COLLEGE OF ENGINEERING & TECHNOLOGY

Module - 8
Theory of Error

Subject:- Surveying Code:-3140601	Prepared by: Asst.Prof. Janki Patel (Civil Department, ACET)	COLLEGE OF ENGINEERING & TECHNOLOGY

Introduction

- Due to faulty instrument, climate condition, and human mistake, it is not possible to measure the observation very precisely.
- Hence the survey measurement have some errors in all measurement.
- Hence the surveyor or civil engineers must know about the errors developed and its elimination.
- Knowledge of theory of error helps a lot to get the correct measurement.

- The errors should be reduced by adopting standard procedure and methods.
- Note that natural errors like wind effect, errors due to earth curvature and refraction due to air can not be controlled by man.

Types of errors

- There are main three types of the errors.

Systematic Errors

Accidental Errors

Human Errors

Systematic Errors

- Following are the systematic errors which are likely to develop while triangulation.

1. Errors due to shrinkage of topographical map.
2. Errors by wrong length of chain, may too long or too short.
3. Errors due to wrong scale of map.
4. Errors caused by atmospheric refraction.
5. Errors because of shrinkage or expansion of aria photograph.

- Systematic errors follow some definite mathematical or physical law.
- Determination of correction and its application can be made by definite mathematical or physical law.
- The errors may be positive or negative and the correction are apply.

Accidental errors

- The errors which remain as it is even after systematic errors and human mistakes are eliminated called accidental error.
- Accidental errors are also termed as random error or compensating errors.
- These errors are occurs because of the combination of various causes and reasons which would beyond the control and capacity of the surveyor or observer.

- They do not follow any specific laws and may be positive or negative.
- Accidental errors follows the law of probabilities.
- Accidental errors occur due to lake of perfection in human eyes. Ex: in measurement
- Random errors are generally small and never be avoided in observation.
- The smaller random error the better the precision of the measurement.

Human mistakes

- These type of the mistake are caused by the following things

1. Improper attention of the observer.
2. Carelessness of the observer.
3. Poor knowledge of handling the instrument.
4. Unskilled and inexperience observer.
5. Poor judgment and confusion in the mind of the observer.
6. Improper communication between the observer and field party.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Definition

- Observation :-
  - The measured numerical value of a quantity is known as observation. Ex: 10m, $10^0 20' 20''$ etc.
  - The observation may be classified in to two type :-

1. Direct observation
2. Indirect observation

1. Direct Observations :-

- An observation is called a direct observation if the value of a quantity are measured directly. Ex : measurement of the angle with theodolite.

2. Indirect observation

- An observation is called indirect observation when the value of quantity is calculated indirectly from the direct observation.

❑Observed value of quantity :

- The observed value of quantity is the value obtained from the observation after applying correction.
- The observed value of the quantity may be classified as

1. Independent quantity
2. Dependent quantity

1. Independent quantity :-

- If the value of an observed quantity is independent of the value of the other quantity it is known as independent quantity.

2. Dependent quantity :-

- If the value of an observed quantity is depend upon the value of the other quantity it is known as dependent quantity.

- ❑ True value of quantity :-

- The value of quantity free from all the error is known as true value of quantity.

- ❑ True error :-

- The difference between the true value and the observed value is known as true error.
- True error = True Value – Observed value
- As the true value can not be find perfectly

❑Most probable value :-

- The most probable value of a quantity is the value which has more chance of bring true.
- Most probable value is nearer to the true value

❑Most probable error :-

- It may be defined as the quantity which is subtracted from or added to most probable vale of the quantity.

❑Residual error :

- The difference between the observed value of a quantities and its most probable value is called the residual error.
- Residual error = Observed value – most probable value

❑Observation equation :-

- An observation equation is the relation between the observed quantity and its numerical value.

Conditioned equation :

- An observation equation is the equation expressing the relation between several dependent quantities.
- Ex Triangle equation $A + B + C = 180^0$

Normal Equation :-

- A normal equation is the one which is formed by multiplying each equation by the coefficient of unknown whose normal equation is to be found by any adding the equation.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of Accidental error

- A close examination of the observation a accidental errors law are required.
- Accidental laws of error follow the law of probability.
- This law defines the occurrence of error and can be expressed in terms of equation which is used to compute the probable value.
- Accidental errors always written after the observation quantity with the plus and minus sign.

- The following point may be noted in the normal probabilities curve

1. Positive and negative errors are equal in size and frequency, as the curve is symmetrical.
2. Small error are more frequent than large error.
3. Very large error seldom occur and are impossible.

Probable error

- Probable error is measurements is calculated from the probability curve of error.
- In large series of observation the probable error is an error of the observation is greater or less.
- If the probable error of an angular observation is 2 second so the probability of the error between the limit of -2 second to +2 second.

From the probability of the curve

1. Probable error of the single measurement is given by

- Es = $\pm 0.6745 \sqrt{\boldsymbol{\Sigma} v^2 / n - 1}$
- Es = probable error of the single observation
- v = difference between any single observation and mean of the series.
- n = number of the observation in the series

Probable error of an average

- Since the average of n observation is the sum of the observation divided by n, the probable error of the average of the n observation is $\sqrt{n}$/n times the probable error of single observation.
- Em = Es / $\sqrt{n}$
- Em = probable error of the mean

Probable error of a sum of observation

- Esm = $\sqrt{E1^1 + \ E2^2}$ ....
- E1, E2 = probable error of several observation which sum up in measurement.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Mean Square Error

- The mean square error is equal to the square root of the arithmetic mean of the individual errors.
- MSE = $\frac{\pm\sqrt{v1^2 + v2^2 + v3^2 + \cdots}}{n}$

- V1, V2 = Individual error
- n = Number of the observation

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Average error

- For as series of observation of equal weight, an observation error is defined as the arithmetic mean of separate error, taken all with the same sign either plus or minus.

Standard Deviation (σ)

- The standard Deviation defined as the
- $\sigma = \pm\sqrt{\boldsymbol{\Sigma} v^2/\mathrm{n}-1}$
- Here v = Residual (Variation)
- n = Number of the observation
- The standard deviation is also known as the root mean square error of the measurement.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Determine the probable error

- Considering the following cases :

1. Direct observation of the equal weights :

a. Probable error of single observation unit weight.
b. Probable error of single observation of weight w.
c. Probable error of single arithmetic mean.

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

2. Direct observation of the unequal weight :

a. Probable error of single observation unit weight.
b. Probable error of single observation of weight w.
c. Probable error of weighted arithmetic mean.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

3. Indirect observation of independent quantities.
4. Indirect observation involving conditional equation.
5. Computed quantities.

Direct observation of the equal weights

a. Probable error of single observation unit weight may be calculated from the following equation.

- $E_s = \pm\, 0.06745 \sqrt{\Sigma w v^2 / n - 1}$

Where

- $\Sigma v^2 = v_1{}^2 + v_2{}^2 + v_3{}^2 + \ldots\ldots$
- $V =$ Residual error

  $=$ Observed value of quantity – Probable value of quantities
- $n =$ Number of observation

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

b. Probable error of single observation of weight w may be calculated from the following equation :

$$E_{sw} = E_s / \sqrt{w}$$

c. Probable error of single arithmetic mean may be calculated by the following formula :

$$E_m = E_s / \sqrt{n}$$

Direct observation of the unequal weight

a. Probable error of single observation unit weight :

$$E_{su} = \pm 0.06745 \sqrt{\Sigma w v^2 / n - 1}$$

b. Probable error of single observation of weight w :

$$E_{suw} = E_{su} / \sqrt{n}$$

or

$$E_{suw} = \pm\, 0.06745 \sqrt{\Sigma w v^2 / w(n-1)}$$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

c. Probable error of weighted arithmetic mean :

$$E_{swm} = \pm\, 0.6745 \sqrt{\Sigma w v^2 / \Sigma w(n-1)}$$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Indirect observation of independent quantities

- The probable error of an observation of unit weight

$$E_{si} = \pm\, 0.06745 \sqrt{\Sigma w v^2/(n - q)}$$

- The probable error of an observation of weight w

$$E_{siw} = E_{si} \,/\, \sqrt{w}$$

Here

n = the number of observation equations

q = the number of unknown quantities

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Indirect observation involving conditional equation

- The probable error of an observed unit weight

$$E_{sic} = \pm\, 0.06745 \sqrt{\Sigma w v^2/(n - q + p)}$$

Where

n = number of observation equations

q = the number of unknown observation

p = the number of conditional equations

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of Weights

- The weight of the quantities is trust worthiness of a quantities.
- The relative precision and trustworthiness of an observation of an observation as compare to the precision of other quantities is known as weight of the observation.
- The weight are always expressed in number.
- Higher the number higher the precision and trust as compare to lesser number.

- Ex : an observation with a weight 6 is three time precise as an observation with a weight of 2.
- Weights are assigned to the observation or quantities observed in direct proportion to the number of times, the quantities is measured.

Laws of weight (1)

- The weight of the arithmetic mean of the number of observation of unit weight is equal to the number of observation.
- Ex : calculate the weight of the arithmetic mean of the following observation of an angle of unit weight.

Angle	Weight
$60^0 30' 10''$	1
$60^0 30' 15''$	1
$60^0 30' 20''$	1

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

- Sol :- here n = 3

$$\text{Arithmetic mean} = \frac{60^0 30' 10'' + 60^0 30' 15'' + 60^0 30' 20''}{3}$$

$$= 60^0 30' 15''$$

The weight of the arithmetic mean $60^0 30' 15''$ is 3.

Laws of weight (2)

- The weight of the weighted arithmetic mean of the number of observation is equal to the sum of the individual weights of observation.
- Ex :- an angle A was observed three times as given below with their respective weights. What is the weight of the weighted arithmetic mean of the angle?

Angle	Weight
$40^0 15' 10''$	1
$40^0 15' 14''$	2
$40^0 15' 12''$	3

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

- Sol :- weighted arithmetic mean of A

$$= \frac{(40^0 15'10'') \times 1 + (40^0 15'14'') \times 2 + (40^0 15'12'') \times 3}{1+2+3}$$

$= 40^0 15'12.33''$

- Sum of the individual weights = 1+2+3 = 6
- The weighted arithmetic mean $40^0 15'12.33''$ has the weight of 6

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of weight (3)

- The weight of algebraic sum of two or more quantities is equal to the reciprocal of the sum of the reciprocal of individual weight.
- Ex : - calculate the weights of (A + B) and (A – B) if the measured values and the weight of A and B respectively are :

$$A = 40^0 50' 30'' \text{ wt. } 3$$

$$B = 30^0 40' 20'' \text{ wt. } 4$$

- Sol :
- $A + B = 40^0 50'30" + 30^0 40'20" = 71^0 30'50"$
- $A - B = 40^0 50'30" - 30^0 40'20" = 10^0 10'10"$
- The weight of (A + B) and (A – B)

$$= \frac{1}{1/w_1 + 1/w_2}$$

$$= 12/7$$

Hence $A + B = 71^0 30'50"$ wt. 12/7

$A - B = 10^0 10'10"$ wt. 12/7

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of weight (4)

- The weight of the any product of the quantities multiplied by a constant is equal to the weight of the quantities divided by the square of that constant.
- Ex :- what is the weight of 3A if A = $30^0 25' 40''$ and its weight is 3?

Sol : -

As A is multiplied by 3, the constant of multiplication C is 3 and weight w of the observation A is 3.

- As per the law 4 the weight of 3 is w/c^2
- Weight of 3A = $91^0 17'00'' = 3/3^2 = 1/3$

Laws of weight (5)

- The weight of the quantient of any quantity divided by a constant is equal to the weight of that quantities multiplied by the square of that constant.
- Ex :- Compute the weight of A/4 if A = $36^0 20' 40''$ of weight 3.
- Sol :-

Here constant C = 4

- As per the law no 5
- Weight of $A/4 = 30^0 20'40''/4 = 3 \times 4^2$

$$A/4 = 9^0 5'10'' \text{ wt. } 48$$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of weight (6)

- The weight of an equation remain unchanged if all the sign of the equation are changed or if the equation added to or subtracted from the constant.
- Ex :- If weight of A + B = $76^0 20'30''$ is 3, what is the weight of - (A + B) or 180^0 – (A + B) ?
- Sol :- from the law 6, the weights of – (A + B) will be remain same.

weight of - (A + B) = - $76^0 20'30''$ wt. 3 or

weight of 180^0 - (A + B) = $103^0 39'30''$ is equal to wt. 3

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Laws of weight (7)

- If an equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of that equation.
- Ex :- calculate the weight of the equation ¾(A + B) if weight of (A + B) is 3/4 . The observed value of (A + B) is $120^0 20'40''$
- Sol :-

Weight of ¾(A + B) = $90^0 15'30''$ = 1/¾ = 4/3

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Theory of least square

- The principle of distributing errors by the method of least squares is to find the most probable value of quantities.
- The fundamental principle of the method of least square may be stated as follows :-
- In observation of equal precision, the most probable value of the observed quantities are those that render the sum of the square of the residual error a minimum.

- If the measurement are of equal weight, the most probable value is that which makes the sum of the square of the residue (v) a minimum.
- Thus, $\boldsymbol{\Sigma} v^2 = \text{a minimum}$
- If the measurement are of unequal weight, the most probable value is that which makes the sum of the products of the weight (w) and the square of the residuals a minimum.
- Thus, $\boldsymbol{\Sigma} wv^2 = \text{a minimum}$

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

- When a quantity is being deduced from a series of observation the residual error will be the difference between the adopted value and the several observed values.
- Let X_1, X_2, X_3, X_4, …. Be the observed values
- Z = most probable value
- Then,

$$Z - X_1 = e_1$$
$$Z - X_2 = e_2$$
$$Z - X_3 = e_3$$

……………………………………………………………………..(1)

……………. = …….

- Where e are the respective residual error of the observed value.

- $$M = \frac{X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots + X_n}{n} \qquad \ldots\ldots\ldots(2)$$

$$M = \Sigma X / n$$

Where n = number of observation

From equation (1), $nZ - \Sigma X = \Sigma e$

$$Z = (\Sigma X/n) + (\Sigma e/n)$$

$$Z = M + (\Sigma e/n) \quad \ldots\ldots\ldots(3)$$

AMIRAJ

COLLEGE OF ENGINEERING & TECHNOLOGY

- If the number of the observation is large, n is very large and e is kept small by making precise measurement, and the second term $(\Sigma e/n)$ become nearly equal to zero
- Thus $Z = M$
- Thus when the number of observation is large, the arithmetic is the true value or most probable value.
- $Z = M$

- Now we calculated the residual error from the mean value. If v_1, v_2, v_3 etc. are the residual errors, then

$$M - X_1 = v_1$$

$$M - X_2 = v_2$$

$$M - X_3 = v_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots$$

Adding the above

$$nM - \Sigma X = \Sigma v$$

$$M = (\Sigma v/n) + (\Sigma X/n)$$

$$M = M + (\Sigma v/n)$$

$$(\Sigma v/n) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

- Hence the sum of residual equals zero and the sum of plus residual equals the sum of minus residuals.
- Let N be any other value of the unknown other than the arithmetic mean, we have

$$N - X_1 = v'_1$$

$$N - X_2 = v'_2$$

$$N - X_3 = v'_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots$$

Rules for giving weight

- The weights of an angle varies directly to the number of observation made on the angle.
- The weights of the level line vary inversely as the length of their route.
- The weight of any angle measured a large number of times, is inversely proportional to the square of the probable error.
- The correction to be applied to various observed quantities are in inversely proportional to their weight.

Distribution of error to the field observations

- When the observation are made there are some error during the observation.
- It is necessary to check the observation made in the field for the closing error.
- The closing error should be distributed to the observed quantities.

Rules should be applied for distribution of the error

1. The correction to be applied to an observation is inversely proportional to the weight of the observation.(in other word greater weight, the smaller correction)
2. The correction to be applied to an observation is directly proportional to the square of the probable error.
3. In case of line of levels, the correction to be applied is proportional to the length or route.
4. If all the observation are of the same weight, the error is distributed to observed quantities equally.

Example : 1

The fooling are three angle observed at a station closing the horizon, along with their probable error of measurement, Determine the correct values.

$A = 85^0 13' 10'' \pm 2''$

$B = 130^0 49' 30'' \pm 3''$

$C = 143^0 57' 10'' \pm 4''$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Solution :-

Sum of three angle =

$$\begin{array}{r} 85^0 13' 10'' \\ +\ 130^0 49' 30'' \\ +\ 143^0 57' 10'' \\ \hline 359^0 59' 50'' \end{array}$$

so here Discrepancy = 10”

Here C1, C2, C3 are the correction applied for angle A, B, C etc.

(As per rule no 2)

$C_1 : C_2 : C_3 = (2^2) : (3^2) : (4^2) = 4 : 9 : 16$ ………(1)

$C_1 + C_2 + C_3$ = 15”……………………………………(2)

$C_1 : C_2 = 4 : 9$

$C_2 = 9/4\ C_1$

$C_1 : C_3 = 4 : 16$

$C_3 = 16/4\ C_1 = 4\ C_1$

Now put the both C_2 , C_3 value in equation no 2

So

$C_1 + 4\ C_1 + 9/4\ C_1 = 15''$

$C_1\ (1 + 4 + 9/4) = 15''$

$C_1 = 15\ (4/29) = 2.07''$

$C_2 = 4\ (C_1) = 4(2.07'') = 8.28''$

$C_3 = 9/4\ (C_1) = 9/4\ (2.07'') = 4.65''$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

- Check :-
- $C_1 + C_2 + C_3 = 2.07'' + 8.28'' + 4.65'' = 15''$

Hence the corrected angle are :

$$85^0 13' 10'' + 2.07''$$
$$130^0 49' 30'' + 8.28''$$
$$\underline{143^0 57' 10'' + 4.65''}$$
$$360^0 00' 00''$$

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Example : 2

The following are the angles observed at a triangulation traverse along with their probable errors. Determine correct value of angles.

$A = 64^0 12' 12'' \pm 02''$

$B = 50^0 48' 30'' \pm 04''$

$C = 64^0 59' 08'' \pm 05''$

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Solution :-

Sum of three angle =

$$
\begin{array}{r}
64^0 12' 12'' \\
+\ 50^0 48' 30'' \\
+\ 64^0 59' 08'' \\
\hline
179^0 59' 50''
\end{array}
$$

so here Discrepancy = 10"

Here C1, C2, C3 are the correction applied for angle A, B, C etc.

(As per rule no 2)

$C_1 : C_2 : C_3 = (2^2) : (4^2) : (5^2) = 4 : 16 : 25$ ………(1)

$C_1 + C_2 + C_3 =$ 10”…………………………………….(2)

$C_1 : C_2 = 4 : 16$

$C_2 = 16/4\ C_1 = 4\ C_1$

$C_1 : C_3 = 4 : 25$

$C_3 = 25/4\ C_1 = 25/4\ C_1$

- Check :-
- $C_1 + C_2 + C_3 = 0.89'' + 3.56'' + 5.55'' = 10''$

Hence the corrected angle are :

$$
\begin{array}{r}
64^0 12' 12'' + 0.89'' \\
+\ 50^0 48' 30'' + 3.56'' \\
+\ 64^0 59' 08'' + 5.55'' \\
\hline
180^0 00' 00''
\end{array}
$$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Now put the both C_2, C_3 value in equation no 2

So

$C_1 + 4\,C_1 + 25/4\,C_1 = 10''$

$C_1\,(1 + 4 + 25/4) = 10''$

$C_1 = 10\,(4/45) = 0.89''$

$C_2 = 4\,(C_1) = 4(0.89'') = 3.56''$

$C_3 = 25/4\,(C_1) = 25/4\,(0.89'') = 5.55''$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

Determination of most probable value of quantities

- The most probable value of quantities is the value which has more chance of being true than any other value.
- This value is derived from the several different value.
- The most probable value of a quantities can be determined from the principle of least squares.

In practice, the following cases may arise of which the most probable value may be require to determine

- Direct observation of equal weight
- Direct observation of unequal weight
- Indirect observations involving unknown of equal weight
- Indirect observations involving unknown of unequal weight
- Observation equation accompanied by condition equation

Case :1 Direct observation of equal weight

- The most probable value of the directly observed quantities of observation of equal weight is the arithmetic mean of the observation.
- Thus if X_1, X_2, X_3, ……..X_n be observed value of a quantities of equal weight and X be the most probable value of that quantities.
- $$X = M = \frac{(X_1 + X_2 + X_3 + \ldots\ldots X_n)}{n}$$

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

- Ex : the following direct measurement of a base line were taken 2523.32m, 2523.25m, 2523.17m, 2523.38m, 2523.47m, 2523.68m, calculate most probable value of the length of the base line.
- Sol :-
- $X = M = \dfrac{(X_1 + X_2 + X_3 + \ldots\ldots X_n)}{n}$

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

$$X = M = \frac{2523.32 + 2523.25 + 2523.17 + 2523.38 + 2523.47 + 2523.68}{6}$$

$M = 2523.378 \text{ m}$

The most probable value of a base line length X = 2523.378 m

Case :2 Direct observation of unequal weight

- The most probable value of the directly observed quantities of observation of equal weights is the weighted arithmetic mean of the observations.
- Thus if X_1, X_2, X_3, ...X_n be observed value of a quantities and w_1, w_2, w_3, ...w_n be their weight, let X be the probable quantities.

$$X = \frac{X_1 w_1 + X_2 w_2 + X_3 w_3 + \ldots\ldots + X_n w_n}{w_1 + w_2 + w_3 + \ldots\ldots + w_n}$$

- Ex :- Find the most probable value of the angle from the following observation

$A = 76^0 35'00''$ wt 1

$A = 76^0 33'40''$ wt 2

Sol :-

$$X = \frac{X_1 w_1 + X_2 w_2}{w_1 + w_2} = \frac{(76^0 35'00'') \times 1 + (76^0 33'40'') \times 2}{1 + 2}$$

$$= 76^0 34' 6.67''$$