$= \frac{P}{w}$

$= \frac{v^2}{gR}$

$\frac{e}{b} = \frac{v^2}{gR}$

$e = \frac{bv^2}{gR}$

b = width of the road



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Length of Transition Curve

- Length of the transition curve introduce between straight and circular curve is calculated following consideration.

1) By rate of super elevation
2) By time rate
3) By rate of change of radial acceleration.

1) By rate of super elevation

- The length of the transition curve is given by
- L = ne
- The value of n may vary from 300 to 1200.
- e = $\frac{bv^2}{gR}$

2) By time rate

- The time taken by a vehicle pass over the transition curve of length L with speed v is
- $t = \frac{L}{v}$
- The super elevation is attained in this time is
- $e = tr = \frac{L}{v} r$
- $L = \frac{ev}{r}$
- Substitute the value of e in above equation
- $L = \frac{v}{r} \frac{bv^2}{gR}$

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3) By the rate of change of radial acceleration

- In this method, the length of transition curve is decided based on the basis of the comfort of the passengers.
- If α is the rate of change of radial acceleration, the radial acceleration a attain during the time vehicle passes over the transition curve, is given by.
- $a = \alpha\, t = \alpha \frac{L}{v}$
- Radial acceleration is given by
- $a = \frac{v^2}{R}$
- $\alpha \frac{L}{v} = \frac{v^2}{R}$
- $L = \frac{v^3}{\alpha R}$

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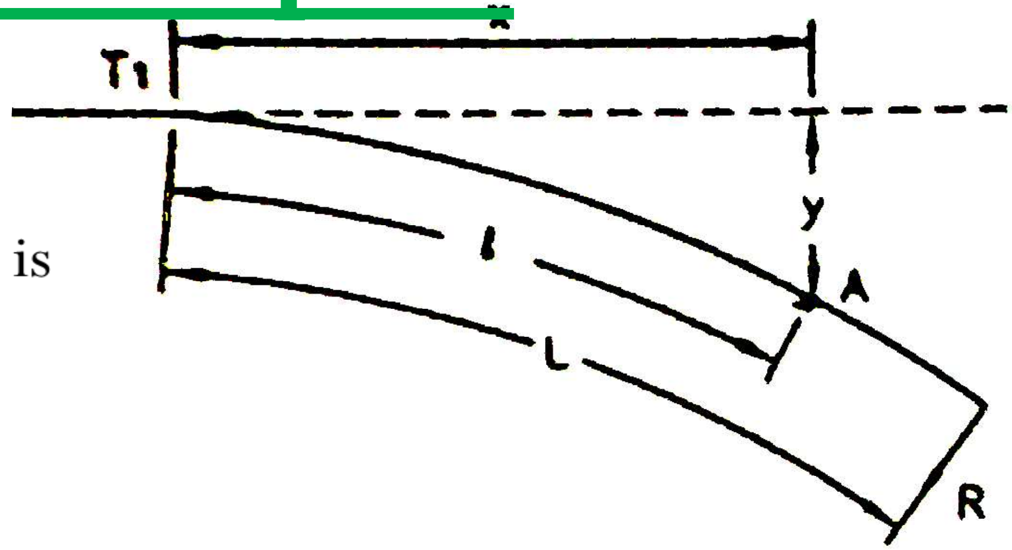
Type of Transition curve

- There are mainly three type of the transition curve.

Type of transition curve	Cubic spiral
	Cubic parabola
	The Lemniscate curve

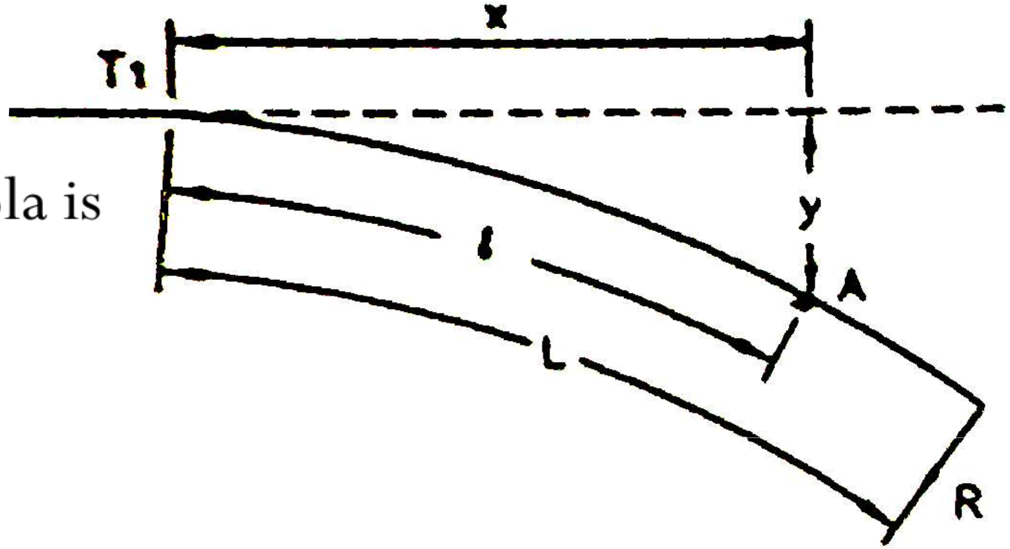
1) Cubic Spiral



- Suited for railway.
- Equation for cubic spiral is
- $y = \dfrac{l^3}{6RL}$

- Where,
- Y = Perpendicular offset from the tangent
- l = distance measure along the curve
- R = Radius of the circular curve
- L = length of the transition curve

2) Cubic Parabola

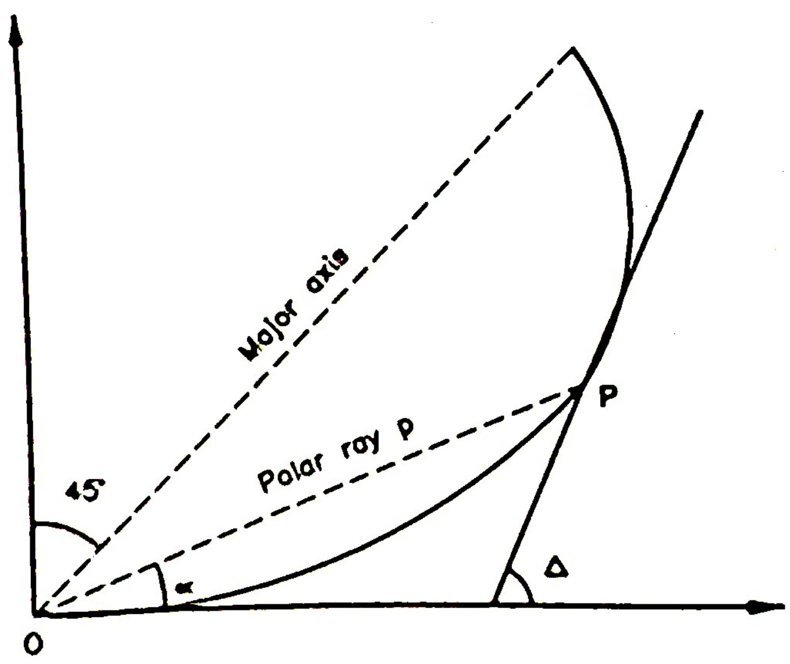


- Suited for railway.
- Equation for cubic Parabola is
- $y = \frac{x^3}{6RL}$

- Where,
- Y = Perpendicular offset from the tangent
- x = distance measure along the tangent
- R = Radius of the circular curve
- L = length of the transition curve

3) The Lemniscate curve

- When the entire curve is provided in the form of the transition curve it is known as Lemniscate curve.
- Used in high way construction.
- Equation for Bernoulli's lemniscate curve is
- $P = K\sqrt{\sin 2\alpha}$
- Where
- P = Polar distance of any point
- α = deflection angle of any point
- K = constant



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Equation for Ideal transition curve

- When a vehicles moving on a straight takes a circular path, super elevation is required to be introduce uniformly from zero to its maximum design value.
- On the other hand speed of the vehicle keep constant speed .
- Requirements are fulfill by

1) Increase the centrifugal force at a constant rate.
2) Varying the distance travelled along transition curve with time.

1) Centrifugal force is directly proportional to the length of transition curve

- P α L
- P = centrifugal force = $\frac{wv^2}{gr}$
- $\frac{wv^2}{gR}$ α L
- But w, v, and g are constant
- $\frac{1}{R}$ α L

2) Super elevation is proportional to the length of transition curve

- e α L α $\frac{wv^2}{gR}$
- But w, v, and g are constant
- L α $\frac{1}{R}$
- LR = Constant

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Vertical Curve

- Vertical curve is provided when there is sudden change in gradient of highway or a railway.
- Vertical curve are provided when highway or railway are at hilly or valley area.
- Gradient is expressed in the form of the percentage.
- EX : +2% means rise by 2m in every 100m.
- Rising gradient is taken as +ve
- Falling gradient taken as -ve

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- General equation of parabola with its axis is vertical is
- $y = ax^2 + bx$

- Slope of the curve is given by
- $\frac{dy}{dx} = 2ax + b$

- The rate of change of slope or gradient is
- $\frac{d^2y}{dx^2} = 2a = \text{Constant}$

Rate of Change of Gradient

- The characteristic of a parabolic curve is that the gradient change from point to point but the rate of change of gradient remain constant.
- Rate of change of gradient r = $\frac{g_2 - g_1}{L}$
- Where,
- g_1 = Percentage of the gradient before the intersecting point.
- g_2 = Percentage of the gradient after the intersecting point.
- L = Length of vertical curve

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Rate of Change of Gradient for Railway

Types of Railway	Curve at	
	Summit	Valley
First class Railway	0.3%	0.15%
Second class railway or branch line	0.6 %	0.3%

Advantage of Vertical curve

- Change in gradient is gradually.
- Improve the appearance of the road.
- Road and railway journey become comfortable.

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Types of Vertical Curve

- There are main two type of the vertical curve.

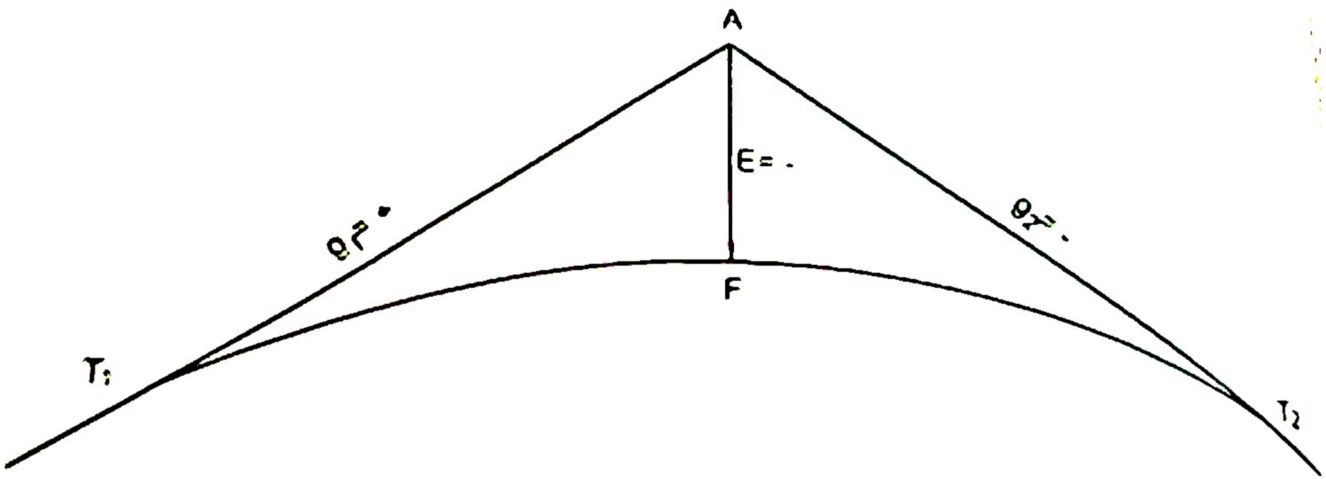
Summit Curve
(Convex curve)

Valley curve

(Concave Curve)

Summit Curve (Convex curve)

- Summit curve is provide in following situation :-
  - An upgrade ($+g_1$) followed by down grade ($-g_2$)
  - An upgrade ($+g_1$) followed by another upgrade ($+g_2$). $g_1 > g_2$
  - An downgrade ($-g_1$) followed by another down grade ($-g_2$). $g_2 > g_1$
  - A plane surface followed by down grade ($-g_1$).



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Valley Curve (Concave curve)

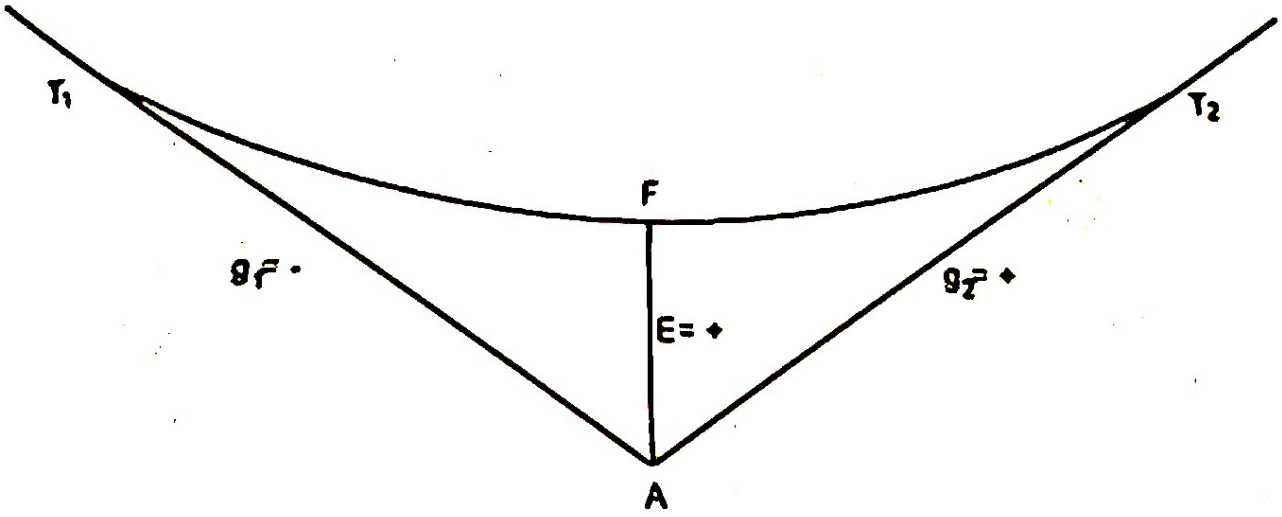
- Valley curve is provide in following situation :-

➢ A Down grade ($-g_1$) followed by up grade ($+g_2$)

➢ A Down grade ($-g_1$) followed by another down grade ($-g_2$). $g_1 > g_2$

➢ An up grade ($+g_1$) followed by another up grade ($+g_2$). $g_2 > g_1$

➢ A plane surface followed by up grade ($+g_1$).



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Length of Vertical Curve

- The length of the vertical curve can be obtained by dividing the algebraic difference of the two grades by the rate of change of grade.
- Length of curve (L) = $\frac{Total\ change\ of\ grade}{Rate\ of\ Change\ of\ Grade}$

$$= \frac{g_2 - g_1}{r}$$

- g_1 and g_2 = grade in %
- R = rate of change of grade (%)

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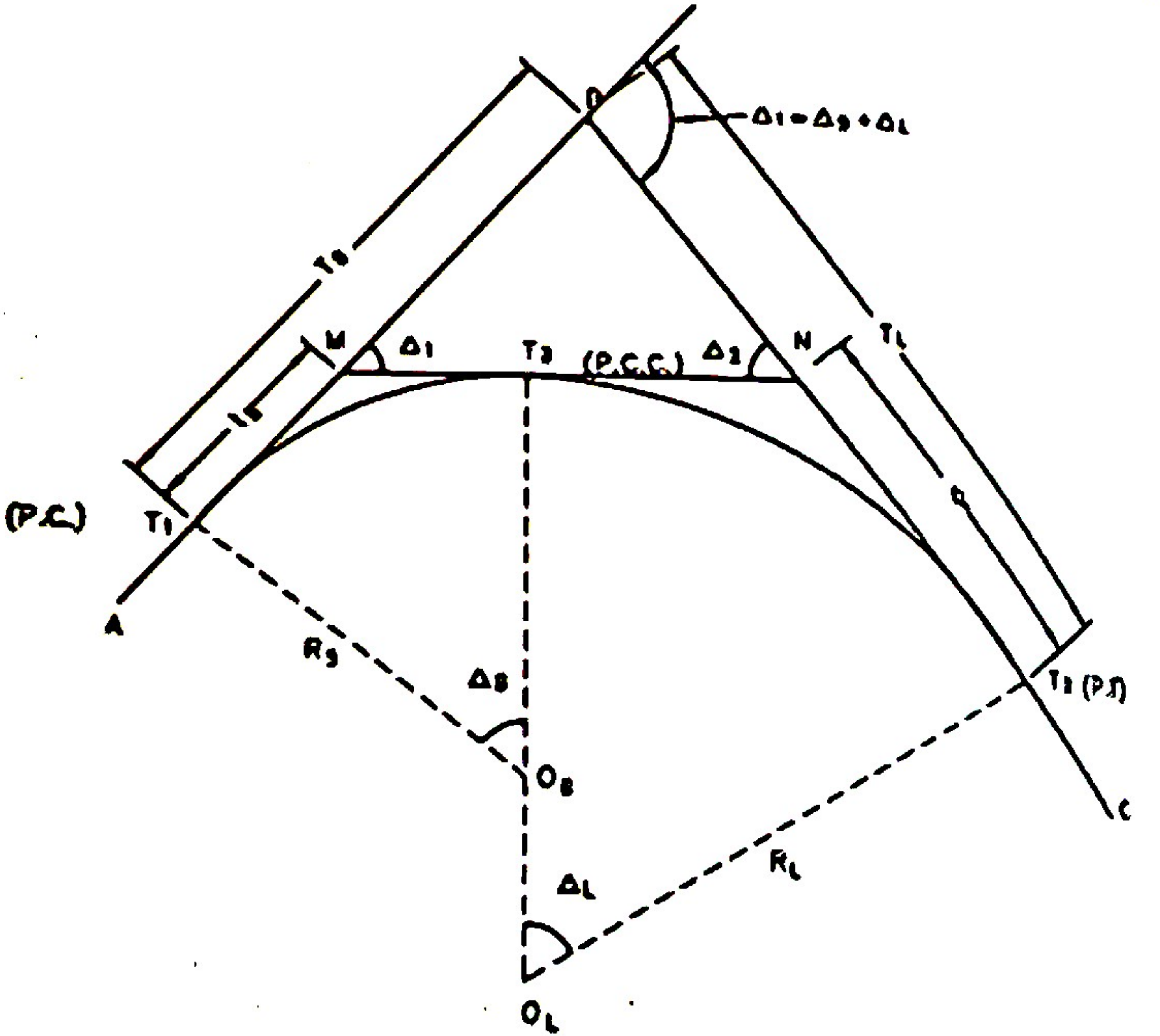
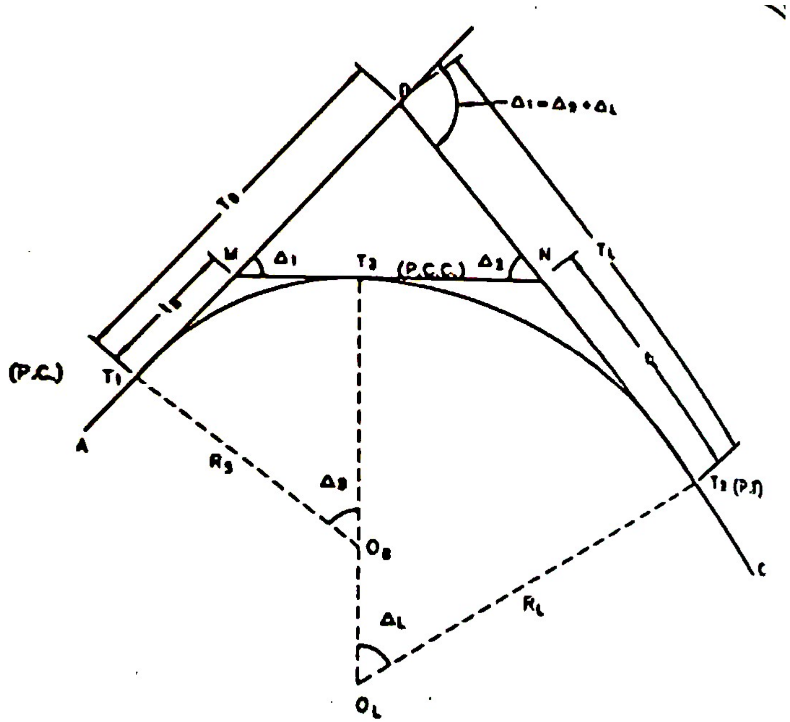
Compound Curve

- A compound curve consist of two or more circular arc of different radiation with their centre of curvature on the same side of the common tangent.

Elements of compound curve



- Fig shows a two centred compound curve $T_1T_3T_2$ having two circular arcs T_1T_3 and T_3T_2 meeting at common point T_3 known as the point of compound curvature (P.C.C).
- T_1 is the point of curve (P.C)
- T_2 is the point of tangency (P.T)
- R_S, R_L = the radius of the curve
- Δ_S, Δ_L = the deflection angle
- l_S, l_L = length of curve
- t_S, t_L = the tangent length
- T_S, T_L = the tangent length



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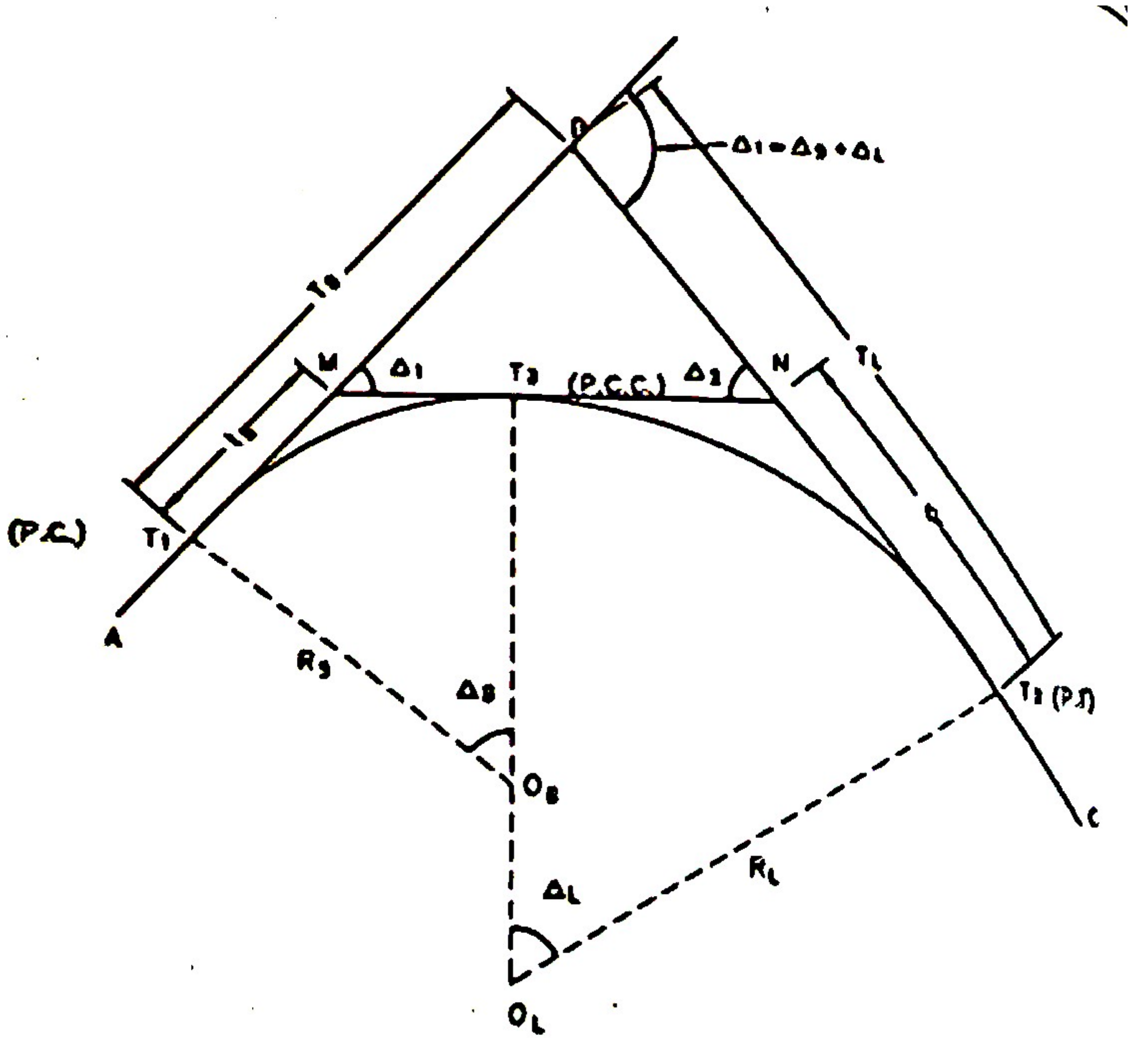
1) Tangent length of Circular curve

- $t_s = T_1M = MT_3 = R_S \tan\frac{\Delta_S}{2}$

- $t_L = T_3M = NT_2 = R_L \tan\frac{\Delta_L}{2}$

- $MN = t_s + t_L$



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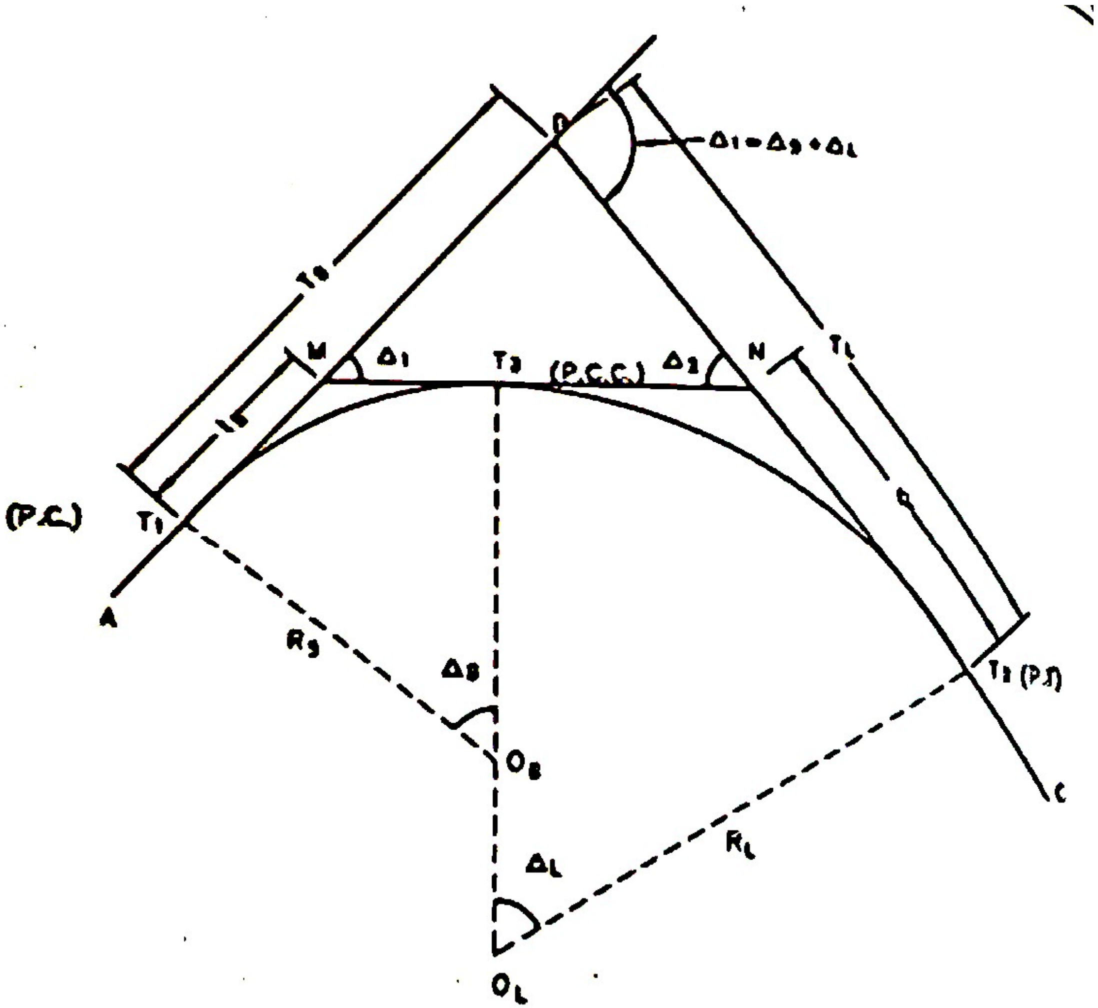
2) Total Deflection angle

- $\Delta = \Delta_S + \Delta_L$



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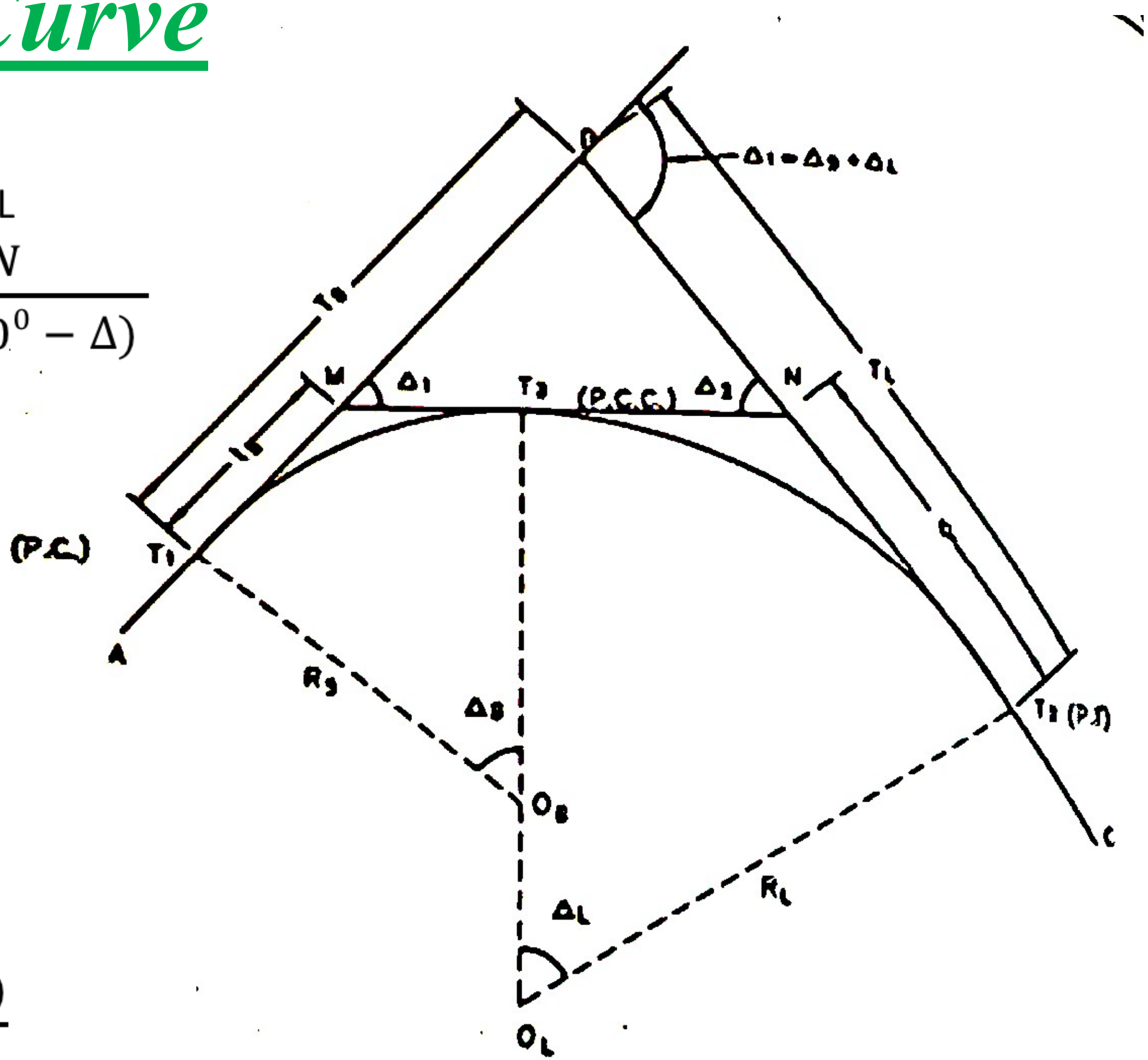
3) Tagent length of Compound Curve

- In ΔMBN
- $MN = MT_3 + T_3N = t_S + t_L$
- $$\frac{MB}{Sin\ \Delta_L} = \frac{BN}{Sin\ \Delta_s} = \frac{MN}{Sin\ (180^0 - \Delta)}$$

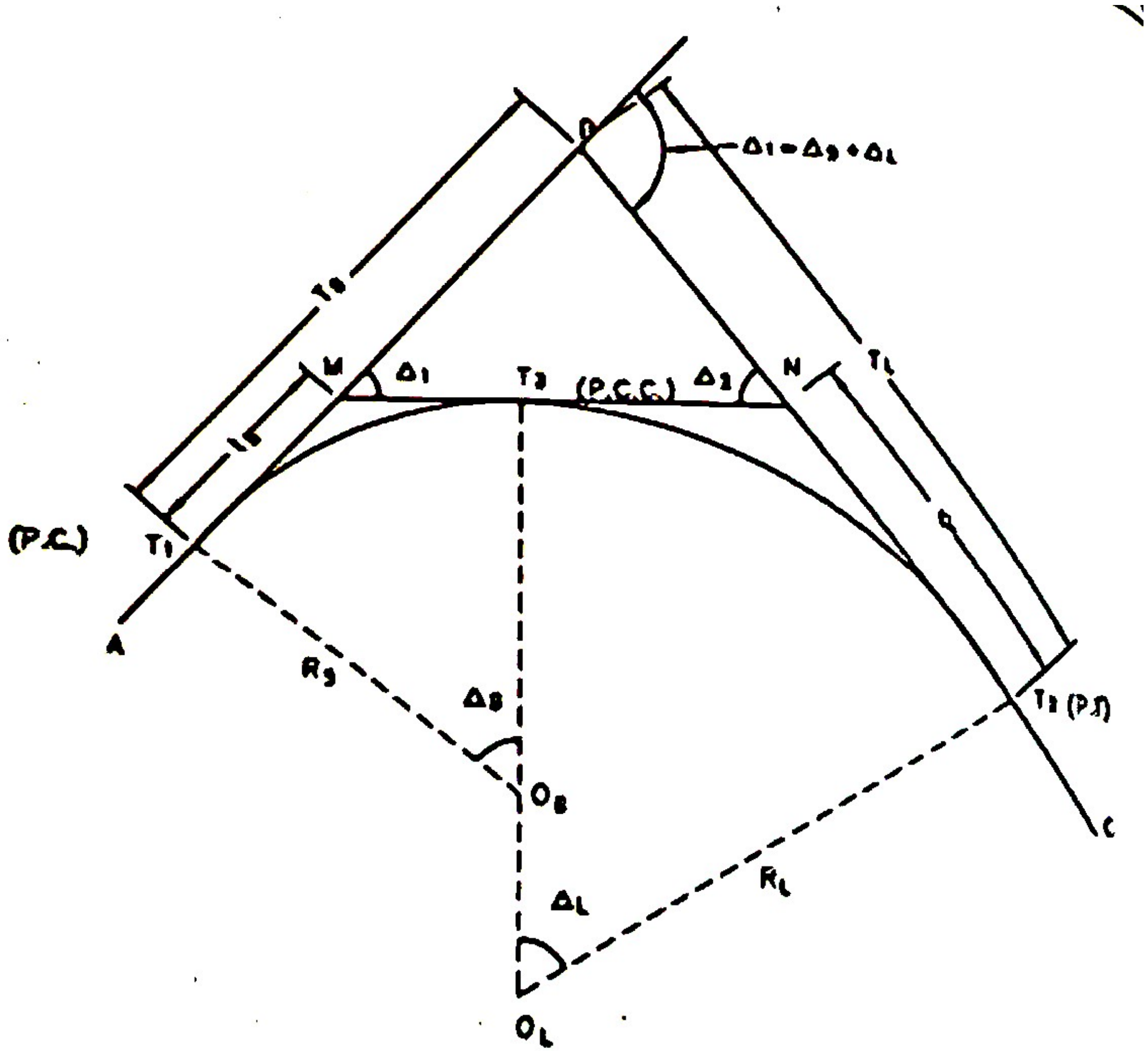


- $$\frac{MB}{Sin\ \Delta_L} = \frac{BN}{Sin\ \Delta_s} = \frac{(t_S + t_L)}{Sin\ \ \Delta}$$

- $\frac{MB}{Sin\ \Delta_L} = \frac{BN}{Sin\ \Delta_s} = \frac{(t_S + tL)}{Sin\ \Delta}$
- $MB = (t_S + t_L)\ \frac{Sin\ \Delta_L}{Sin\ \Delta}$
- $BN = (t_S + t_L)\ \frac{Sin\ \Delta_S}{Sin\ \Delta}$
- $T_S = t_s + MB$

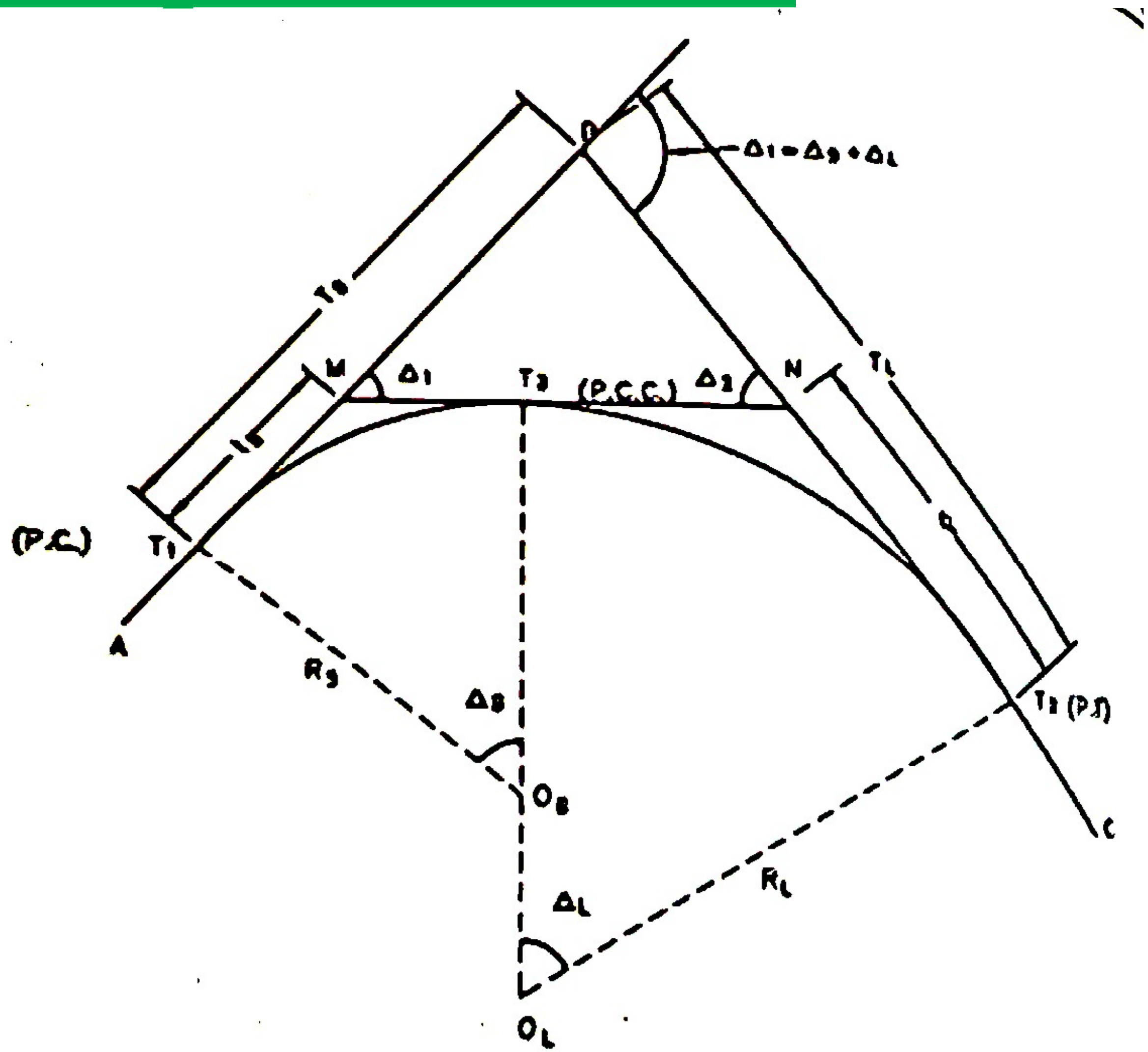
  $= t_s + (t_S + t_L)\ \frac{Sin\ \Delta_L}{Sin\ \Delta}$
- $T_L = t_L + BN$

  $= t_L + (t_S + t_L)\ \frac{Sin\ \Delta_S}{Sin\ \Delta}$



4) Length of Compound Curve

- $l = l_S + l_L$
- But we have
- $l_S = \frac{\pi R_S \Delta_S}{180^0}$
- $l_L = \frac{\pi R_L \Delta_L}{180^0}$



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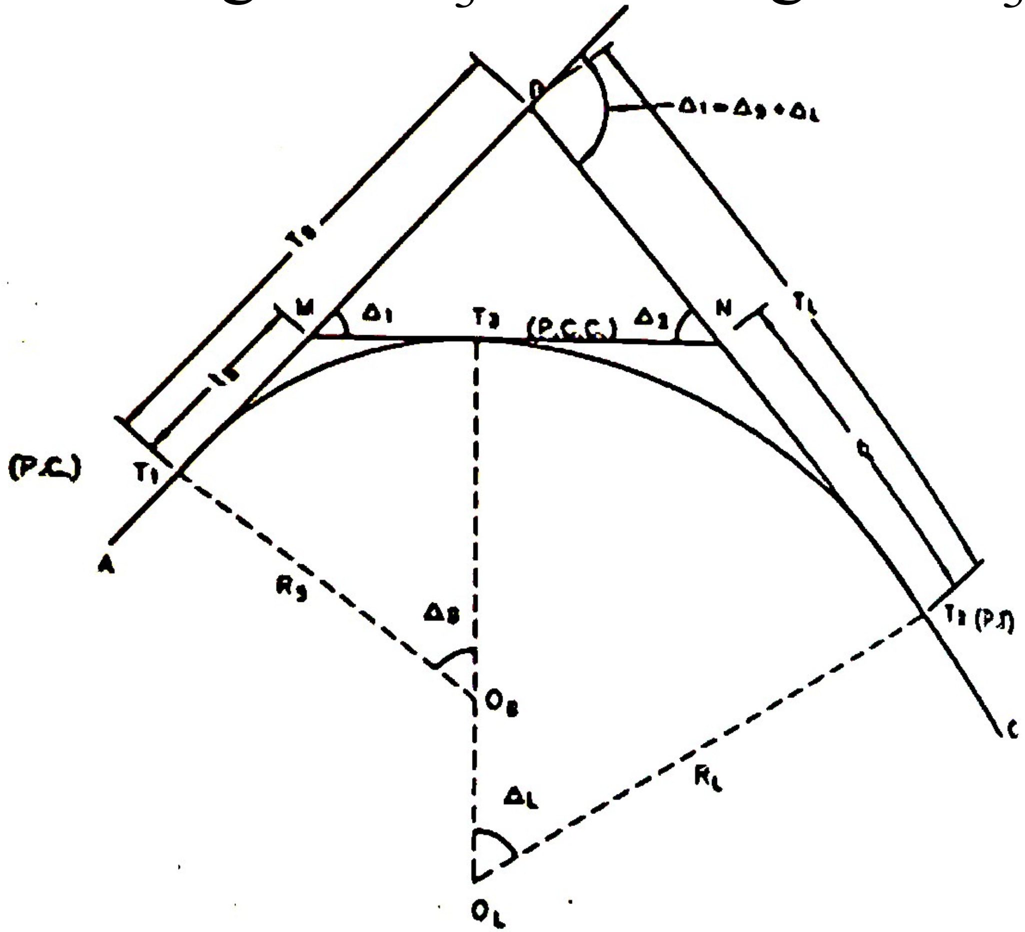
5) Chainages

- Chainage of T_1 = Chainage of B – T_S
- Chainage of T_3 = Chainage of $T_1 + l_S$
- Chainage of T_3 = Chainage of $T_3 + l_L$



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