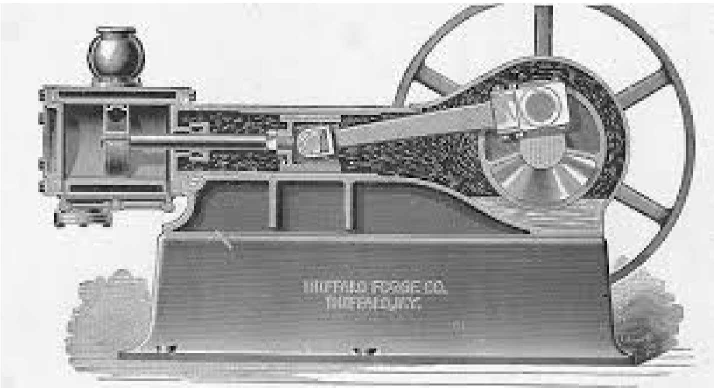
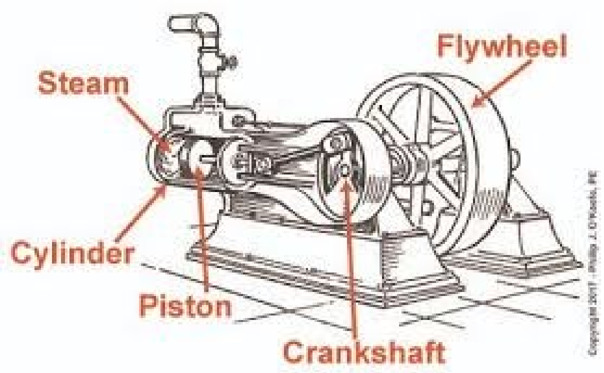
COLLEGE OF ENGINEERING & TECHNOLOGY

Dynamic force analysis of mechanisms





Subject:- DOM Code:- 3151911	Prepared by: Asst.Prof.Chirag Mevada (Mechanical Department,ACET)	AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering
Subject Code: 3151911
Semester – V
DYNAMICS OF MACHINERY

Type of course: Professional Core

Prerequisite: Kinematics and theory of machines

Rationale: The course is designed to give fundamental knowledge of behavior of machines under dynamic condition.

Teaching and Examination Scheme:

Teaching Scheme			Credits	Examination Marks				Total Marks
L	T	P	C	Theory Marks		Practical Marks		
				ESE (E)	PA (M)	ESE (V)	PA (I)	
4	0	2	5	70	30	30	20	150

Content:

Sr. No.	Content	Total Hrs
1	**Dynamic force analysis of mechanisms:** Introduction, D'alembert's principle, equivalent offset inertia force, dynamic analysis of four link mechanism, dynamic analysis of slider crank mechanism, velocity & acceleration of piston, angular velocity & angular acceleration of connecting rod, engine force analysis, dynamically equivalent system inertia of the connecting rod, inertia force in reciprocating engines.	04
2	**Turning moment diagrams and flywheel** Turning moment diagram for various type of engines, fluctuation of energy, fluctuation of speed, flywheel, energy stored in flywheel, dimensions of flywheel rims, flywheel in punching presses	04
3	**Balancing:** Introduction, static balancing, dynamic balancing, transference of force from one plane to another plane, balancing of several masses in different planes, force balancing of linkages, balancing of reciprocating mass, balancing of locomotives, Effects of partial balancing in locomotives, secondary balancing, balancing of inline engines, balancing of v-engines, balancing of radial engines, balancing machines.	11
4	**Gyroscope:** Angular velocity, angular acceleration, gyroscopic torque, gyroscopic effect on naval ships, aero plane, stability of an automobile, stability of two wheel vehicle	05
5	**Free vibrations and damped free vibrations:** Types of vibrations, elements constituting vibration, spring mass system, free undamped vibrations, equation of motion, equivalent spring stiffness, free damped vibrations, equation of motion for viscous damper, damping factor, under damped system, critically damped system, over damped system, logarithmic decrement, free torsional vibration of a two and three rotor system, torsionally equivalent shaft, torsional vibration of a geared system.	12
6	**Forced damped vibrations:** Analytical solution of forced damped vibration, vector representation of forced vibrations, Magnification factor, force transmissibility, forced vibration with rotating and reciprocating unbalance, forced vibration due to excitation of support, vibration frequency measurement.	08
7	**Critical speeds of shafts:** Critical speed of shaft carrying single rotor and having no damping, Critical speed of shaft carrying single rotor and having damping, secondary critical speeds in horizontal shafts, critical speed of shaft having multiple rotors.	05

❑ Introduction

- Dynamic forces are associated with accelerating masses.
- As all machines have some accelerating parts, dynamic forces are always present when the machines operate.
- In situations where dynamic forces are dominant or comparable with magnitudes of external forces and operating speeds are high, dynamic analysis has to be carried out.
- For example, in case of rotors which rotate at speeds more than 80,000 rpm, even the slightest eccentricity of the centre of mass from the axis of rotation produces very high dynamic forces.
- This may lead to vibrations, wear, noise or even machine failure.

❑ D' ALEMBERT'S PRINCIPLE

- D' Alembert's principle states that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.
- Inertia is a property of matter by virtue of which a body resists any change in velocity.

$$\text{Inertia force } \mathbf{F}_i = -m\,\mathbf{f}_g$$

where m = mass of body

$\mathbf{f}_g$ = acceleration of centre of mass of the body

- The negative sign indicates that the force acts in the opposite direction to that of the acceleration. The force acts through the centre of mass of the body.
- Similarly, an inertia couple resists any change in the angular velocity.

Inertia couple,

$$\mathbf{C}_i = -I_g\alpha$$

where I_g = moment of inertia about an axis passing through the centre of mass G and perpendicular to plane of rotation of the body

α = angular acceleration of the body

Let $\Sigma\mathbf{F} = \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, etc. = external forces on the body

and $\Sigma\mathbf{T} = \mathbf{T}_{g1}, \mathbf{T}_{g2}, \mathbf{T}_{g3}$, etc. = external torques on the body about the centre of mass G.

D' ALEMBERT'S PRINCIPLE

- According to D' Alembert's principle, the vector sum of forces and torques (or couples) has to be zero, i.e.,

$$\Sigma \mathbf{F} + \mathbf{F}_i = 0$$

- and

$$\Sigma \mathbf{T} + \mathbf{C}_i = 0$$

These equations are similar to the equation of a body in static equilibrium, i.e., $\Sigma \mathbf{F} = 0$ and $\Sigma \mathbf{T} = 0$.

- This suggests that first the magnitudes and the directions of inertia forces and couples can be determined, after which they can be treated just like static loads on the mechanism. Thus, a dynamic analysis problem is reduced to one requiring static analysis.

❑ EQUIVALENT OFFSET INERTIA FORCE

- In plane motions involving accelerations, the inertia force acts on a body through its centre of mass.
- However, if the body is acted upon by forces such that their resultant does not pass through the centre of mass, a couple also acts on the body.
- In graphical solutions, it is possible to replace inertia force and inertia couple by an equivalent offset inertia force which can account for both. This is done by displacing the line of action of the inertia force from the centre of mass.
- The perpendicular displacement h of the force from the centre of mass is such that the torque so produced is equal to the inertia couple acting on the body,

i.e. $$T_i = C_i$$

or $$F_i \times h = C_i$$

or $$h = \frac{C_i}{F_i} = \frac{-I_g\alpha}{-mf_g} = \frac{mk^2\alpha}{mf_g} = \frac{k^2\alpha}{f_g}$$

h is taken in such a way that the force produces a moment about the centre of mass, which is opposite in sense to the angular acceleration α.

•S.S. Ratan, Theory of Machines, 3rd

❑ DYNAMIC ANALYSIS OF FOUR-LINK MECHANISMS

For dynamic analysis of four-link mechanisms, the following procedure may be adopted:

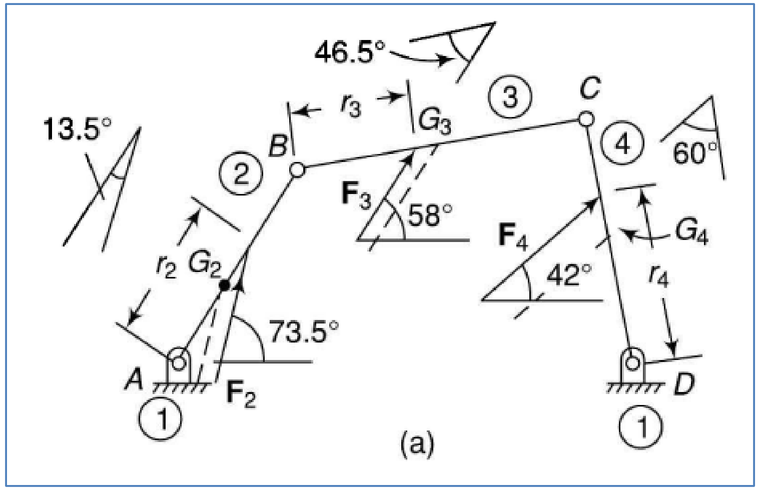
1. Draw the velocity and acceleration diagrams of the mechanism from the configuration diagram by usual methods.
2. Determine the linear acceleration of the centres of masses of various links, and also the angular accelerations of the links.
3. Calculate the inertia forces and inertia couples from the relations $\mathbf{F}_i = -m\mathbf{f}_g$ and $\mathbf{C}_i = -I_g\alpha$.
4. Replace $\mathbf{F}_i$ with equivalent offset inertia force to take into account $\mathbf{F}_i$ as well as $\mathbf{C}_i$.
5. Assume equivalent offset inertia forces on the links as static forces and analyse the mechanism by any of the methods.

•S.S. Ratan, Theory of Machines, 3rd

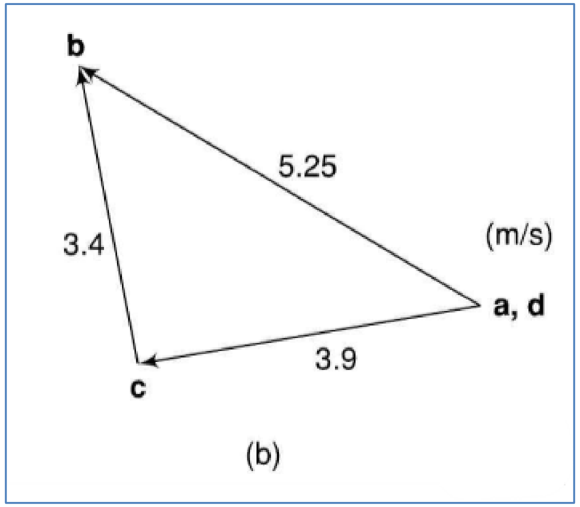
❑ Example:-

The dimensions of a four-link mechanism are AB = 500 mm, BC = 660 mm, CD= 560 mm and AD= 1000 mm. The link AB has an angular velocity of 10. 5 rad/s counter-clockwise and an angular retardation of 26 rad/s^2 at the instant when it makes an angle of 60° with AD, the fixed link. The mass of the links BC and CD is 4.2 kg/ m length. The link AB has a mass of 3.54 kg, the centre of which lies at 200 mm from A and a moment of inertia of 88,500 kg.mm^2. Neglecting gravity and friction effects, determine the instantaneous value of the drive torque required to be applied on AB to overcome the inertia forces.

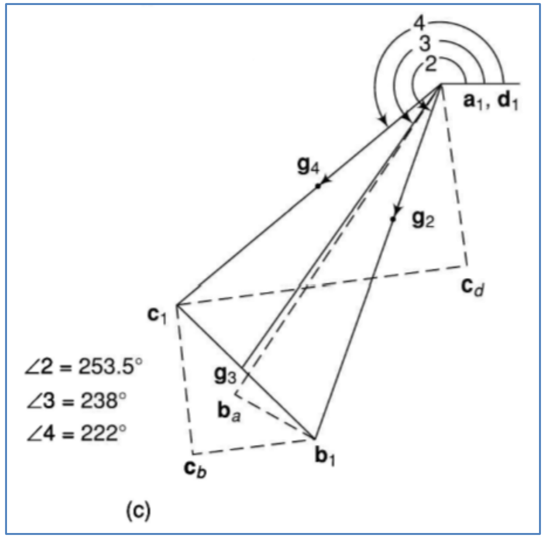
Solution:- The required input torque 23.5 N.m (counterclockwise)



Configuration Diagram



Velocity Diagram

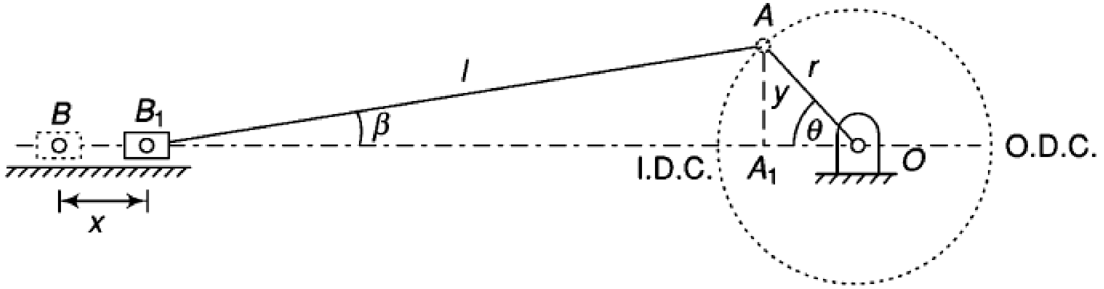


Acceleration Diagram

DYNAMIC ANALYSIS OF SLIDER-CRANK MECHANISMS

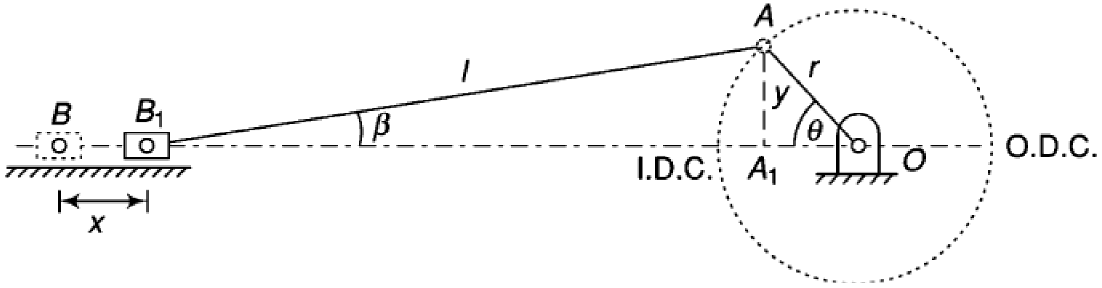
- The steps outlined for dynamic analysis of a four-link mechanism also hold good for a slider-crank mechanism and the analysis can be carried out in exactly the same manner.
- However, an analytical approach is also being described in detail in the following sections.

VELOCITY AND ACCELERATION OF A PISTON



- Figure shows a slidercrank mechanism in which the crank OA rotates in the clockwise direction. l and r are the lengths of the connecting rod and the crank respectively.
- Let x = displacement of piston from inner-dead centre
- At the moment when the crank has turned through angle θ from the inner-dead centre,

❑ VELOCITY AND ACCELERATION OF A PISTON



$$x = B_1B = BO - B_1O$$
$$= BO - (B_1A_1 + A_1O)$$
$$= (l + r) - (l\cos\beta + \cos\theta)$$
$$= (nr + r) - (nr\cos\beta + r\cos\theta) \qquad \text{(taking } l/r = n\text{)}$$
$$= r[(n + 1) - (n\cos\beta + \cos\theta)]$$

where
$$\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \frac{y^2}{l^2}} = \sqrt{1 - \frac{(r\sin\theta)^2}{l^2}} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

If the connecting rod is very large as compared to the crank, n^2 will be large and the maximum value of $\sin^2\theta$ can be unity. Then $\sqrt{n^2 - \sin^2\theta}$ will be approaching $\sqrt{n^2}$ or n, and

$$x = r(1 - \cos\theta)$$

This is the expression for a simple harmonic motion. Thus, the piston executes a simple harmonic motion when the connecting rod is large.

❑ VELOCITY OF PISTON

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}[r\{(1-\cos\theta)+n-(n^2-\sin^2\theta)^{1/2}\}]\frac{d\theta}{dt}$$

$$= r[(0+\sin\theta)+0-\frac{1}{2}(n^2-\sin^2\theta)^{1/2}(-2\sin\theta\cos\theta)]\omega$$

$$= r\omega\left[\sin\theta+\frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

If n^2 is large compared to $\sin^2\theta$,

$$v = r\omega\left[\sin\theta+\frac{\sin 2\theta}{2n}\right]$$

If $\frac{\sin 2\theta}{2n}$ can be neglected (when n is quite large),

$$v = r\omega\sin\theta$$

❑ ACCELERATION OF PISTON

$$f = \frac{dv}{dt} = \frac{dv d\theta}{d\theta dt}$$

$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]\omega$$

$$= r\omega\left(\cos\theta + \frac{2\cos 2\theta}{2n}\right)\omega$$

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

If n is very very large,

$f = r\omega^2 \cos\theta$ as in case of SHM

When $\theta = 0°$, i.e., at IDC, $f = r\omega^2\left(1 + \frac{1}{n}\right)$

When $\theta = 180°$, i.e., at ODC, $f = r\omega^2\left(-1 + \frac{1}{n}\right)$

At $\theta = 180°$, when the direction of motion is reversed,

$$f = r\omega^2\left(1 - \frac{1}{n}\right)$$

Note that this expression of acceleration has been obtained by differentiating the approximate expression for the velocity. It is, usually, very cumbersome to differentiate the exact expression for velocity. However, this gives satisfactory results.

❑ ANGULAR VELOCITY AND ANGULAR ACCELERATION OF CONNECTING ROD

As $y = l \sin\beta = r \sin\theta$

$$\therefore \quad \sin\beta = \frac{\sin\theta}{n}$$

Differentiating with respect to time,

$$\cos\beta \frac{d\beta}{dt} = \frac{1}{n}\cos\theta \frac{d\theta}{dt}$$

$$\frac{d\beta}{dt} = \frac{\cos\theta}{n\cos\beta}\omega$$

or

$$\omega_c = \omega \frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}}$$

where ω_c is the angular velocity of the connecting rod

$$= \omega \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Let α_c = angular acceleration of the connecting rod

$$= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}[\cos\theta\,(n^2 - \sin^2\theta)^{-1/2}]\omega$$

$$= \omega^2[-\cos\theta \frac{1}{2}(n^2 - \sin^2\theta)^{-3/2}(-2\sin\theta\cos\theta) + (n^2 - \sin^2\theta)^{-1/2}(-\sin\theta)]$$

$$= \omega^2 \sin\theta \left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$= -\omega^2 \sin\theta \left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β. Thus, in the given case, the angular acceleration of the connecting rod is clockwise.

•S.S. Ratan, Theory of Machines, 3rd

❑ ENGINE FORCE ANALYSIS

- An engine is acted upon by various forces such as weight of reciprocating masses and connecting rod, gas forces, forces due to friction and inertia forces due to acceleration and retardation of engine elements, the last being dynamic in nature.
- In this section, the analysis is made of the forces neglecting the effect of the weight and the inertia effect of the connecting rod.

(i) Piston Effort (Effective Driving Force)

- The piston effort is termed as the net or effective force applied on the piston.
- In reciprocating engines, the reciprocating masses accelerate during the first half of the stroke and the inertia force tends to resist the same.
- Thus, the net force on the piston is decreased.
- During the later half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of the applied gas pressure and thus, increases the effective force on the piston.
- In a vertical engine, the weight of the reciprocating masses assists the piston during the outstroke (down stroke), thus, increasing the piston effort by an amount equal to the weight of the piston.
- During the instroke (upstroke), the piston effort is decreased by the same amount.

(i) Piston Effort (Effective Driving Force)

Let A_1 = area of the cover end

A_2 = area of the piston rod end

P_1 = pressure on the cover end

P_2 = pressure on the rod end

m = mass of the reciprocating parts

Force on the piston due to gas pressure, $F_p = p_1 A_1 - p_2 A_2$

Inertia force, $F_b = mf = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$

which is in the opposite direction to that of the acceleration of the piston.

Net (effective) force on the piston, $F = F_p - F_b$

In case friction resistance F_f is also taken into account,

Force on the piston, $F = F_p - F_b - F_f$

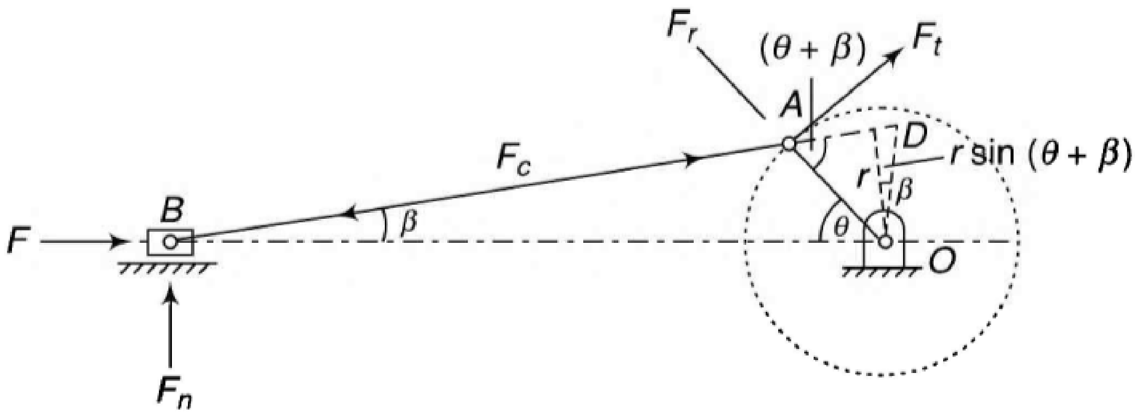
In case of vertical engines, the weight of the piston or reciprocating parts also acts as force and thus force on the piston, $F = F_p + mg - F_b - F_f$

(ii) Force (thrust) along the Connecting Rod

Let F_c = Force in the connecting rod

Then equating the horizontal components of forces,

$$F_c \times \cos\beta = F \quad \text{or} \quad F_c = \frac{F}{\cos\beta}$$



(iii) Thrust on the Sides of Cylinder

It is the normal reaction on the cylinder walls.

$$F_n = F_c \sin\beta = F \tan\beta$$

•S.S. Ratan, Theory of Machines, 3rd

(iv) Crank Effort

Force is exerted on the crankpin as a result of the force on the piston. *Crank effort* is the net effort (force) applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft.

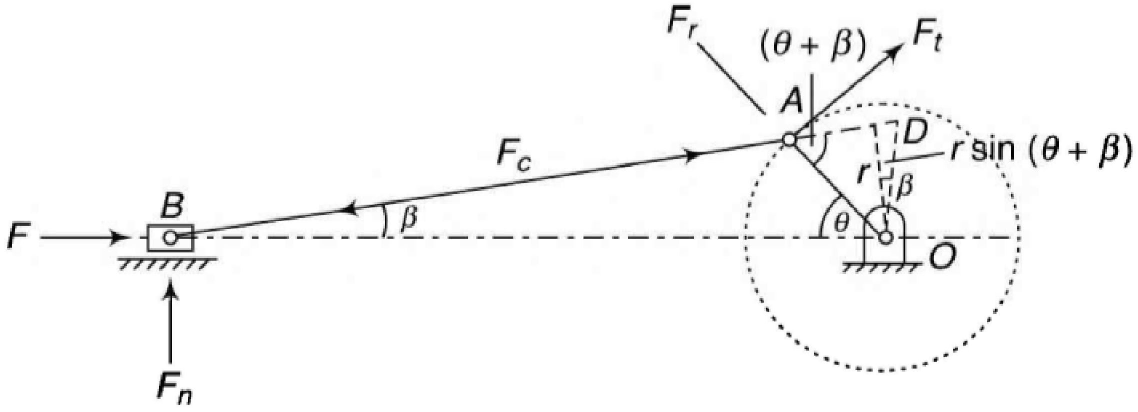
Let F_t = crank effort

As

$$F_t \times r = F_c \, r \sin(\theta + \beta)$$

$$F_t = F_c \sin(\theta + \beta)$$

$$= \frac{F}{\cos\beta} \sin(\theta + \beta)$$



(v) Thrust on the Bearings

The component of F_c along the crank (in the radial direction) produces a thrust on the crankshaft bearings.

$$F_r = F_c \cos(\theta + \beta) = \frac{F}{\cos\beta} \cos(\theta + \beta)$$

❑ TURNING MOMENT ON CRANKSHAFT

$$T = F_t \times r = \frac{F}{\cos\beta}\sin(\theta+\beta)\times r$$

$$= \frac{Fr}{\cos\beta}(\sin\theta\cos\beta + \cos\theta\sin\beta)$$

$$= Fr\left(\sin\theta + \cos\theta\sin\beta\frac{1}{\cos\beta}\right)$$

$$= Fr\left(\sin\theta + \cos\theta\frac{\sin\theta}{n}\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{2\sin\theta\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

Also, as $r\sin(\theta+\beta) = OD\cos\beta$

$$T = F_t \times r$$

$$= \frac{F}{\cos\beta} r\sin(\theta+\beta)$$

$$= \frac{F}{\cos\beta}(OD\cos\beta)$$

$$= F \times OD$$

Example *A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m² and 60 kN/m² respectively. Diameter of the piston rod is 40 mm. Determine*

(i) turning moment on the crank shaft

(ii) thrust on the bearings

(iii) acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW

Solution

$r = 0.44/2 = 0.22$ m $\quad l = 0.924$ m

$N = 210$ rpm $\quad m = 20$ kg

$\theta = 30°$

$n = l/r = 0.924/0.22 = 4.2$

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 30°}{4.2} = 0.119 \quad \text{or} \quad \beta = 6.837°$$

$$F_p = (p_1A_1 - p_2A_2)$$

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

Piston effort, $F = F_p - F_b$

Example *A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m² and 60 kN/m² respectively. Diameter of the piston rod is 40 mm. Determine*

(i) turning moment on the crank shaft

(ii) thrust on the bearings

(iii) acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW

Solution

$r = 0.44/2 = 0.22$ m $l = 0.924$ m

$N = 210$ rpm $m = 20$ kg

$\theta = 30°$

$n = l/r = 0.924/0.22 = 4.2$

(i) Turning moment, $T = \dfrac{F}{\cos\beta}\sin(\theta + \beta) \times r$

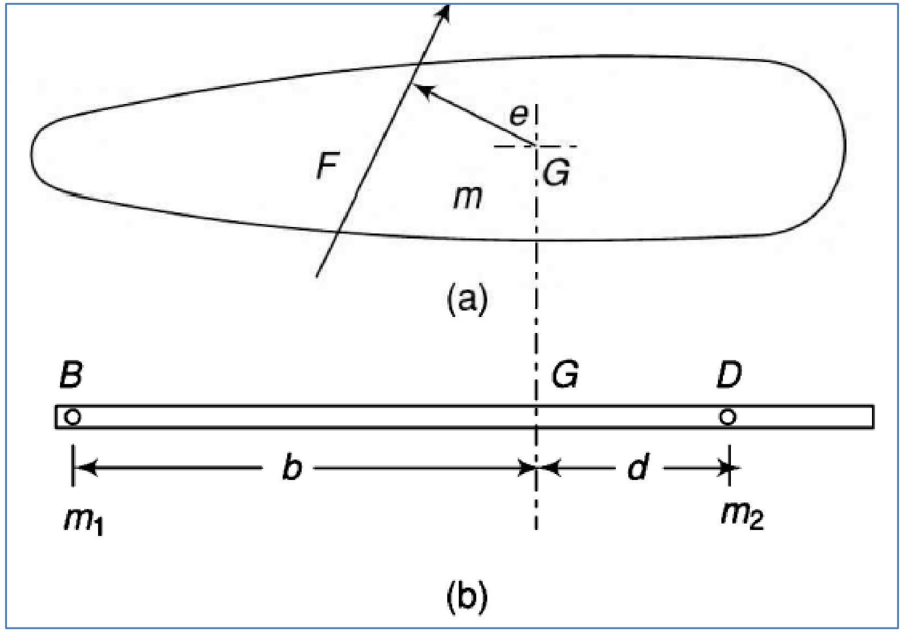
❑ DYNAMICALLY EQUIVALENT SYSTEM

- In the previous section, the expression for the turning moment of the crankshaft has been obtained for the net force F on the piston.
- This force F may be the gas force with or without the consideration of inertia force acting on the piston.
- As the mass of the connecting rod is also significant, the inertia due to the same should also be taken into account.
- As neither the mass of the connecting rod is uniformly distributed nor the motion is linear, its inertia cannot be found as such.
- Usually, the inertia of the connecting rod is taken into account by considering a dynamically-equivalent system.
- A dynamically equivalent system means that the rigid link is replaced by a link with two point masses in such a way that it has the same motion as the rigid link when subjected to the same force, i.e., the centre of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.

•S.S. Ratan, Theory of Machines, 3rd

❑ DYNAMICALLY EQUIVALENT SYSTEM

- Figure(a) shows a rigid body of mass *m* with the centre of mass at G. Let it be acted upon by a force F which produces linear acceleration *f* of the centre of mass as well as the angular acceleration of the body as the force F does not pass through G.



As we know, $F = m.f$ and $F.e = I.\,\alpha$

Acceleration of G, $f = \dfrac{F}{m}$

Angular acceleration of the body, $\alpha = \dfrac{F.e}{I}$

where e = perpendicular distance of F from G

and I = moment of inertia of the body about perpendicular axis through G

- Now to have the dynamically equivalent system, let the replaced massless link [Fig. (b)] has two point masses m_1 (at B and m_2 at D) at distances b and d respectively from the centre of mass G as shown in Fig. (b).

❑ DYNAMICALLY EQUIVALENT SYSTEM

1. To satisfy the first condition, as the force *F* is to be same, the sum of the equivalent masses m_1 and m_2 has to be equal to *m* to have the same acceleration. Thus,

$$m = m_1 + m_2$$

2. To satisfy the second condition, the numerator *F*e* and the denominator *I* must remain the same. *F* is already taken same, Thus, *e* has to be same which means that the perpendicular distance of *F* from *G* should remain same or the combined centre of mass of the equivalent system remains at *G*. This is possible if

$$m_1 b = m_2 d$$

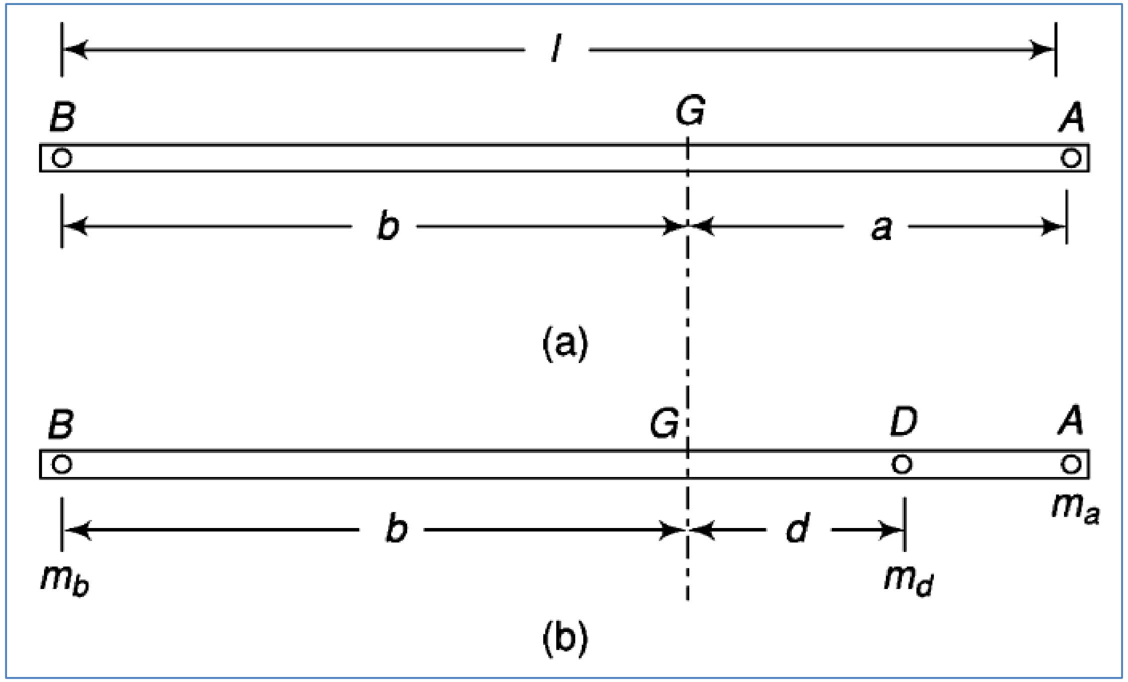
- To have the same moment of inertia of the equivalent system about perpendicular axis through their combined centre of mass *G*, we must have

$$I = m_1 b_2 + m_2 d_2$$

- Thus, any distributed mass can be replaced by two point masses to have the same dynamical properties if the following conditions are fulfilled:
  - (i) The sum of the two masses is equal to the total mass.
  - (ii) The combined centre of mass coincides with that of the rod.
  - (iii) The moment of inertia of two point masses about the perpendicular axis through their combined centre of mass is equal to that of the rod.

❑ INERTIA OF THE CONNECTING ROD

- Let the connecting rod be replaced by an equivalent massless link with two point masses as shown in Fig.
- Let m be the total mass of the connecting rod and one of the masses be located at the small end B.
- Let the second mass be placed at D and



m_b = mass at B

m_d = mass at D

Take, $BG = b$ and $DG = d$

Then

$$m_b + m_d = m$$

and $$m_b.b = m_d.d$$

From (i) and (ii)

$$m_b + \left(m_b \frac{b}{d}\right) = m$$

or $$m_b\left(1 + \frac{b}{d}\right) = m$$ or $$m_b = m\frac{d}{b+d}$$

Similarly,

$$m_d = m\frac{b}{b+d}$$

❑ INERTIA OF THE CONNECTING ROD

$$I = m_b b^2 + m_d d^2$$

$$= m\frac{d}{b+d}b^2 + m\frac{b}{b+d}d^2$$

$$= mbd\left(\frac{b+d}{b+d}\right) \quad = mbd$$

- Let k = radius of gyration of the connecting rod about an axis through the centre of mass G perpendicular to the plane of motion.
- Then $mk^2 = mbd$
- or $k^2 = bd$
- This result can be compared with that of an equivalent length of a simple pendulum in the following manner:
- The equivalent length of a simple pendulum is given by

$$L = \frac{k^2}{b} + b = d + b \qquad \left(\frac{k^2}{b} = d\right)$$

- where b is the distance of the point of suspension from the centre of mass of the body and k is the radius of gyration.
- Thus, in the present case, d + b (= L) is the equivalent length if the rod is suspended from the point B, and D is the centre of oscillation or percussion.
- However, in the analysis of the connecting rod, it is much more convenient if the two point masses are considered to be located at the centre of the two end bearings, i.e., at A and B.

Let m_a = mass at A, distance $AG = a$

$$m_a = m\frac{b}{a+b} = m\frac{b}{l}$$

(l = length of rod)

$$m_b = m\frac{a}{a+b} = m\frac{a}{l}$$

$$I' = mab$$

Assuming $a > d, I' > I$

❑ INERTIA OF THE CONNECTING ROD

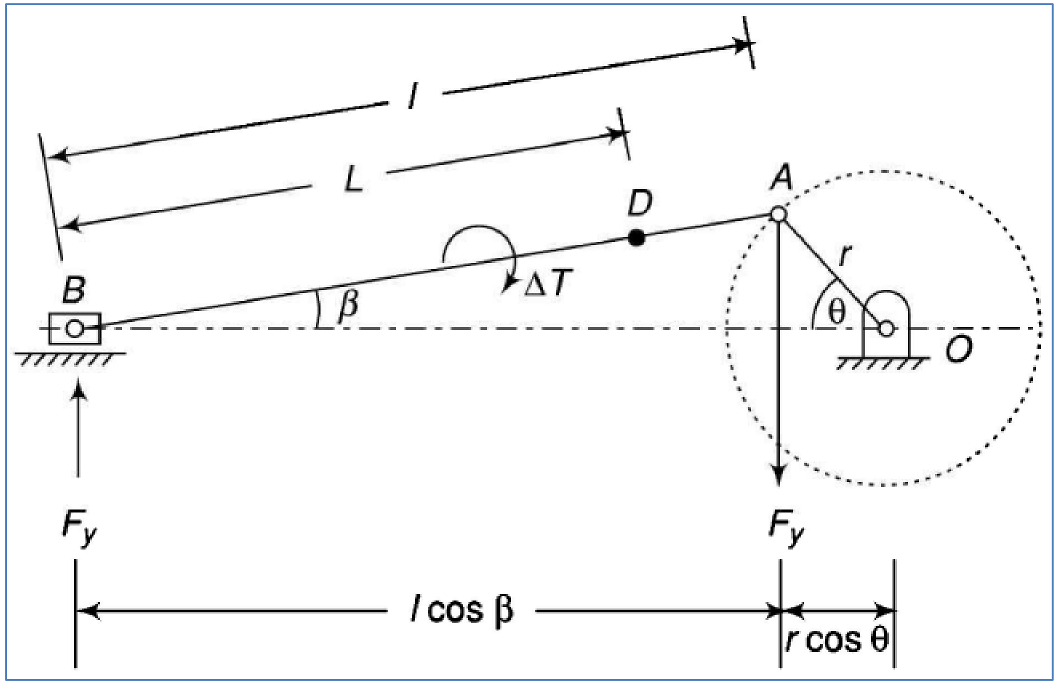
This means that by considering the two masses at A and B instead of at D and B, the inertia torque is increased from the actual value ($T = I\alpha_c$). The error is corrected by incorporating a correction couple.

Then,

$$\text{correction couple, } \Delta T = \alpha_c\,(mab - mbd)$$
$$= mb\alpha_c\,(a - d)$$
$$= mb\alpha_c\,[(a + b) - (b + d)]$$
$$= mb\alpha_c\,(l - L) \qquad \text{(taking } b + d = L\text{)}$$

This correction couple must be applied in the opposite direction to that of the applied inertia torque. As the direction of the applied inertia torque is always opposite to the direction of the angular acceleration, the direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β.

The correction couple will be produced by two equal, parallel and opposite forces F_y acting at the gudgen pin and crankpin ends perpendicular to the line of stroke Fig. The force at B is taken by the reaction of guides.



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❑ INERTIA OF THE CONNECTING ROD

Turning moment at crankshaft due to force at A or correction torque,

$$T_c = F_y \times r\cos\theta$$

$$= \frac{\Delta T}{l\cos\beta} \times r\cos\theta \qquad (\because \Delta T = F_y l\cos\beta)$$

$$= \frac{\Delta T}{(l/r)}\frac{\cos\theta}{\cos\beta} = \Delta T\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2-\sin^2\theta}} = \Delta T\frac{\cos\theta}{\sqrt{n^2-\sin^2\theta}}$$

This correction torque is to be deducted from the inertia torque acting on the crankshaft.

Also, due to the weight of the mass at A, a torque is exerted on the crankshaft which is given by

$$T_a = (m_a g) r\cos\theta$$

In case of vertical engines, a torque is also exerted on the crankshaft due to the weight of mass at B and the expression will be

$$T_b = (m_b g)\, r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

•S.S. Ratan, Theory of Machines, 3rd

❑ INERTIA OF THE CONNECTING ROD

The net torque or turning moment on the crankshaft will be the algebraic sum of the

(i) turning moment due to the force of gas pressure (T)

(ii) inertia torque due to the inertia force at the piston as a result of inertia of the reciprocating mass including the mass of the portion of the connecting rod (T_b)

(iii) inertia torque due to the weight (force) of the mass at the crank pin which is the portion of the mass of the connecting rod taken at the crank pin (T_a).

(iv) inertia torque due to the correction couple (T_c)

(v) turning moment due to the weight (force) of the piston in case of vertical engines

Usually, it is convenient to combine the forces at the piston occurring in (ii) and (v).

❑ INERTIA FORCE IN RECIPROCATING ENGINES (GRAPHICAL METHOD)

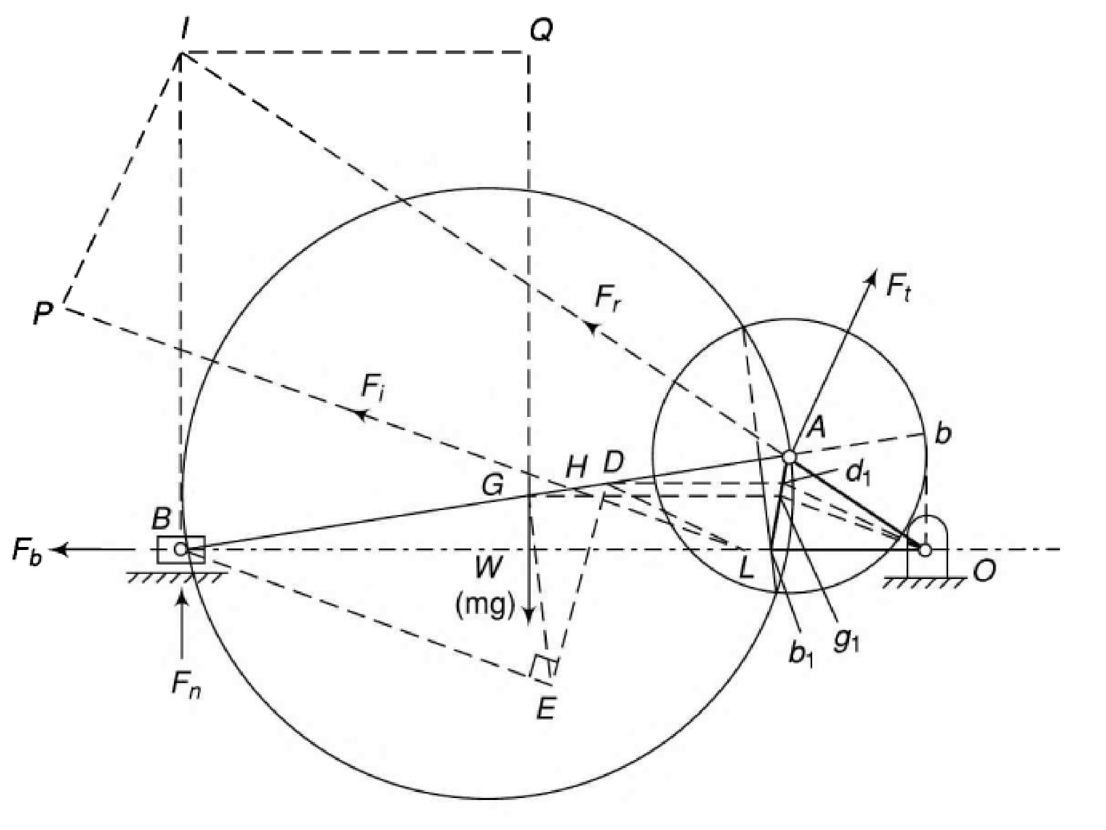
The inertia forces in reciprocating engines can be obtained graphically as follows

1. Draw the acceleration diagram by Klein's construction

   Remember that the acceleration diagram is turned through 180° from the actual diagram and therefore, the directions of accelerations are towards *O*

2. Replace the mass of the connecting rod by a dynamically equivalent system of two masses. If one mass is placed at *B*, the other will be at *D* given by $d = k^2/b$, where *k* is the radius of gyration and *b* and *d* are the distances of the centre of mass from *B* and *D* respectively.

   Point D can also be obtained graphically. Draw $GE \perp AB$ at *G* and take $GE = k$. Make $\angle BED = 90°$, and obtain the point *D* on *AB*.



(a)

3. Obtain the accelerations of points *G* and *D* from the acceleration diagram by locating the points g_1 and d_1 on Ab_1 which represents the total acceleration of the connecting rod.

   As Ad_1/AD and Ag_1/AG are equal to Ab_1/AB, Dd_1 and Gg_1 can be drawn parallel to *OB*. Thus, d_1O and g_1O represent accelerations of points *D* and *G* respectively.

•S.S. Ratan, Theory of Machines, 3rd

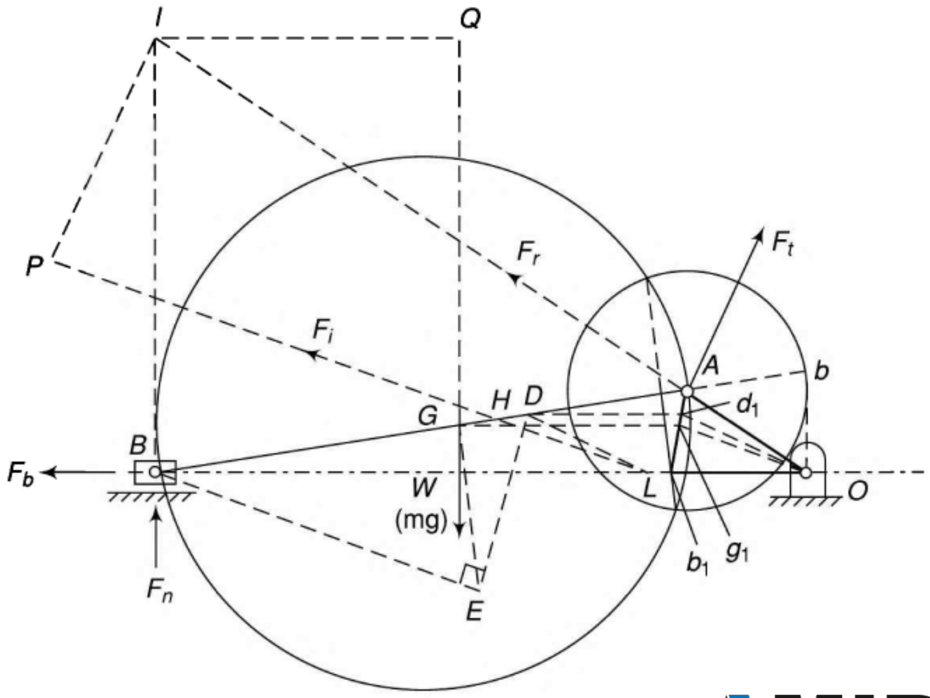
❑ INERTIA FORCE IN RECIPROCATING ENGINES (GRAPHICAL METHOD)

4. The acceleration of the mass at B is along BO and in the direction B to O. Therefore, the inertia force due to this mass acts in the opposite direction.
5. The acceleration of the mass at D is parallel to d_1O and in the direction d_1 to O, therefore, the inertia force due to this mass acts in the opposite direction through D. Draw a line parallel to Od_1 through D to represent the direction of the inertia force.

Let the lines of action of the two inertia forces due to masses at B and D meet at L. Then the resultant of the forces which is the total inertia force of the connecting rod and is parallel to Og_1 must also pass through the point L. Therefore, draw a line parallel to Og_1 through L to represent the direction of the inertia force of the connecting rod.

Now, the connecting rod is under the action of the following forces:

- Inertia force of reciprocating part $\mathbf{F}_b$ along OB
- The reaction of the guide $\mathbf{F}_n$ (magnitude and direction sense unknown)
- Inertia force of the connecting rod $\mathbf{F}_i$
- The weight of the connecting rod $\mathbf{W}$ (= mg)
- Tangential force $\mathbf{F}_t$ at the crank pin (to be found)
- Radial force $\mathbf{F}_r$ at the crank pin along OA (magnitude and direction sense unknown).



(a)

❑ INERTIA FORCE IN RECIPROCATING ENGINES (GRAPHICAL METHOD)

Produce the lines of action of $\mathbf{F}_i$ and $\mathbf{F}_n$ to meet at I, the instantaneous centre of the connecting rod. Draw IP and IQ perpendicular to the lines of action of $\mathbf{F}_i$ and the weight $\mathbf{W}$ respectively.

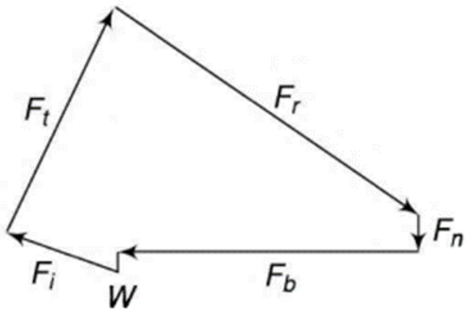
For the equilibrium of the connecting rod, taking moments about I,

$$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$$

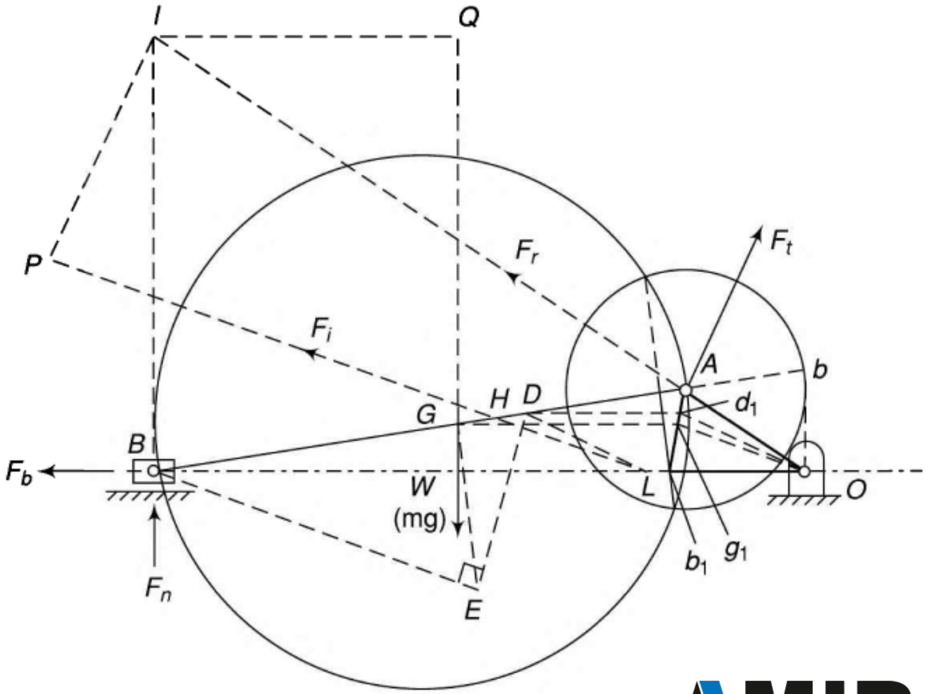
Obtain the value of F_t from it and draw the force polygon to find the magnitudes and directions of forces $\mathbf{F}_r$ and $\mathbf{F}_n$

In the above equation, F_t is the force required for the static equilibrium of the mechanism or it is the force required at the crank pin to overcome the inertia of the reciprocating parts and of the connecting rod. If it indicates a clockwise torque, then

$$\text{Inertia torque on the crankshaft} = F_i \times OA \text{ counter-clockwise}$$



(b)



(a)

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Example *The following data relate to a horizontal reciprocating engine:*

Mass of reciprocating parts	= *120 kg*
Crank length	= *90 mm*
Engine speed	= *600 rpm*
Connecting rod:	
Mass	= *90 kg*
Length between centres	= *450 mm*
Distance of centre of mass from big end centre	= *180 mm*
Radius of gyration about an axis through centre of mass	= *150 mm*

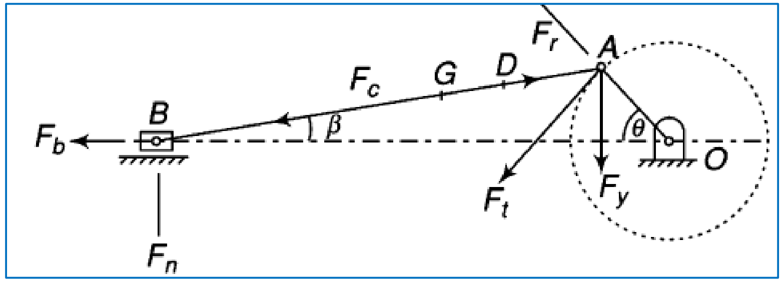
Find the magnitude and the direction of the inertia torque on the crankshaft when the crank has turned 30° from the inner-dead centre.

Solution It is required to find the inertia torque, or turning moment, on the crankshaft due to the inertia of the piston as well as of the connecting rod. This can be obtained by analytical or graphical methods.

Analytical Method

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.8 \text{ rad/s}$$

Divide the mass of the connecting rod into two parts



Mass at crank pin,

$$m_a = 90 \times \left(\frac{450-180}{450}\right) = 54 \text{ kg}$$

Mass at gudgeon pin, $m_b = 90 - 54 = 36$ kg

Total mass of reciprocating parts, $m = 120 + 36 = 156$ kg

Acceleration of the reciprocating parts,

$$f = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

As θ is less than 90°, it is towards the right and Thus, the inertia force is towards left.

$$\text{Inertia force, } F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 156 \times 0.09 \times (62.8)^2\left(\cos 30° + \frac{\cos 60°}{5}\right)$$

$$= 53\,490 \text{ N}$$

Inertia torque due to reciprocating parts

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

$$= 53\,490 \times 0.09\left(\sin 30° + \frac{\sin 60°}{2\sqrt{(5)^2 - \sin^2 30°}}\right)$$

$$= 2826 \text{ N.m}$$

(counter-clockwise as inertia force is towards left)

Correction couple due to assumed second mass of connecting rod at A,

$$\Delta T = m\alpha_c b(l - L)$$

where $b = 450 - 180 = 270$ mm

$l = 450$ mm

$$\text{and} \quad L = b + \frac{k^2}{b} = 270 + \frac{(150)^2}{270} = 353.3 \text{ mm}$$

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2-1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$= -(62.8)^2 \sin 30°\left[\frac{5^2-1}{(25 - \sin^2 30°)^{3/2}}\right]$$

$$= -384.7 \text{ rad/s}^2$$

$\therefore\ \Delta T = 90 \times (-384.7) \times 0.27 \times (0.45 - 0.3533)$

$= -903.97$ N.m

The direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β

Thus, it is clockwise.

•S.S. Ratan, Theory of Machines, 3rd

Example *The following data relate to a horizontal reciprocating engine:*

Mass of reciprocating parts	= *120 kg*
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Distance of centre of mass from big end centre	= *180 mm*
Radius of gyration about an axis through centre of mass	= *150 mm*

Find the magnitude and the direction of the inertia torque on the crankshaft when the crank has turned 30° from the inner-dead centre.

$\therefore$ correction torque on the crankshaft,

$$T_c = \Delta T \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -903.97 \times \frac{\cos 30°}{\sqrt{25 - \sin^2 30°}}$$

$$= -157.4 \text{ N.m}$$

Correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards left of the crankshaft, the correction torque is counter-clockwise.

Torque due to weight of mass at A,

$$T_a = (m_a g)\, r\cos\theta$$
$$= 54 \times 9.81 \times 0.09 \times \cos 30°$$
$$= 41.3 \text{ N.m counter-clockwise}$$

$\therefore$ total inertia torque on the crankshaft

$$= T_b - T_c + T_a$$
$$2826 - (-157.4) + 41.3$$
$$= \underline{3024.7 \text{ N. m}} \text{ counter-clockwise}$$

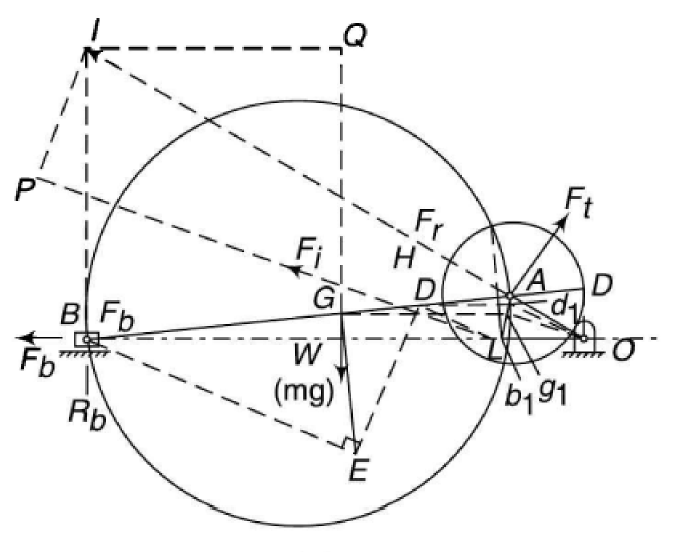
Graphical Method

Draw the configuration diagram OAB of the engine mechanism to a convenient scale and its velocity and acceleration diagrams by Klein's construction

$$v_a = \omega r = 62.8 \times 0.09 = 5.65 \text{ m/s}$$
$$f_a = \omega^2 r = (62.8)^2 \times 0.09 = 355 \text{ m/s}^2$$

Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram represents the acceleration of A relative to O, i.e., 355 m/s^2, therefore, f_b can be obtained from

$$f_b = 355 \times \frac{\text{length } b_1O}{\text{length } OA}$$



(a)

It is found to be $f_b = 343.2$ m/s^2

Similarly, $f_g = 345$ m/s^2

$$F_b = m_b \times f_b = 120 \times 343.2 = 41186 \text{ N}$$
$$F_i = m \times f_g = 90 \times 345 = 31050 \text{ N}$$

Complete the diagram of Fig.

Taking moments about I,

$$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$$

$$F_t \times 515 = 41\,186 \times 300 + 31\,050 \times 152 + 90 \times 9.81 \times 268$$

$$F_t = 33615.5 \text{ N.m}$$

•S.S. Ratan, Theory of Machines, 3rd

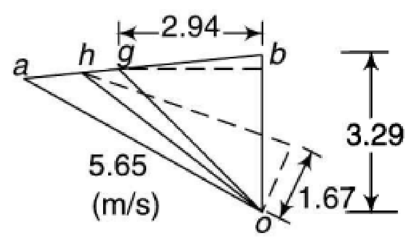
Example *The following data relate to a horizontal reciprocating engine:*

Mass of reciprocating parts	*= 120 kg*
Crank length	*= 90 mm*
Engine speed	*= 600 rpm*
Connecting rod:	
Mass	*= 90 kg*
Length between centres	*= 450 mm*
Distance of centre of mass from big end centre	*= 180 mm*
Radius of gyration about an axis through centre of mass	*= 150 mm*

Find the magnitude and the direction of the inertia torque on the crankshaft when the crank has turned 30° from the inner-dead centre.

Graphical Method

Instead of taking moments about the I-centre, the principle of virtual work can also be applied to obtain the torque as follows:



(b)

On the velocity diagram locate the points *b*, *h* and *g* corresponding to *B*, *H* and *G* respectively and take the components of velocities in the directions of forces F_b, F_i and mg. In Klein's construction, the velocity diagram in turned through 90°. Then

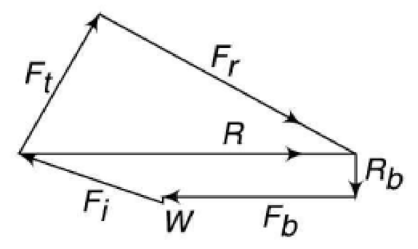
$$T \times \omega = F_b \times v_b + F_i \times v_h + mg \times v_g$$

$T \times 62.8 = 41\ 186 \times 3.29 + 31\ 050 \times 1.67 + 90 \times 9.81 \times 2.94$

$T = 2157.6 + 825.7 + 41.3$

$= \underline{3024.6 \text{ N.m}}$

If it is desired to find the resultant force on the crank, complete the force diagram as shown in Fig



(c)

Resultant force on the crank pin, $R = \underline{70\,000}$ N at 0°

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Thank you

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