CHAPTER : COMPLEX NUMBER AND ANALYTIC FUNCTION	
	MODOLUS AND ARGUMENT OF COMPLEX NUMBER
(1)	Find the principal value of $\arg i$. [$Arg\ i = \frac{\pi}{2}$]
(2)	Find the principal argument of $z = \frac{-2}{1+i\sqrt{3}}$.
(3)	Determine the modulus and argument of Z^5 Where $Z = 1+i\sqrt{3}$ [$\arg\left(z^5\right) = -\frac{\pi}{3}$]
(4)	To calculate principal value of argument of following complex number a) $\sqrt{3}+i$ b) $-\sqrt{3}+i$ c) $-\sqrt{3}-i$ d) $\sqrt{3}-i$ [a) $\frac{\pi}{6}$ b) $\frac{5\pi}{6}$ c) $-\frac{5\pi}{6}$ d) $\frac{-\pi}{6}$]
(5)	Find the value of $\text{Re}(f(z))$ and $\text{Im}(f(z))$ at the indicate point Where $f(z) = \frac{1}{1-z}$ at $7+2i$.
(6)	Is $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$? Justify.
	SOLUTION OF QUADRATIC EQUATION
(7)	Find the roots of the equation $z^2 + 2iz + (2-4i) = 0$ [$z = 1+i$ or $z = -1-3i$]
(8)	Solve the Equation of $z^2 - (5+i)z + 8 + i = 0$.[$z = 3+2i$ or $z = 2-i$]
(9)	Find the roots of the equation $z^2 - (3-i)z + (2-3i) = 0$ [$z = 2+i$ or $z = 1-2i$]
	De Moirve's theorem & ROOTS OF COMPLEX NUMBER
(10)	Find and plot the square root of $4i$ [$\sqrt{4i} = \pm\left(\sqrt{2}+i\sqrt{2}\right)$]
(11)	Find and plot aii root of $\sqrt[3]{8i}$.
(12)	Show that if c is any n^{th} root of Unity other than Unity itself , then $1+c+c^2+.........+c^{n-1} = 0.$
(13)	Find and plot all the roots of $(1+i)^{1/3}$.
(14)	Find real and imaginary part of $(-1-i)^7 + (-1+i)^7$.[$\text{Real} = -\sqrt{2}$ $\text{Im}g = 0$]
	ELIMENTRY FUNCTIONS AND EXAMPIE.
(15)	Define 1)Exponential function 2) Trigonometric function 3) Hyperbolic function 4) Logarithmic function 5)Inverse trigonometric and Inverse hyperbolic function 6)Relation between hyperbolic and trigonometric functions 7)Hyperbolic identity.
(16)	Prove that $\tan^{-1} z = \frac{i}{2}\log\frac{i+z}{i-z}$.
(17)	Define $\log(x+iy)$ Determine $\log(1-i)$.
(18)	Show that $\cos(i\bar{z}) = \overline{\cos(iz)}$ for all z.
(19)	Expand $\cosh(z_1+z_2)$.

(20)	Prove that $\left\|e^{(-2z)}\right\| < 1$ if and only if $\operatorname{Re} z > 0$.
(21)	Find all Solution of sinz=2.
(22)	Show that the set of values of $\log(i^2)$ is not the same as the set of values $2\log i$.
(23)	Find the principal value of $\left[\frac{e}{2}\left(-1-i\sqrt{3}\right)\right]^{3\pi i}$.
(24)	Find all root s of the Equation $\log z = \frac{\pi}{2}$.
	FUNCTION OF COMPLEX VARIABLE
(25)	Define 1) Limit of function 2)continuous function 3)Differentiable function.
(26)	Prove $\lim_{z\to 1}\frac{iz}{3} = \frac{i}{3}$ by definition.
(27)	Use the $\varepsilon - \delta$ definition of limit to Show that where $\lim_{z\to 3i}(3x+iy^2) = 9i$ Where $z = x+iy$.
(28)	Show that the limit of the function does not exist $f(z)=\begin{cases}\frac{\operatorname{Im} g(z)}{\|z\|} & ,z\neq 0\\ 0 & ,z=0\end{cases}$
(29)	Find out and (given reason) Where $f(z)$ is continuous at $z=0$ if $f(z)=\begin{cases}\frac{\operatorname{Re}(z^2)}{\|z\|} & ,z\neq 0\\ 0 & ,z=0\end{cases}$.
(30)	Find the derivative of $\frac{z-i}{z+i}$ at i.
(31)	Show that $f(z)=z\operatorname{Im}(z)$ is differential only at $z=0$ and $f'(0)=0$.
	ANALYTIC FUNCTION
(32)	Define 1)Analytic function 2)Entire function 3)C-R Equation 4)Harmonic function.
(33)	State necessary and sufficient Condition for function to be analytics and prove that necessary Condition.
(34)	The function $f(z)=\begin{cases}\frac{\bar{z}^2}{z} & ,When\ z\neq 0.\\ 0 & ,When\ z=0.\end{cases}$ Satisfies C-R equation at the origin but $f'(0)$. fails to exist.
(35)	Check Whether the function is analytics or not. $f(z)=\bar{z}$.
(36)	Check Whether the function is analytics or not at any point. $f(z)=2x+ixy^2$
(37)	Check Whether the function is analytics or not at any point. $f(z)=e^{\bar{z}}$
(38)	Verify that $f(z)=z^2$ is analytic everywhere.
(39)	Check Whether the function is analytics or not. $f(z)=z^{\frac{5}{2}}$

(40)	Check Whether the function $f(z)=\sin z$ is analytics or not. if analytic find it's derivative.
(41)	Find the all analytic function $f(z)=u+iv$ if $u-v=(x-y)(x^2+4xy+y^2)$.
(42)	Show that if $f(z)$ is analytics in a domain D and $\lvert f(z)\rvert=k$ constant in D then show that $f(z)=const$ in D.
(43)	Let a function $f(z)$ be analytic in a domain D prove that $f(z)$ must be constant in D in each of following cases. a) if $f(z)$ is real value for all z in D b) if $\overline{f(z)}$ is analytic in D.
(44)	Define harmonic function. Show that $u=x\sin x\cosh y-y\cos x\sinh y$ is harmonic
(45)	Determine 'a' and 'b' such that $u=ax^3+bxy$ is harmonic and find Conjugate harmonic.
(46)	Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find a harmonic Conjugate $v(x,y)$.
(47)	Determine the analytic function whose imaginary part is $e^x(x\cos y-y\sin y)$.
(48)	Determine the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$.

	CHAPTER:COMPLEX VARIABLE INTEGRATION	
	LINE INTEGRATION	
(1)	Evaluate $\int_c \boldsymbol{Re(z^2)dz}$,Where c is the boundary of the square with vertices $\mathbf{0}, \boldsymbol{i}, \mathbf{1}+\boldsymbol{i}, \mathbf{1}$ in the clockwise direction.	$-1-i$
(2)	Evaluate $\int_c f(z)dz$,Where $f(z)$ is defined by $f(z)=\begin{cases}1; y<0\\ 4y; y>0\end{cases}$ & c is the arc from $z=-1-i$ to $z=1+i$ along the curve $y=x^3$	$2+3i$
(3)	Evaluate $\int_0^{4+2i} \bar{z}\, dz$ along the curve $z=t^2+it$	$10-\frac{8}{3}i$
(4)	Evaluate $\int_c \bar{z}\, dz$ from $z=1-i$ to $z=3+2i$ along the straight line.	$\frac{11}{2}+5i$
(5)	Evaluate $\int_c (x^2-iy^2)dz$,along the parabola $y=2x^2$ from $(1,2)$ to $(2,8)$	$\frac{511}{3}-\frac{49}{5}i$
(6)	Evaluate $\int_c (x-y+ix^2)dz$,Where c is a straight line from $z=0$ to $z=1+i$	$\frac{i(1+i)}{3}$
(7)	Evaluate $\int_c (x-y+ix^2)dz$,Where c is along the imaginary axis from $z=0$ to $z=i$, $z=1$ to $z=1+i$ & $z=1+i$ to $z=0$	$\frac{3i-1}{6}$
(8)	Evaluate $\int_c (x-y+ix^2)dz$,Where c is along the parabola $y^2=x$	$-\frac{11}{30}+\frac{i}{6}$
(9)	Evaluate $\int_c z^2dz$,Where c is the path joining the points $1+i$ and $2+4i$ along (i) the parabola $x^2=y$ (ii) the curve $x=t, y=t^2$	$-\frac{86}{3}-6i$
(10)	Evaluate $\int_c Re(z)dz$,Where c is a straight line from $(1,1)$ to $(3,1)$ & then from $(3,1)$ to $(3,2)$	$4+3i$
	ML-inequality	
	ML-inequality: If $f(z)$ is continuous on a contour C,then $\left\lvert\int_C f(z)dz\right\rvert \le ML$. where $\lvert f(z)\rvert \le M, z\in C$ and L is the length of the curve (contour)C.	
(11)	Find an upper bound for the absolute value of the integral $\int_c e^z dz$, where c is the line segment joining the points $(0,0)$ and $(1,2\sqrt{2})$	$3e$
(12)	Find an upper bound for the absolute value of the integral $\int_c \frac{dz}{z^4}$, where c is the line segment i to1, without actually evaluating the integral.	$4\sqrt{2}$
(13)	Find an upper bound for the absolute value of the integral $\int_c \frac{dz}{z^2+1}$, where c is the arc of a circle $\lvert z\rvert=2$ that lies in the first quadrant.	$\frac{\pi}{3}$
	Cauchy's integral theorem(Cauchy Goursat's theorem)	
	Theorem:- If $f(z)$ is an analytic function in a simply connected domain D and $f'(z)$ is continuous at each point within and on a simple closed curve C in D then $\oint_C f(z)dz=0$	
(14)	State and prove Cauchy integral theorem.	
(15)	Evaluate $\oint_C (z^2-2z-3)dz$, where C is the circle $\lvert z\rvert=2$	0

(16)	Evaluate $\oint_C \frac{z}{z-3} dz$, where C is the unit circle $\lvert z \rvert = 1$	0
(17)	Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $\lvert z+1 \rvert = 1$	0
	Cauchy's integral formula	
	Theorem: - If $f(z)$ is an analytic within and on a simple closed curve C and z_0 is any point interior to C,then $\oint_C \frac{f(z)}{z-z_0} dz = 2\pi i\, f(z_0)$ the integration being taken counterclockwise.	
(18)	Evaluate $\oint_C \frac{dz}{z^2+1}$, where C is $\lvert z+i \rvert = 1$, counterclockwise.	$-\pi$
(19)	Evaluate $\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $\lvert z \rvert = 3$.	$4\pi i$
(20)	Evaluate $\oint_C \frac{\sin 3z}{z+\frac{\pi}{2}} dz$, where C the circle is $\lvert z \rvert = 5$.	$2\pi i$
(21)	Evaluate $\oint_C \frac{e^z}{z(1-z)^3} dz$, where C is (a)$\lvert z \rvert = \frac{1}{2}$ (b)$\lvert z-1 \rvert = \frac{1}{2}$	$2\pi i, -\pi i e$
(22)	Evaluate $\oint_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $\lvert z \rvert = 3$.	$8\pi i$
(23)	Find the value of the integral $\int_C \frac{2z^2+2}{(z-1)(z^2+9)} dz$ taken counterclockwise around the circle C: $\lvert z-2 \rvert = 2$	$\frac{4}{5}\pi i$

CHAPTER: LAURENT'S SERIES , SINGULARITIES & RESIDUE		
SERIES		
Radius of convergence:-Let $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ be a power series with radius of convergence R,where $R = \lim_{n\to\infty}\left\|\frac{a_n}{a_{n+1}}\right\|$ or $R = \lim_{n\to\infty}\|a_n\|^{\frac{-1}{n}}$		
(1)	Find the radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$	$R = 0$
(2)	Discuss the convergence of $\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}(z-3i)^n$ and also find the radius of convergence.	$R = \frac{1}{4}$
(3)	Find the radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{2^n}{n!}z^n$	$R = \infty$
(4)	Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2} z^n$	$R = \frac{1}{e}$
	Taylor's series, Maclaurin series and Laurent's series	
(5)	Derive the Taylor's series representation $\frac{1}{1-z} = \sum_{n=0}^{\infty}\frac{(z-i)^n}{(1-i)^{n+1}}$,where $(\|z-i\| < \sqrt{2})$	
(6)	Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$.	
(7)	Develop $f(z) = \sin^2 z$ in a Maclaurin series and find the radius of convergence.	
(8)	Find the Maclaurin series representation of $f(z) = \sin z$ in the region $\|z\| < \infty$	
(9)	Show that when $o < \|z-1\| < 2$, $\frac{z}{(z-1)(z-3)} = \frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}}$	
(10)	Find the series of $f(z) = \frac{z}{(z-1)(z-4)}$ in terms of $(z+3)$ valid for $\|z+3\| < 4$	
(11)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the region (i)$\|z\| < 2$ (ii)$2 < \|z\| < 4$ (iii)$\|z\| > 4$	
(12)	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series in the interval $1 < \|z\| < 3$	
(13)	Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < \|z+1\| < 3$	
(14)	Expand $f(z) = -\frac{1}{(z-1)(z-2)}$ in the region (i)$\|z\| < 1$ (ii)$1 < \|z\| < 2$ (iii)$\|z\| > 2$	
(15)	Write the two Laurent series expansion in powers of z that represent the function $f(z) = \frac{1}{z^2(1-z)}$ in certain domains, and also specify domains.	
(16)	Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about $z = 0$ and identify the singularity.	
	Singularities, poles and residue	
(17)	**Definition :** **(i)Singular point:-**A point z_0 is a singular point if a function $f(z)$ is not analytic at z_0 but is analytic at some points of each neighbourhood of z_0. **(ii)Isolated point:-**A singular point z_0 of $f(z)$ is said to be isolated point if there is a neighbourhood of z_0 which contains no singular points of $f(z)$ except z_0.i.e. $f(z)$ is analytic in some deleted neighbourhood, $0 < \|z-z_0\| < \varepsilon$. For example: - $f(z) = \frac{z^2+1}{(z-1)(z-2)}$ has two isolated point $z = 1$ & $z = 2$. **(iii)Poles:-**If principal part of Laurent's series has finite number of terms, i.e.,	

	$f(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n + \frac{b_1}{z-z_0} + \frac{b_2}{(z-z_0)^2} + \ldots\ldots + \frac{b_n}{(z-z_0)^n}$, then the singularity $z = z_0$ is said to be pole of order n. If $b_1 \neq 0$ and $b_2 = b_3 = \cdots\ldots = b_n = 0$,then $f(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n + \frac{b_1}{z-z_0}$ the singularity $z = z_0$ is said to be pole of order 1 or a simple pole. **(iv)Types of singularities:-** **(a)Removable singularity:-**If in the Laurent's series expansion, the principal part is zero; i.e.,$f(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n + 0$ then the singularity $z = z_0$ is said to be removable singularity. (i.e.,$f(z)$ is not defined at $z = z_0$ but $\lim_{z\to 0} f(z)$ exists.) For example: - $f(z) = \frac{\sin z}{z}$ is undefined at $z = 0$ but $\lim_{z\to 0}\frac{\sin z}{z} = 1$.so,$z = 0$ is a removable singularity. **(b)Essential singularity:-**If in the Laurent's series expansion, the principal part contains an infinite number of terms, then the singularity $z = z_0$ is said to be an essential singularity. For example: - $f(z) = \sin\frac{1}{z}$ has an essential singularity at $z = 0$,since $\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!z^3} + \frac{1}{5!z^5} + \cdots\ldots$ **(v)Residue of a function:-** If $f(z)$ has a pole at the point $z = z_0$ then the coefficient b_1 of the term $(z-z_0)^{-1}$ in the Laurent's series expansion of $f(z)$ at $z = z_0$ is called the residue of $f(z)$ at $z = z_0$.Residue of $f(z)$ at $z = z_0$ is denoted by $\underset{z=z_0}{Res} f(z)$.	
(18)	Classify the poles of $\boldsymbol{f(z) = \frac{1}{z^2-z^6}}$	
(19)	Define residue at simple pole and find the sum of residues of the function $\boldsymbol{f(z) = \frac{\sin z}{z\cos z}}$ at its poles inside the circle $\boldsymbol{\|z\| = 2}$.	
(20)	Find the residue at $z = 0$ of $\boldsymbol{(z) = z\cos\frac{1}{z}}$.	
(21)	Show that the singular point of the function $\boldsymbol{f(z) = \frac{1-\cosh z}{z^3}}$ is a pole. Determine the order m of that pole and corresponding residue.	
(22)	Find the residue at $z = 0$ of $\boldsymbol{(z) = \frac{1-e^z}{z^3}}$.	
Cauchy Residue Theorem and Application of Residues		
Cauchy's residue theorem:-If $f(z)$ is analytic in a closed curve C except at a finite number of singular points within C,then $\int_C f(z)\,dz = 2\pi i(sum\ of\ the\ residue\ at\ the\ singular\ points)$		
(23)	Using residue theorem, evaluate $\oint_C \frac{z^2\sin z}{4z^2-1}dz, C: \|z\| = 2$.	$\frac{\pi i}{4}\sin\frac{1}{2}$
(24)	State Cauchy's residue theorem and evaluate $\int_C \frac{5z-2}{z(z-1)}dz$,where C is the circle $\|z\| = 2$	$10\pi i$
(25)	Evaluate $\int_C \frac{dz}{(z^2+1)^2}$, where $C: \|z+i\| = 1$	$\frac{\pi}{2}$
(26)	Determine the poles of the function $\boldsymbol{f(z) = \frac{z^2}{(z-1)^2(z+2)}}$ and residue at each pole.	$2\pi i$

	Hence evaluate $\int_C f(z)\,dz$ where C is the circle $\lvert z\rvert = 3$.	
(27)	Evaluate $\oint_C \frac{dz}{\sinh 2z}$,where $C: \lvert z\rvert = 2$	$-\pi i$
(28)	Use residues to evaluate the integrals of the function $\frac{\exp(-z)}{z^2}$ around the circle $\lvert z\rvert = 3$ in the positive sense.	$-\pi i$
(29)	Find the value of the integral $\int_C \frac{2z^2+2}{(z-1)(z^2+9)} dz$ taken counterclockwise around the circle $C: \lvert z-2\rvert = 2$	$\frac{4}{5}\pi i$
(30)	Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$	$\frac{3\pi}{2}$
(31)	Using the residue theorem, evaluate $\int_0^{2\pi} \frac{4\,d\theta}{5+4\sin\theta}$	$\frac{2\pi}{3}$
(32)	Evaluate $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta}$.	
(33)	Evaluate $\int_0^{\pi} \frac{d\theta}{17-8\cos\theta}$, by integrating around a unit circle.	$\frac{\pi}{15}$
(34)	Use residues to evaluate $\int_0^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$	$\frac{\pi}{6}$
(35)	Let $a > b > 0$.Prove that $\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a^2-b^2}\left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a}\right)$	
Rouche's Theorem		
Theorem: If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C and if $\lvert g(z)\rvert < \lvert f(z)\rvert$ on C,then $f(z) + g(z)$and $f(z)$ have the same number of zeros inside C.		
(36)	Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $\lvert z\rvert = 1$ and $\lvert z\rvert = 2$ using Rouche's theorem.	
(37)	Use Rouche's theorem to determine the number of zeros of the polynomial $z^6 - 5z^4 + z^3 - 2z$ inside the circle $\lvert z\rvert = 1$.	