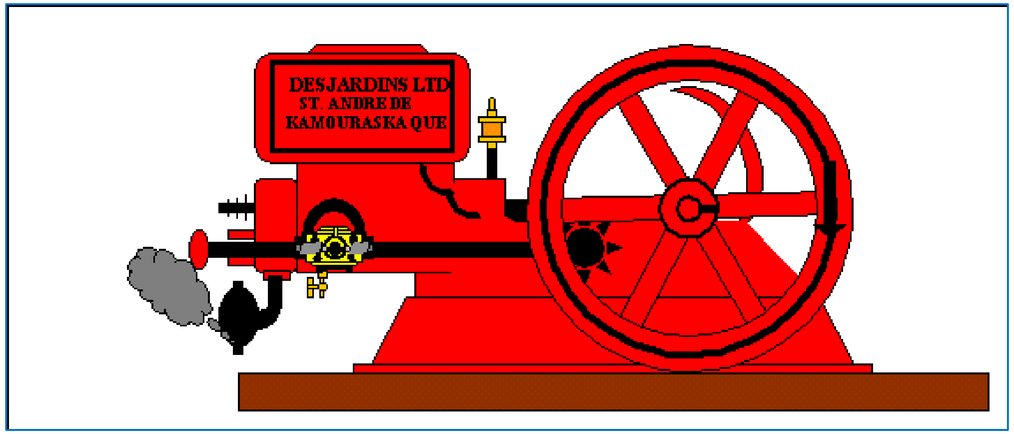
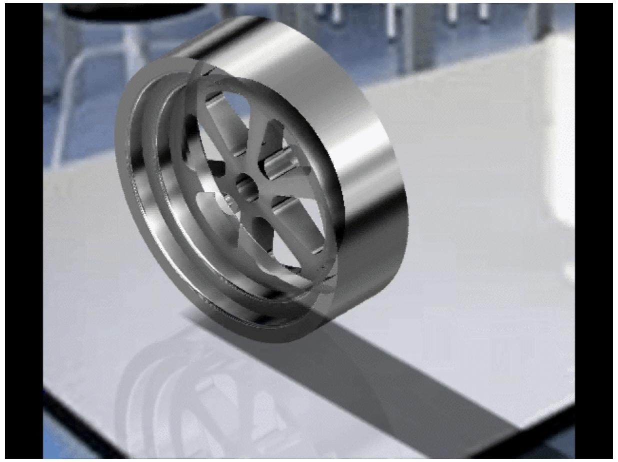
AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

Turning moment diagrams and flywheel



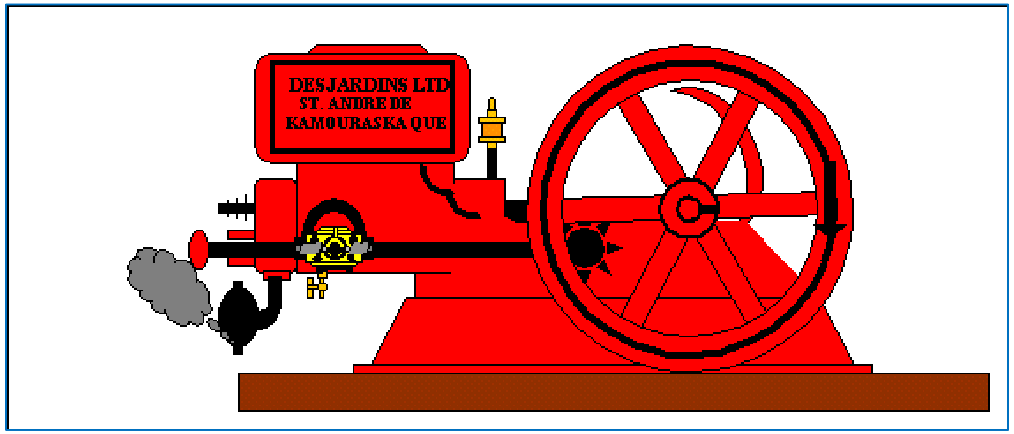


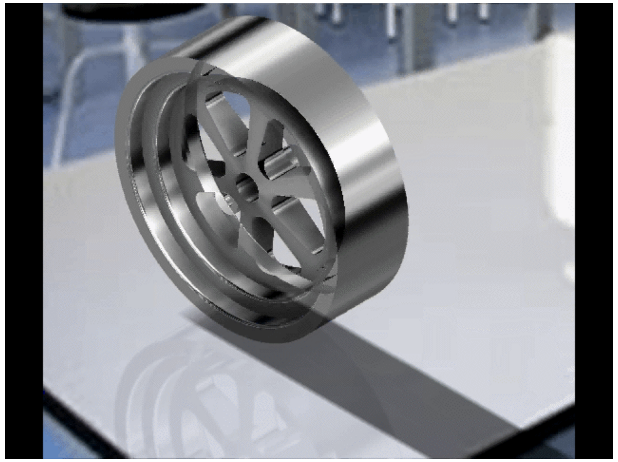
Subject:- DOM Code:- 3151911	Prepared by: Asst.Prof.Chirag Mevada (Mechanical Department,ACET)	AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering
Subject Code: 3151911
Semester – V
DYNAMICS OF MACHINERY

2	**Turning moment diagrams and flywheel** Turning moment diagram for various type of engines, fluctuation of energy, fluctuation of speed, flywheel, energy stored in flywheel, dimensions of flywheel rims, flywheel in punching presses	04





AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

❑ Introduction

- The turning moment diagram (also known as crankeffort diagram) is the graphical representation of the turning moment or crank-effort for various positions of the crank.
- It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

❑ Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- A turning moment diagram for a single cylinder double acting steam engine is shown in Fig.
- The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.
- We have discussed in Chapter 1 that the turning moment on the crankshaft,

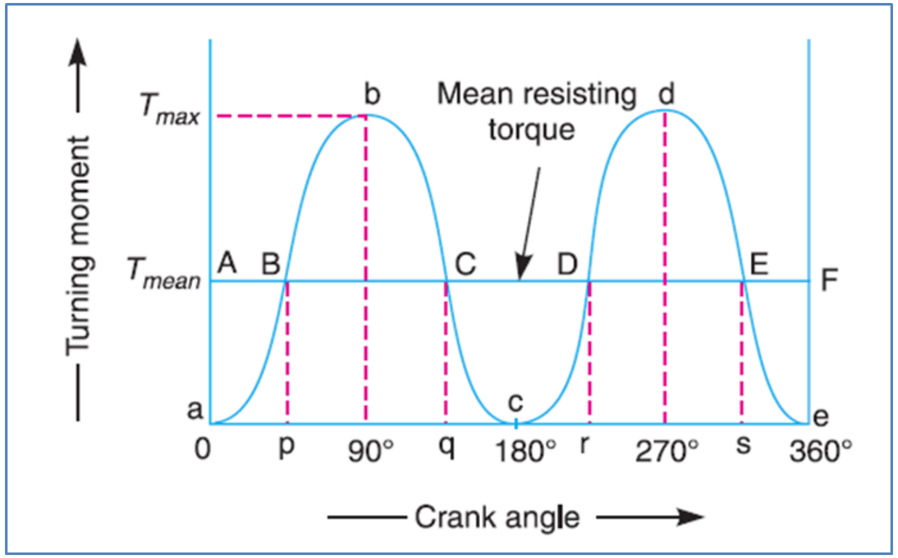
$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

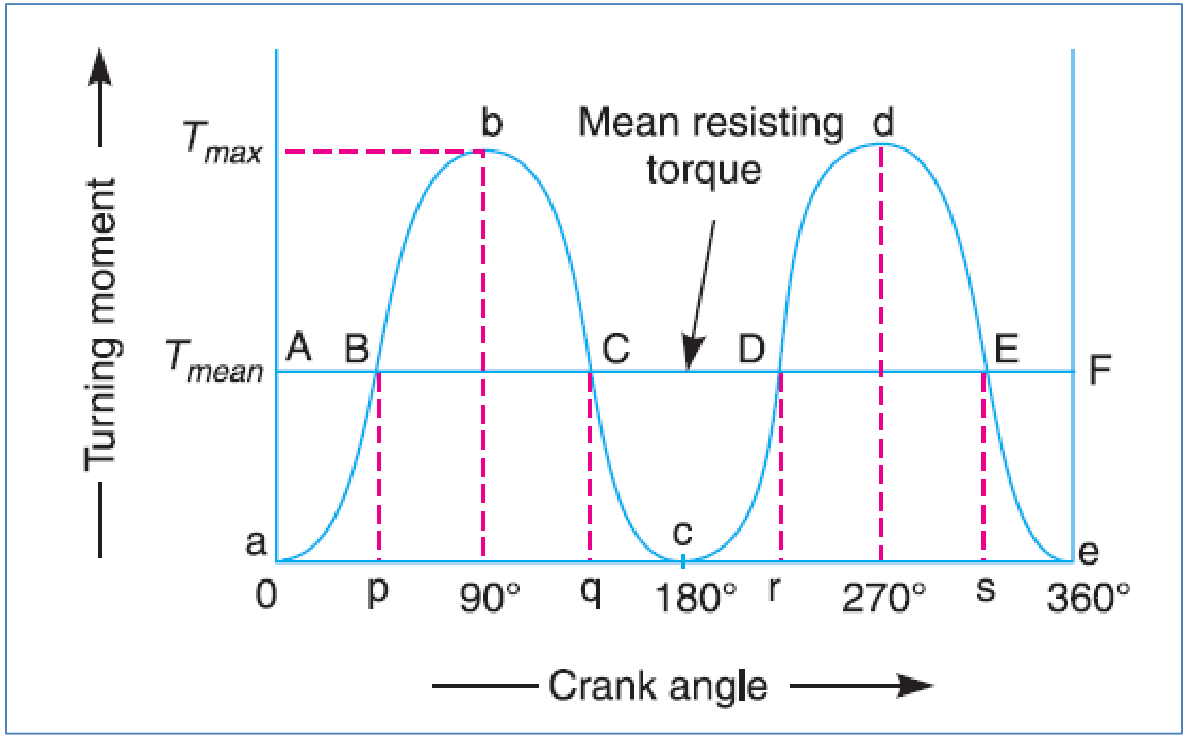
θ = Angle turned by the crank from inner dead centre.



•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.
- This is shown by the curve abc in Fig. and it represents the turning moment diagram for outstroke. The curve ***cde*** is the turning moment diagram for instroke and is somewhat similar to the curve ***abc***.
- Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution.



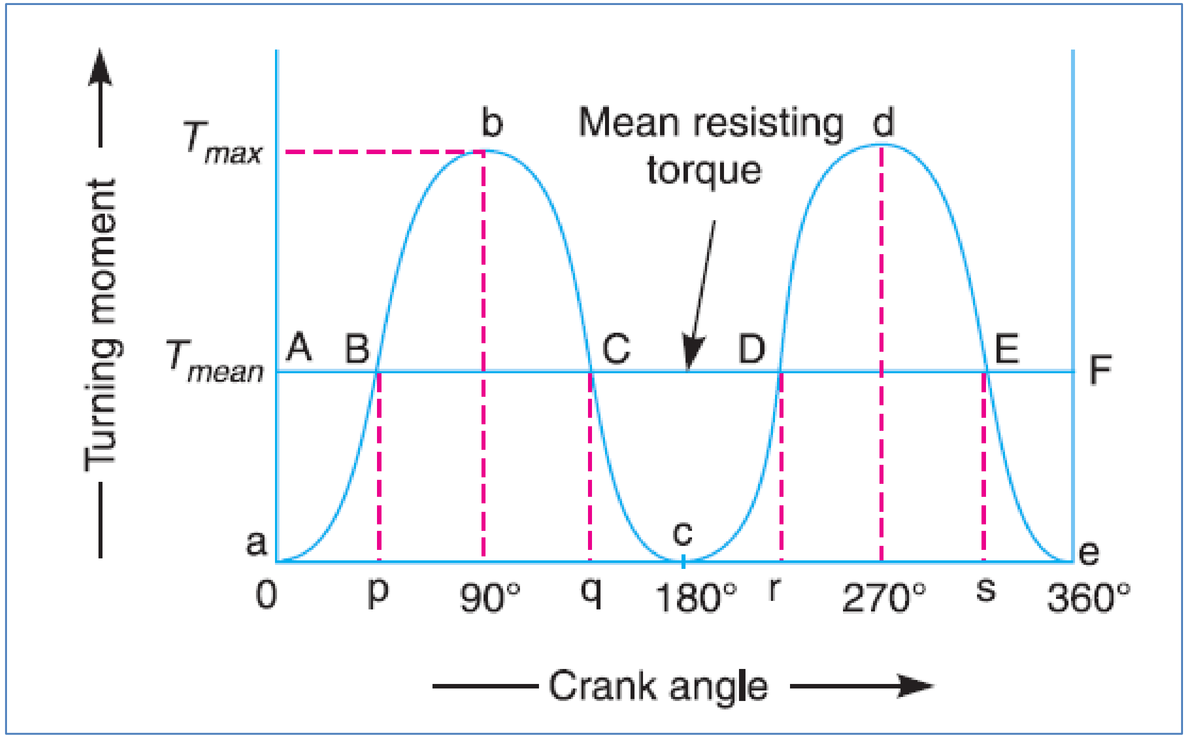
- In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line AF.
- The height of the ordinate a ***A*** represents the mean height of the turning moment diagram.
- Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle ***aAFe*** is proportional to the work done against the mean resisting torque.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

❖ ***Notes:***

1. When the turning moment is positive (i.e. when the engine torque is more than the mean resisting torque) as shown between points ***B*** and ***C*** (or ***D*** and ***E***) in Fig., the crankshaft accelerates and the work is done by the steam.
2. When the turning moment is negative (i.e. when the engine torque is less than the mean resisting torque) as shown between points ***C*** and ***D*** in Fig., the crankshaft retards and the work is done on the steam.
3. If T = Torque on the crankshaft at any instant, and T_{mean} = Mean resisting torque.
   Then accelerating torque on the rotating parts of the engine = $T - T_{mean}$
4. If $(T - T_{mean})$ is positive, the flywheel accelerates and if $(T - T_{mean})$ is negative, then the flywheel retards.

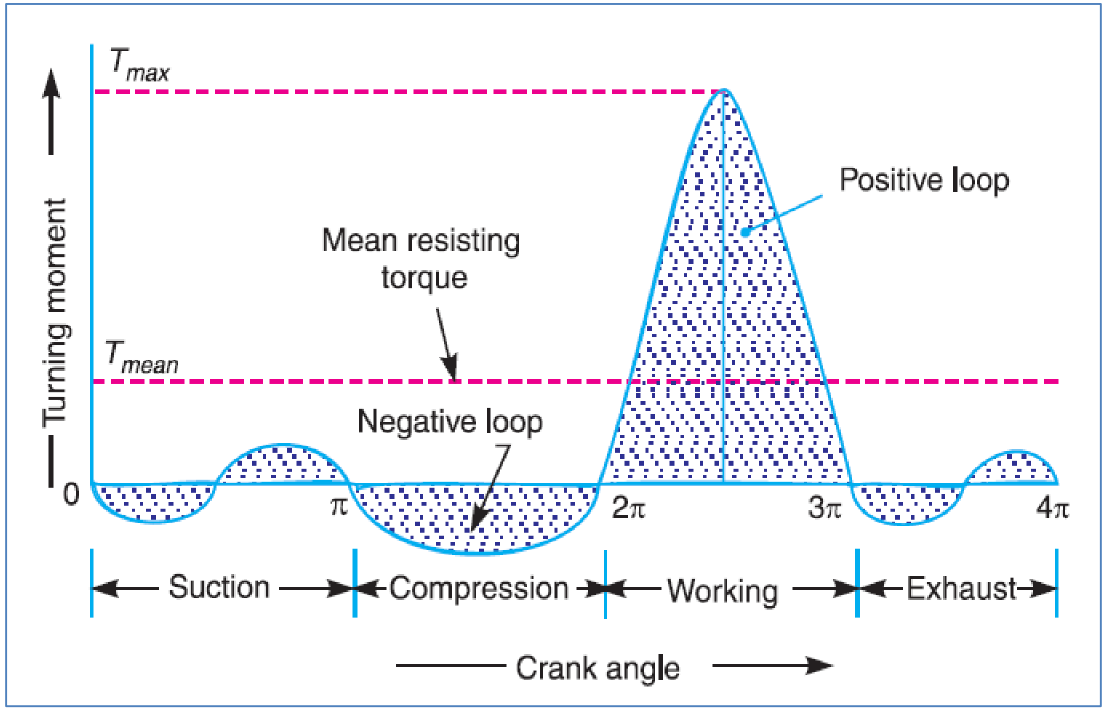


•Khurmi, R. et al.; Theory of Machines, 14th ed.

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

❑ Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine

- A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig.
- We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4π radians).
- Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig.
- During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained.
- During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained.
- In this stroke, the work is done by the gases.
- During exhaust stroke, the work is done on the gases, therefore a negative loop is formed.
- It may be noted that the effect of the inertia forces on the piston is taken into account in Fig.

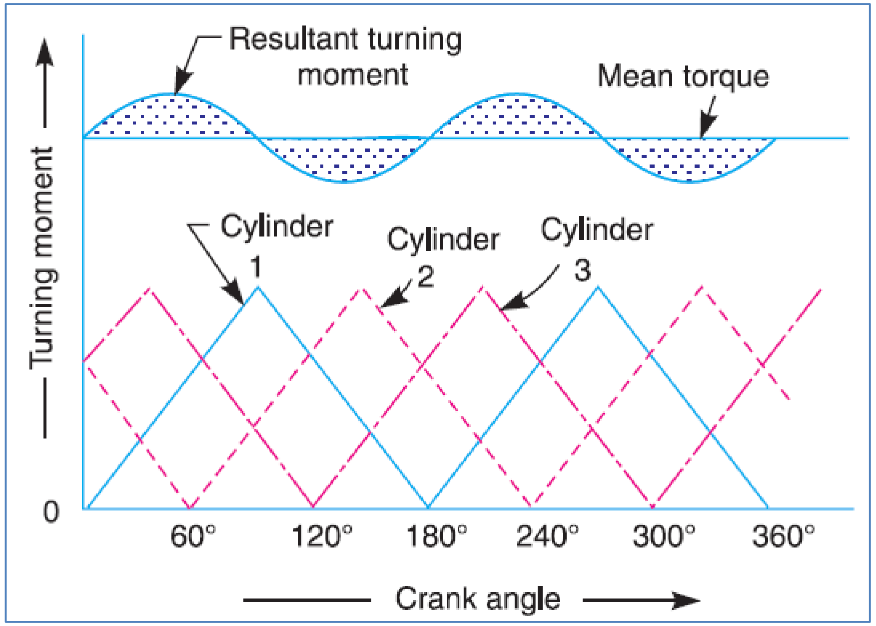


•Khurmi, R. et al.; Theory of Machines, 14th ed.

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

❑ Turning Moment Diagram for a Multi-cylinder Engine

- A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig.

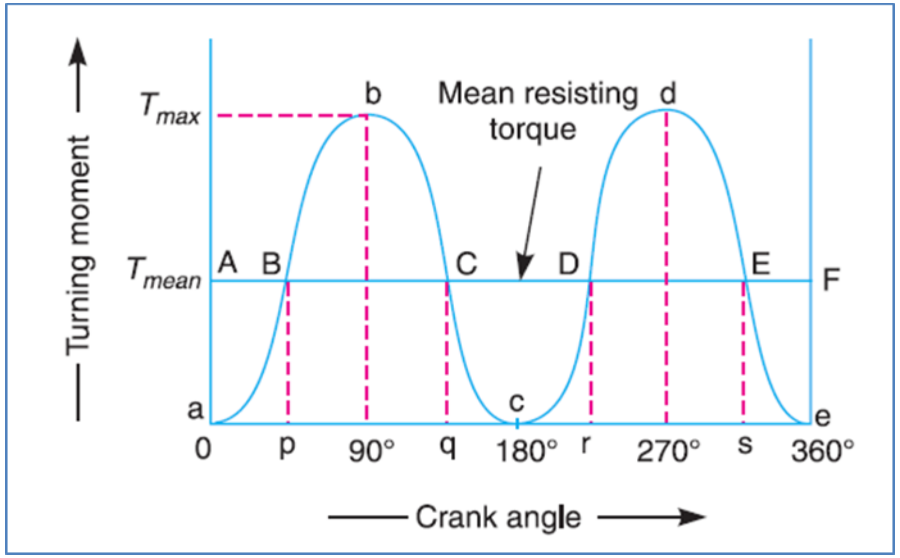


- The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders.
- It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder.
- The cranks, in case of three cylinders, are usually placed at 120° to each other.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

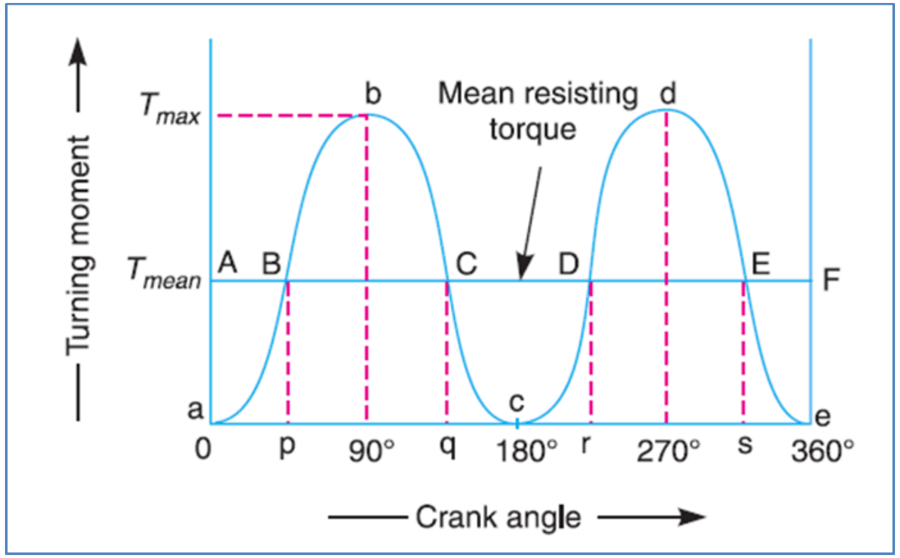
❑ Fluctuation of Energy

- The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation.
- Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig.
- We see that the mean resisting torque line AF cuts the turning moment diagram at points B, C, D and E.
- When the crank moves from a to p, the work done by the engine is equal to the area aBp, whereas the energy required is represented by the area aABp.
- In other words, the engine has done less work (equal to the area a AB) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases.
- Now the crank moves from p to q, the work done by the engine is equal to the area pBbCq, whereas the requirement of energy is represented by the area pBCq.
- Therefore, the engine has done more work than the requirement. This excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from p to q.



❑ Fluctuation of Energy

- Similarly, when the crank moves from q to r, more work is taken from the engine than is developed.
- This loss of work is represented by the area C c D.
- To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r.
- As the crank moves from r to s, excess energy is again developed given by the area D d E and the speed again increases.
- As the piston moves from s to e, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy.***
- The areas BbC, CcD, DdE, etc. represent fluctuations of energy.
- A little consideration will show that the engine has a maximum speed either at q or at s.
- This is due to the fact that the flywheel absorbs energy while the crank moves from p to q and from r to s.
- On the other hand, the engine has a minimum speed either at p or at r. The reason is that the flywheel gives out some of its energy when the crank moves from a to p and q to r.
- The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy.***

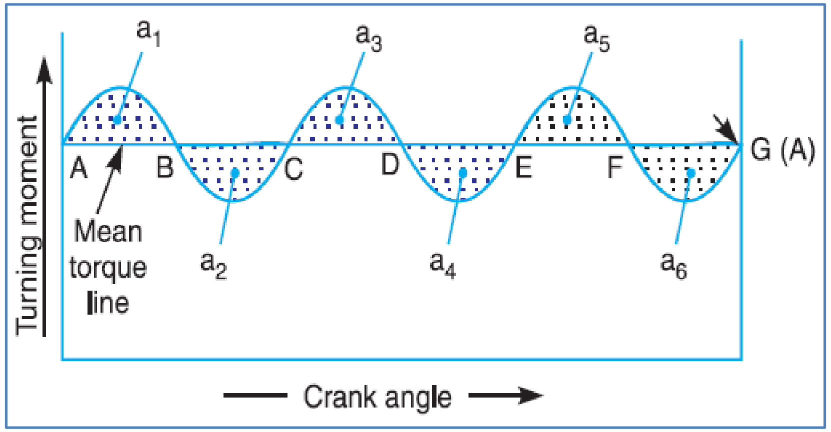


•Khurmi, R. et al.; Theory of Machines, 14th ed.

AMIRAJ COLLEGE OF ENGINEERING & TECHNOLOGY

❑ Determination of Maximum Fluctuation of Energy

- A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig.
- The horizontal line AG represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line.
- These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.
- Let the energy in the flywheel at A = E, then from Fig., we have



$$\text{Energy at } B = E + a_1$$
$$\text{Energy at } C = E + a_1 - a_2$$
$$\text{Energy at } D = E + a_1 - a_2 + a_3$$
$$\text{Energy at } E = E + a_1 - a_2 + a_3 - a_4$$
$$\text{Energy at } F = E + a_1 - a_2 + a_3 - a_4 + a_5$$
$$\text{Energy at } G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$$
$$= \text{Energy at } A \;\; (i.e. \text{ cycle repeats after } G)$$

- Let us now suppose that the greatest of these energies is at B and least at E. Therefore,
- Maximum energy in flywheel = $E + a_1$
- Minimum energy in the flywheel = $E + a_1 - a_2 + a_3 - a_4$

∴ Maximum fluctuation of energy,

ΔE = Maximum energy – Minimum energy = $(E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Coefficient of Fluctuation of Energy

- It may be defined as the ***ratio of the maximum fluctuation of energy to the work done per cycle.*** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

- The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle = $T_{mean} \times \theta$

   where T_{mean} = Mean torque, and

   θ = Angle turned (in radians), in one revolution.

   = 2θ, in case of steam engine and two stroke internal combustion engines

   = 4θ, in case of four stroke internal combustion engines.

- The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N/60$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Coefficient of Fluctuation of Energy

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

➢ The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Table Coefficient of fluctuation of energy (C_E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy* (C_E)
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinders, single acting, four stroke gas engine	0.066
5.	Six cylinders, single acting, four stroke gas engine	0.031

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Flywheel

- ***A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.***
- In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke.
- For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines.
- The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.
- A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases.
- Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- In other words, ***a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.***

❑ Flywheel

- In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle.
- Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Note:

- The function of a ***governor*** in an engine is entirely different from that of a ***flywheel***.
- It regulates the mean speed of an engine when there are variations in the load, e.g., when the load on the engine increases, it becomes necessary to increase the supply of working fluid.
- On the other hand, when the load decreases, less working fluid is required.
- The ***governor*** automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed of the engine within certain limits.
- As discussed above, the ***flywheel*** does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by the varying load.

❑ Coefficient of Fluctuation of Speed

- The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed.***
- The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed.***

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ *Coefficient of fluctuation of speed,*

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

- The coefficient of fluctuation of speed is a limiting factor in the design of flywheel.
- It varies depending upon the nature of service to which the flywheel is employed.
- ***Note***:- The reciprocal of the coefficient of fluctuation of speed is known as ***coefficient of steadiness*** and is denoted by ***m***.

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Energy Stored in a Flywheel

- A flywheel is shown in Fig. We have discussed that when a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

  Let m = Mass of the flywheel in kg,

  k = Radius of gyration of the flywheel in metres,

  I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = m.k^2,

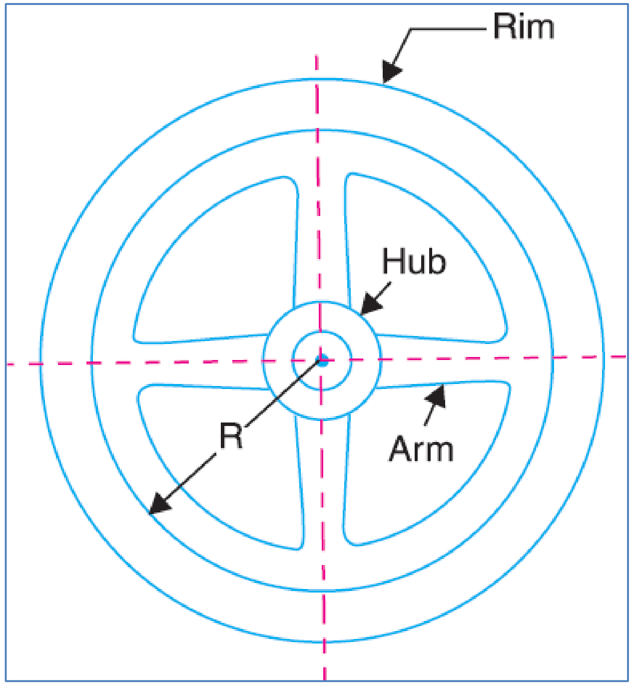
  N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

  ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$



- We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Energy Stored in a Flywheel

- As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,
- ΔE = Maximum K.E. – Minimum K.E.

$$= \frac{1}{2} \times I\,(\omega_1)^2 - \frac{1}{2} \times I\,(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I\,(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \quad \ldots(i) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \ldots \text{(Multiplying and dividing by } \omega)$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \quad \ldots(\because I = m.k^2) \quad \ldots(ii)$$

$$= 2.E.C_S \text{ (in N–m or joules)} \quad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots (iii)$$

- The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting k = R, in equation *(ii)*, we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

where v = Mean linear velocity (i.e. at the mean radius) in m/s = $\omega \cdot R$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Energy Stored in a Flywheel

Notes:-

1. Since $= 2\pi N/60$, therefore equation **(*i*)** may be written as

$$\Delta E = I \times \frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right) = \frac{4\pi^2}{3600} \times I \times N\left(N_1 - N_2\right)$$

$$= \frac{\pi^2}{900} \times m.k^2.N\left(N_1 - N_2\right)$$

$$= \frac{\pi^2}{900} \times m.k^2.N^2.C_S \qquad \ldots\left(\because C_S = \frac{N_1 - N_2}{N}\right)$$

2. In the above expressions, only the mass moment of inertia of the flywheel rim (*I*) is considered and the mass moment of inertia of the hub and arms is neglected.

- This is due to the fact that the major portion of the mass of the flywheel is in the rim and a small portion is in the hub and arms.
- Also the hub and arms are nearer to the axis of rotation, therefore the mass moment of inertia of the hub and arms is small.

❑ **Example 1:** ***A horizontal cross compound steam engine develops 300 kW at 90 r.p.m. The coefficient of fluctuation of energy as found from the turning moment diagram is to be 0.1 and the fluctuation of speed is to be kept within ± 0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2 metres.***

Solution. Given : $P = 300$ kW $= 300 \times 10^3$ W; $N = 90$ r.p.m.; $C_E = 0.1$; $k = 2$ m

We know that the mean angular speed,

$$\omega = 2\pi N/60 = 2\pi \times 90/60 = 9.426 \text{ rad/s}$$

Let ω_1 and ω_2 = Maximum and minimum speeds respectively.

Since the fluctuation of speed is ± 0.5% of mean speed, therefore total fluctuation of speed,

$\omega_1 - \omega_2 = 1\%\ \omega = 0.01\ \omega$ and coefficient of fluctuation of speed, $C_S = \dfrac{\omega_1 - \omega_2}{\omega} = 0.01$

We know that work done per cycle

$$= P \times 60 / N = 300 \times 10^3 \times 60 / 90 = 200 \times 10^3 \text{ N-m}$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Work done per cycle} \times C_E$$

$$= 200 \times 10^3 \times 0.1 = 20 \times 10^3 \text{ N-m}$$

Let m = Mass of the flywheel.

We know that maximum fluctuation of energy (ΔE),

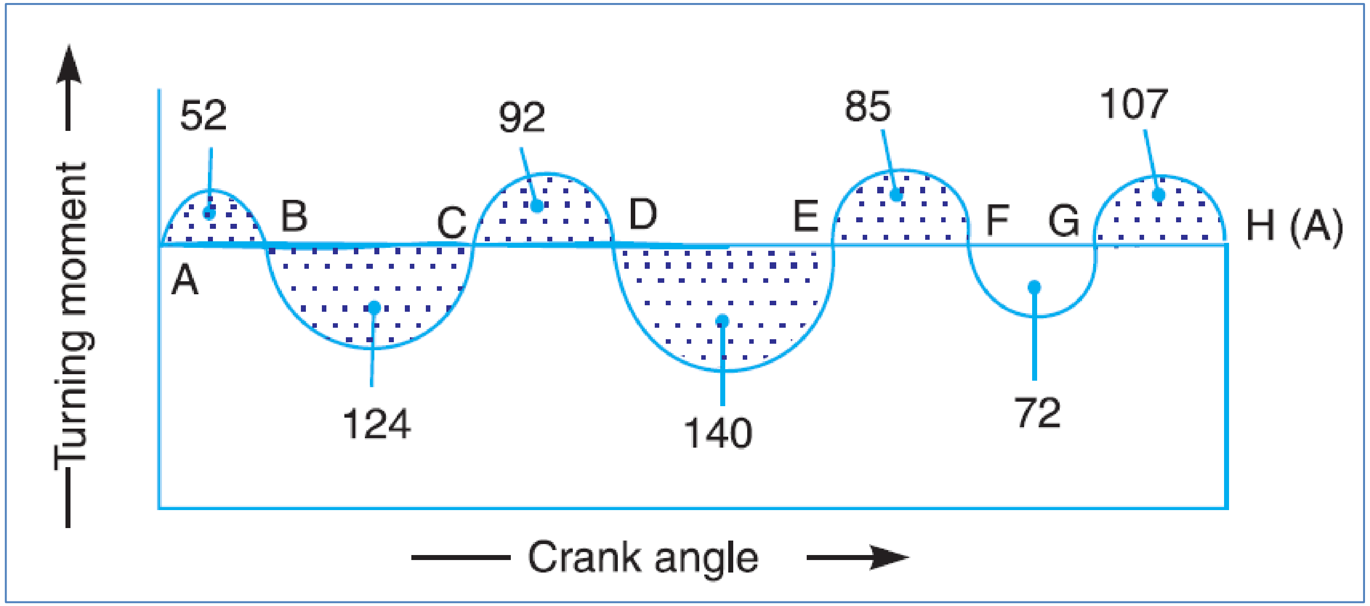
$$20 \times 10^3 = m.k^2.\omega^2.C_S = m \times 2^2 \times (9.426)^2 \times 0.01 = 3.554\ m$$

$$m = 20 \times 10^3/3.554 = 5630 \text{ kg}$$ **Ans.**

❑ **Example 2:** *The turning moment diagram for a multicylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows :*

+ 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm², when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.

Solution. Given : $N = 600$ r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $R = 0.5$ m



•Khurmi, R. et al.; Theory of Machines, 14th ed.

…to be continued

Solution. Given : $N = 600$ r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $R = 0.5$ m

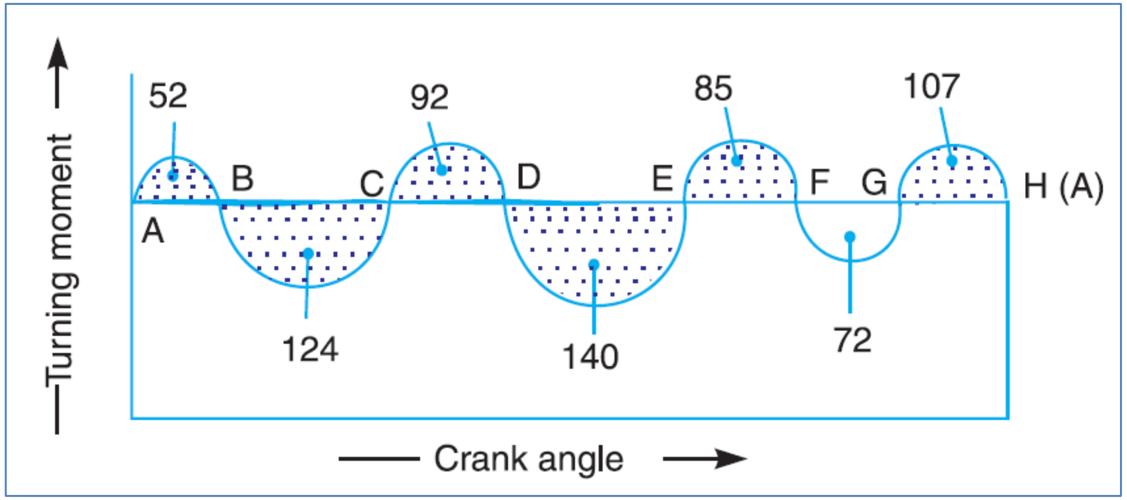
Since the total fluctuation of speed is not to exceed ± 1.5% of the mean speed, therefore

$$\omega_1 - \omega_2 = 3\%\ \omega = 0.03\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

The turning moment diagram is shown in Fig.



Since the turning moment scale is 1 mm = 600 N-m

and crank angle scale is 1 mm = 3°

$= 3° \times \pi/180 = \pi / 60$ rad, therefore

1 mm^2 on turning moment diagram

$= 600 \times \pi/60 = 31.42$ N-m

Let the total energy at $A = E$,

Energy at $B = E + 52$ …(Max. Energy)

Energy at $C = E + 52 - 124 = E - 72$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$ …(Min. Energy)

Energy at $F = E - 120 + 85 = E - 35$

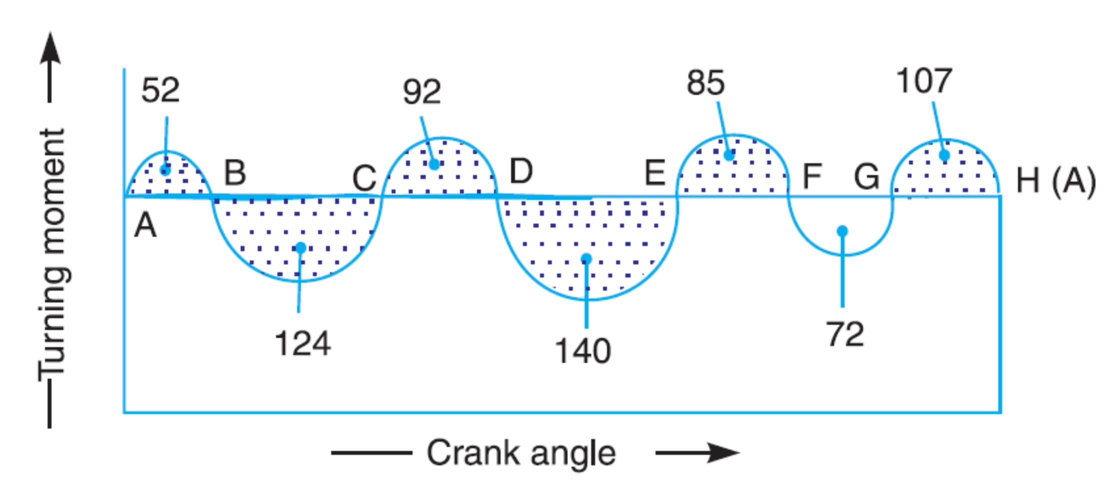
Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E =$ Energy at A

•Khurmi, R. et al.; Theory of Machines, 14th ed.

…to be continued

Solution. Given : $N = 600$ r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $R = 0.5$ m



Energy at $B = E + 52$ …(Max. Energy)

Energy at $E = E + 20 - 140 = E - 120$ …(Min. Energy)

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 52) - (E - 120) = 172 = 172 \times 31.42$$

$$= 5404 \text{ N-m}$$

Let m = Mass of the flywheel in kg.

We know that maximum fluctuation of energy (ΔE),

$$5404 = m.R^2.\omega^2.C_S = m \times (0.5)^2 \times (62.84)^2 \times 0.03 = 29.6\, m$$

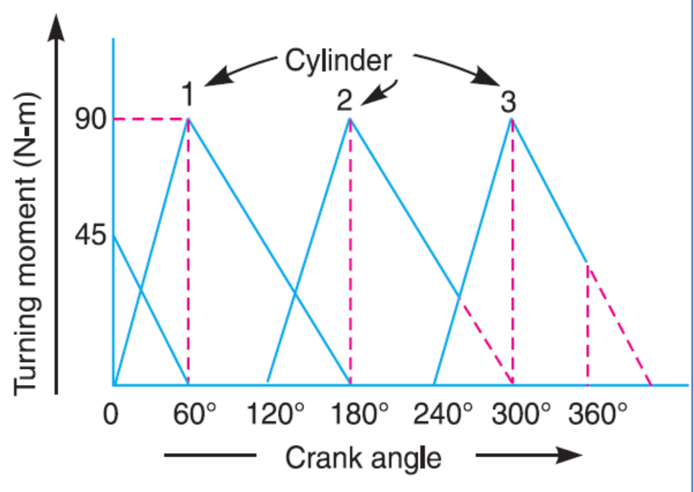
$$m = 5404 / 29.6 = 183 \text{ kg}$$ **Ans.**

•Khurmi, R. et al.; Theory of Machines, 14th ed.

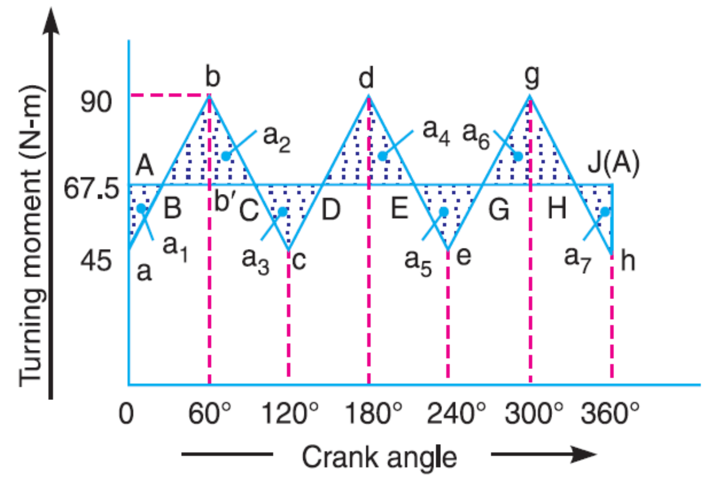
❑ Example 3: ***A three cylinder single acting engine has its cranks set equally at 120° and it runs at 600 r.p.m. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque of 90 N-m at 60° from dead centre of corresponding crank. The torque on the return stroke is sensibly zero. Determine : 1. power developed. 2. coefficient of fluctuation of speed, if the mass of the flywheel is 12 kg and has a radius of gyration of 80 mm, 3. coefficient of fluctuation of energy, and 4. maximum angular acceleration of the flywheel.***

Solution. Given : $N = 600$ r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s; $T_{max} = 90$ N-m; $m = 12$ kg; $k = 80$ mm $= 0.08$ m

The torque-crank angle diagram for the individual cylinders is shown in Fig. (a), and the resultant torque-crank angle diagram for the three cylinders is shown in Fig. (b).



(*a*)



(*b*)

•Khurmi, R. et al.; Theory of Machines, 14th ed.

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY

…to be continued

1. *Power developed*

We know that work done/cycle

$$= \text{Area of three triangles} = 3 \times \frac{1}{2} \times \pi \times 90 = 424 \text{ N-m}$$

and mean torque, $$T_{mean} = \frac{\text{Work done / cycle}}{\text{Crank angle / cycle}} = \frac{424}{2\pi} = 67.5 \text{ N-m}$$

$$\therefore \quad \text{Power developed} = T_{mean} \times \omega = 67.5 \times 62.84 = 4240 \text{ W} = 4.24 \text{ kW} \quad \textbf{Ans.}$$

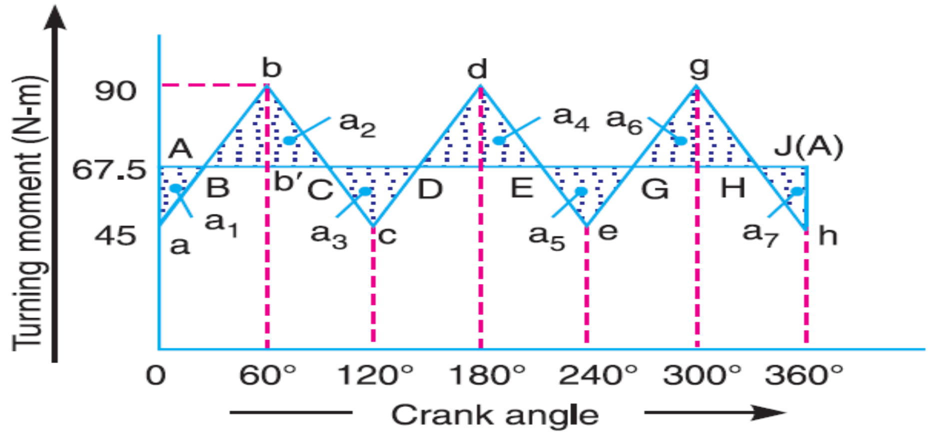
2. *Coefficient of fluctuation of speed*

Let C_S = Coefficient of fluctuation of speed.

First of all, let us find the maximum fluctuation of energy (ΔE).

From Fig.

$$a_1 = \text{Area of triangle } AaB = \frac{1}{2} \times AB \times Aa$$



(*b*)

$$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45) = 5.89 \text{ N-m} = a_7$$

...($\because AB = 30° = \pi / 6$ rad)

$$a_2 = \text{Area of triangle } BbC = \frac{1}{2} \times BC \times bb'$$

$$= \frac{1}{2} \times \frac{\pi}{3}(90 - 67.5) = 11.78 \text{ N-m}$$

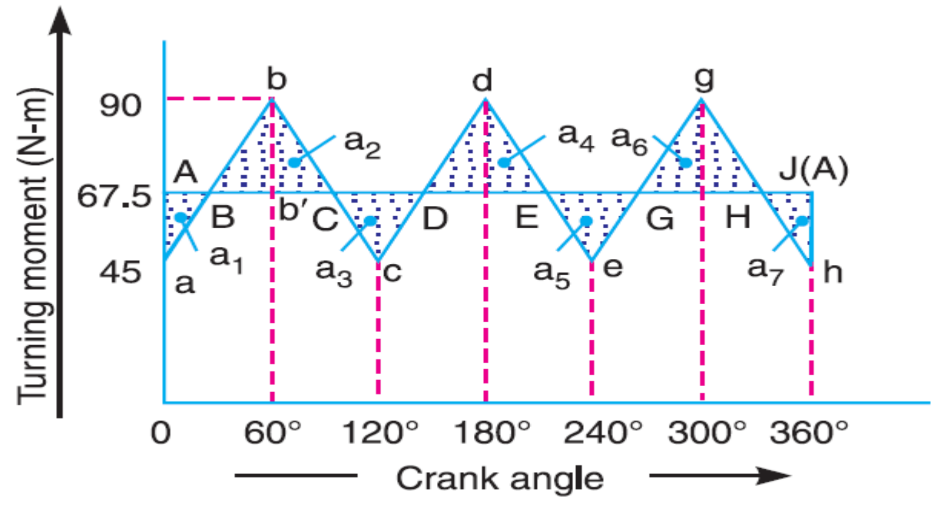
...($\because BC = 60° = \pi/3$ rad)

$$= a_3 = a_4 = a_5 = a_6$$

…to be continued

Now, let the total energy at $A = E$

$$\text{Energy at } B = E - 5.89$$
$$\text{Energy at } C = E - 5.89 + 11.78 = E + 5.89$$
$$\text{Energy at } D = E + 5.89 - 11.78 = E - 5.89$$
$$\text{Energy at } E = E - 5.89 + 11.78 = E + 5.89$$
$$\text{Energy at } G = E + 5.89 - 11.78 = E - 5.89$$
$$\text{Energy at } H = E - 5.89 + 11.78 = E + 5.89$$
$$\text{Energy at } J = E + 5.89 - 5.89 = E = \text{Energy at } A$$



(*b*)

From above we see that maximum energy

$$= E + 5.89$$

and minimum energy $= E - 5.89$

$\therefore$ * Maximum fluctuation of energy,

$$\Delta E = (E + 5.89) - (E - 5.89) = 11.78 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$11.78 = m.k^2.\omega^2.C_S = 12 \times (0.08)^2 \times (62.84)^2 \times C_S = 303.3\ C_S$$

$$\therefore \quad C_S = 11.78 / 303.3 = 0.04 \text{ or } 4\%$$ **Ans.**

•Khurmi, R. et al.; Theory of Machines, 14th ed.

…to be continued

3. *Coefficient of fluctuation of energy*

We know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Max. fluctuation of energy}}{\text{Work done/cycle}} = \frac{11.78}{424} = 0.0278 = 2.78\% \quad \textbf{Ans.}$$

4. *Maximum angular acceleration of the flywheel*

Let α = Maximum angular acceleration of the flywheel.

We know that,

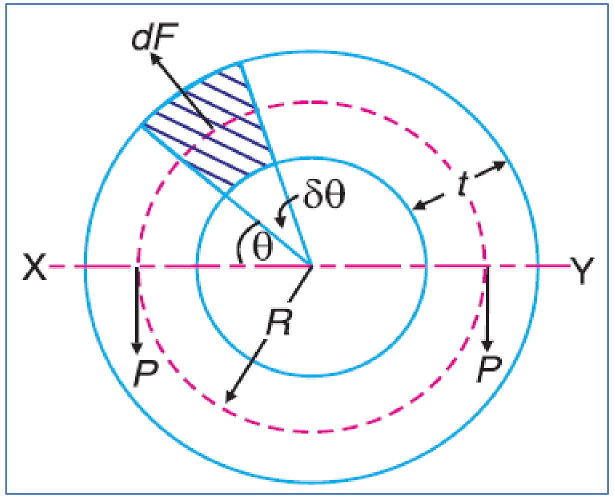
$$T_{max} - T_{mean} = I.\alpha = m.k^2.\alpha$$

$$90 - 67.5 = 12 \times (0.08)^2 \times \alpha = 0.077\ \alpha$$

$$\alpha = \frac{90 - 67.5}{0.077} = 292 \text{ rad/s}^2 \quad \textbf{Ans.}$$

❑ Dimensions of the Flywheel Rim

Consider a rim of the flywheel as shown in Fig.



Let D = Mean diameter of rim in metres,

R = Mean radius of rim in metres,

A = Cross-sectional area of rim in m^2,

ρ = Density of rim material in kg/m^3,

N = Speed of the flywheel in r.p.m.,

ω = Angular velocity of the flywheel in rad/s,

v = Linear velocity at the mean radius in m/s $= \omega . R = \pi D.N/60$, and

σ = Tensile stress or hoop stress in N/m^2 due to the centrifugal force.

Consider a small element of the rim as shown shaded in Fig.

Let it subtends an angle $\delta\theta$ at the centre of the flywheel.

Volume of the small element

$$= A \times R.\delta\theta$$

$\therefore$ Mass of the small element

$$dm = \text{Density} \times \text{volume} = \rho.A.R.\delta\theta$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

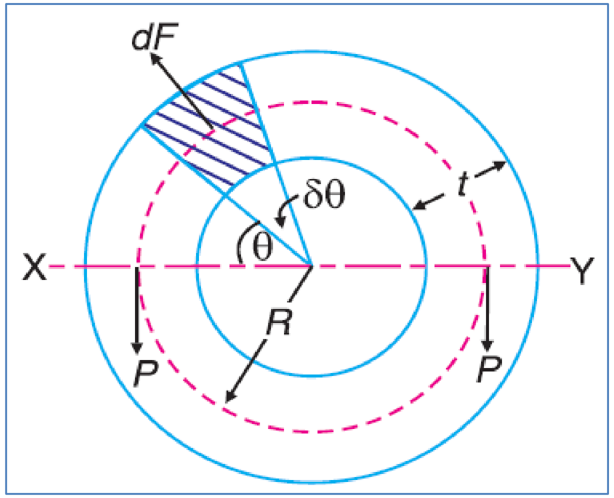
❑ Dimensions of the Flywheel Rim

and centrifugal force on the element, acting radially outwards,

$$dF = dm.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF

$$= dF.\sin\theta = \rho.A.R^2.\omega^2.\delta\theta.\sin\theta$$



∴ Total vertical upward force tending to burst the rim across the diameter $X\ Y$.

$$= \rho.A.R^2.\omega^2 \int_0^\pi \sin\theta.d\theta = \rho.A.R^2.\omega^2 [-\cos\theta]_0^\pi$$

$$= 2\rho.A.R^2.\omega^2 \quad \ldots (i)$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by $2P$, such that

$$2P = 2\sigma.A \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

$$\sigma = \rho.R^2.\omega^2 = \rho.v^2 \quad \ldots (\because v = \omega.R)$$

$$v = \sqrt{\frac{\sigma}{\rho}} \quad \ldots (iii)$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi D.A.\rho$$

$$A = \frac{m}{\pi.D.\rho} \quad \ldots (iv)$$

From equations (*iii*) and (*iv*), we may find the value of the mean radius and cross-sectional area of the rim.

❑ ***Example 1:*** ***An Otto cycle engine develops 50 kW at 150 r.p.m. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed 0.5% of mean on either side. Find the mean diameter of the flywheel and a suitable rim cross section having width four times the depth so that the hoop stress does not exceed 4 MPa. Assume that the flywheel stores 16/15 times the energy stored by the rim and the work done during power stroke is 1.40 times the work done during the cycle. Density of rim material is 7200 kg/m³.***

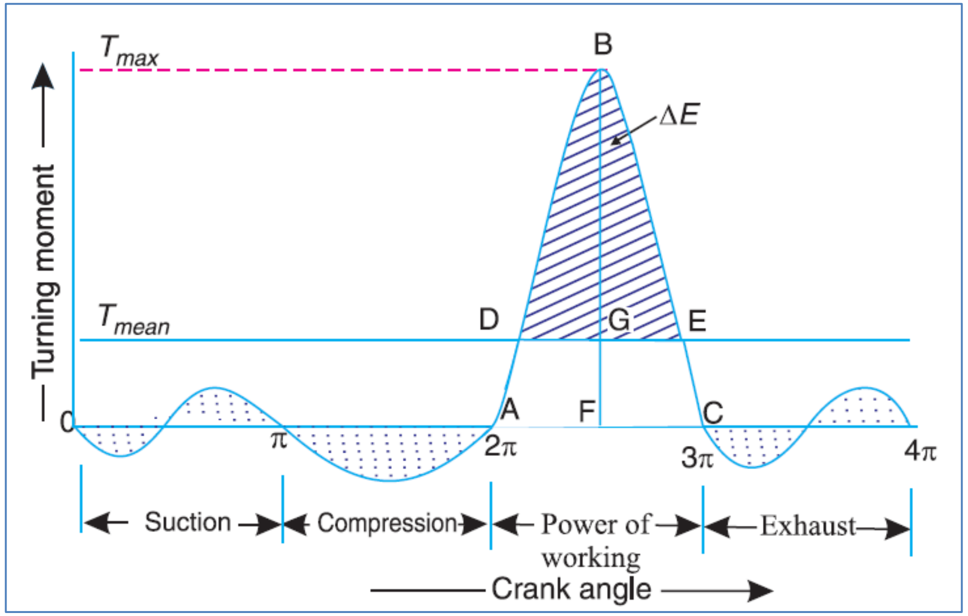
Solution. Given : $P = 50$ kW $= 50 \times 10^3$ W; $N = 150$ r.p.m. or $\omega = 2\pi \times 150/60 = 15.71$ rad/s; $n = 75$; $\sigma = 4$ MPa $= 4 \times 10^6$ N/m^2; $r = 7200$ kg/m^3

First of all, let us find the mean torque (T_{mean}) transmitted by the engine or flywheel. We know that the power transmitted (P),

$$50 \times 10^3 = T_{mean} \times \omega = T_{mean} \times 15.71$$

$$T_{mean} = 50 \times 10^3/15.71 = 3182.7 \text{ N-m}$$

Since the explosions per minute are equal to $N/2$, therefore, the engine is a four stroke cycle engine. The turning moment diagram of a four stroke engine is shown in Fig.



We know that *work done per cycle

$$= T_{mean} \times \theta = 3182.7 \times 4\pi = 40\,000 \text{ N-m}$$

…to be continued

∴ Workdone during power or working stroke

$$= 1.4 \times \text{work done per cycle}$$
$$= 1.4 \times 40\,000 = 56\,000 \text{ N-m} \quad \ldots(i)$$

The workdone during power stroke is shown by a triangle ABC in Fig. in which base $AC = \pi$ radians and height $BF = T_{max}$.

∴ Work done during working stroke

$$= \frac{1}{2} \times \pi \times T_{max} = 1.571\, T_{max} \quad \ldots (ii)$$

From equations (i) and (ii), we have

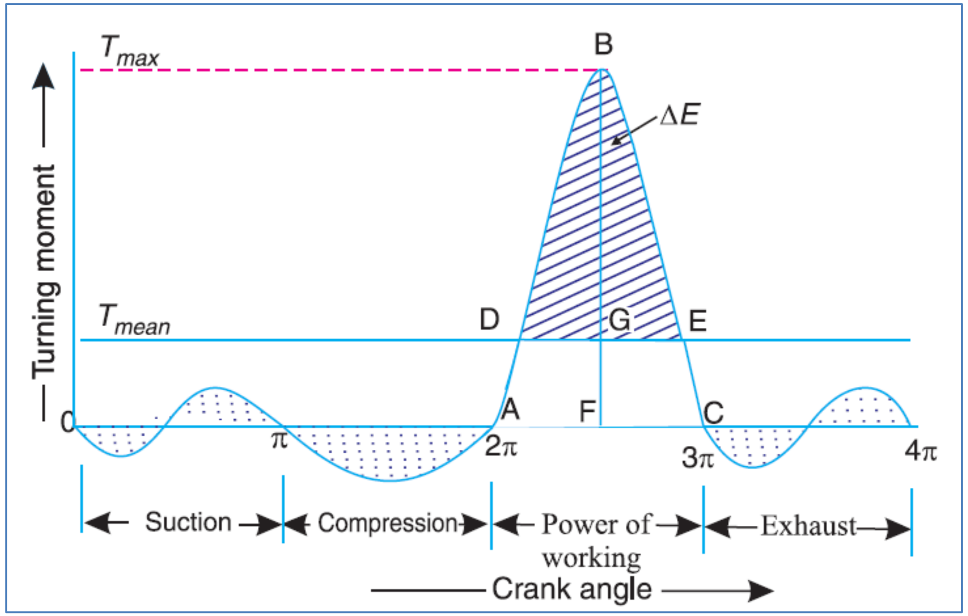
$$T_{max} = 56\,000/1.571 = 35\,646 \text{ N-m}$$

We know that the excess torque,

$$T_{excess} = BG = BF - FG = T_{max} - T_{mean}$$
$$= 35\,646 - 3182.7 = 32\,463.3 \text{ N-m}$$

Now, from similar triangles BDE and ABC,

$$\frac{DE}{AC} = \frac{BG}{BF} \quad \text{or} \quad DE = \frac{BG}{BF} \times AC = \frac{32\,463.3}{35\,646} \times \pi = 0.9107\,\pi$$



We know that maximum fluctuation of energy,

$$\Delta E = \text{Area of triangle } BDE = \frac{1}{2} \times DE \times BG$$
$$= \frac{1}{2} \times 0.9107\,\pi \times 32\,463.3 = 46\,445 \text{ N - m}$$

…to be continued

Mean diameter of the flywheel

Let D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that hoop stress (σ),

$$4 \times 10^6 = \rho . v^2 = 7200\, v^2 \quad \text{or} \quad v^2 = 4 \times 10^6/7200 = 556$$

∴ $v = 23.58$ m/s

We know that $v = \pi DN/60$ or $D = v \times 60/N = 23.58 \times 60/\pi \times 150 = 3$ m **Ans.**

Cross-sectional dimensions of the rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $4\,t$

∴ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2$$

First of all, let us find the mass of the flywheel rim.

Let m = Mass of the flywheel rim in kg, and

E = Total energy of the flywheel in N-m.

Since the fluctuation of speed is 0.5% of the mean speed on either side, therefore total fluctuation of speed,

$$N_2 - N_1 = 1\% \text{ of mean speed} = 0.01\,N \text{ and coefficient of fluctuation of speed,}$$

$$C_S = \frac{N_1 - N_2}{N} = 0.01$$

We know that the maximum fluctuation of energy (ΔE),

$$46\ 445 = E \times 2C_S = E \times 2 \times 0.01 = 0.02\ E$$

$$\therefore \quad E = 46\ 445 / 0.02 = 2322 \times 10^3 \text{ N-m}$$

Since the energy stored by the flywheel is $\frac{16}{15}$ times the energy stored by the rim, therefore,

the energy of the rim, $$E_{rim} = \frac{15}{16} E = \frac{15}{16} \times 232 \times 10^3 = 2177 \times 10^3 \text{ N-m}$$

We know that energy of the rim (E_{rim}),

$$2177 \times 10^3 = \frac{1}{2} \times m \times v^2 = m\,(23.58)^2 = 278\ m$$

$$\therefore \quad m = 2177 \times 10^3 / 278 = 7831 \text{ kg}$$

We also knonw that mass of the flywheel rim (m), $7831 = \pi D \times A \times \rho = \pi \times 3 \times 4t^2 \times 7200 = 271\ 469 t^2$

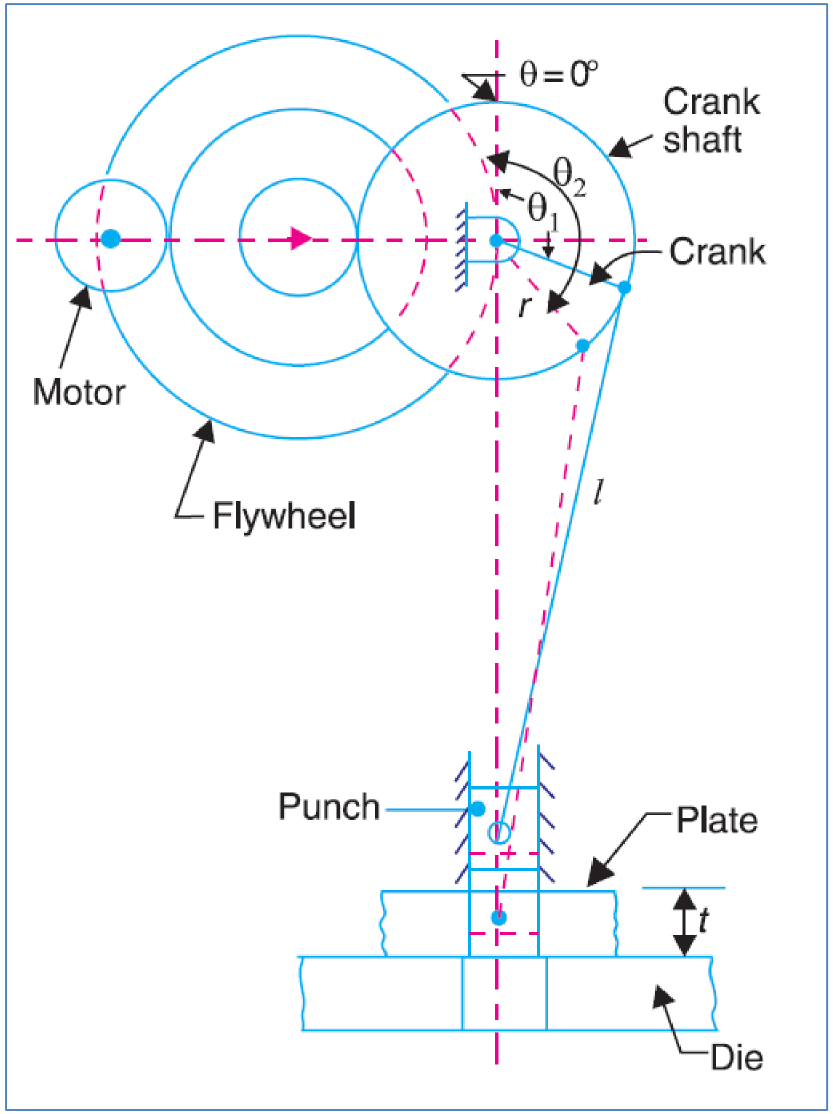
$$t^2 = 831 / 271\ 469 = 0.0288 \text{ or } t = 0.17 \text{ m} = 170 \text{ mm} \textbf{ Ans.}$$

$$b = 4\,t = 4 \times 170 = 680 \text{ mm} \textbf{ Ans.}$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Flywheel in Punching Press

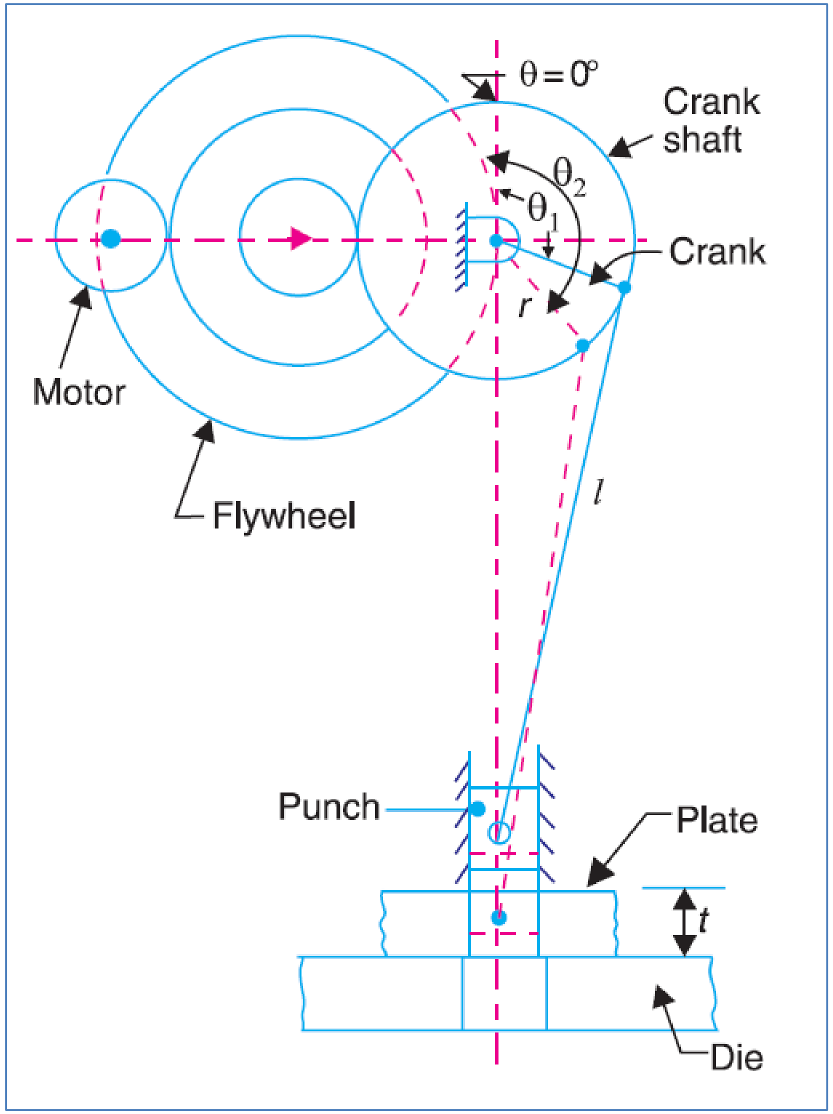
- ➢ The function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle.
- ➢ The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle.
- ➢ Such an application is found in punching press or in a rivetting machine.
- ➢ A punching press is shown diagrammatically in Fig.
- ➢ The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism.
- ➢ From Fig., we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle.



•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Flywheel in Punching Press

- Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied.
- On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from $\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy supplied.
- Thus the flywheel has to absorb excess energy available at one stage and has to make up the deficient energy at the other stage to keep to fluctuations of speed within permissible limits.
- This is done by choosing the suitable moment of inertia of the flywheel.



❑ Flywheel in Punching Press

- Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

  Let d_1 = Diameter of the hole punched,

  t_1 = Thickness of the plate, and

  τ_u = Ultimate shear stress for the plate material.

  ∴ Maximum shear force required for punching,

$$F_S = \text{Area sheared} \times \text{Ultimate shear stress} = \pi d_1 . t_1 \, \tau_u$$

- It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

  ∴ Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

- Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1. The energy supplied by the motor to the crankshaft during actual punching operation,

$$E_2 = E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right)$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Flywheel in Punching Press

∴ Balance energy required for punching

$$= E_1 - E_2 = E_1 - E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right) = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

- This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

- The values of θ_1 and θ_2 may be determined only if the crank radius (r), length of connecting rod (l) and the relative position of the job with respect to the crankshaft axis are known. In the absence of relevant data, we assume that

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

where t = Thickness of the material to be punched,

s = Stroke of the punch = 2 × Crank radius = 2r .

- By using the suitable relation for the maximum fluctuation of energy (ΔE) as discussed in the previous articles, we can find the mass and size of the flywheel.

❑ ***Example 1:*** ***A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 r.p.m. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour; each punching operation takes 2 second and requires 15 kN-m of energy. Find the power of the motor and the minimum mass of the flywheel if speed of the same is not to fall below 200 r. p. m.***

Solution. Given $N_1 = 225$ r.p.m ; $k = 0.5$ m ; Hole punched = 720 per hr; $E_1 = 15$ kN-m $= 15 \times 10^3$ N-m ; $N_2 = 200$ r.p.m.

Power of the motor

We know that the total energy required per second

= Energy required / hole × No. of holes / s

$= 15 \times 10^3 \times 720/3600 = 3000$ N-m/s

∴ Power of the motor = 3000 W = 3 kW **Ans.** (∵ 1 N-m/s = 1 W)

Minimum mass of the flywheel

Let m = Minimum mass of the flywheel.

Since each punching operation takes 2 seconds, therefore energy supplied by the motor in 2 seconds,

$$E_2 = 3000 \times 2 = 6000 \text{ N-m}$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

… to be continued

$\therefore$ Energy to be supplied by the flywheel during punching or maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 15 \times 10^3 - 6000 = 9000 \text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2} = 212.5 \text{ r.p.m}$$

We know that maximum fluctuation of energy (ΔE),

$$9000 = \frac{\pi^2}{900} \times m.k^2 . N (N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m \times (0.5)^2 \times 212.5 \times (225 - 200) = 14.565\, m$$

$m = 9000/14.565 = 618$ kg **Ans.**

❑ **Example 2:** ***A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10,000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute?***

Solution. Given : $P = 3$ kW ; $m = 150$ kg ; $k = 0.6$ m ; $N_1 = 300$ r.p.m. or

$\omega_1 = 2\pi \times 300/60 = 31.42$ rad/s

Speed of the flywheel immediately after riveting

Let ω_2 = Angular speed of the flywheel immediately after riveting.

We know that energy supplied by the motor,

$$E_2 = 3\,\text{kW} = 3000\ \text{W} = 3000\ \text{N-m/s} \qquad (\because 1\ \text{W} = 1\ \text{N-m/s})$$

But energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\ 000\ \text{N-m}$$

∴ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\ 000 - 3000 = 7000\ \text{N-m}$$

… to be continued

We know that maximum fluctuation of energy (ΔE),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150 \times (0.6)^2 \times \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

$$\therefore \quad (\omega_2)^2 = 987.2 - 7000/27 = 728 \text{ or } \omega_2 = 26.98 \text{ rad/s}$$

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60/2\pi = 257.6 \text{ r.p.m.}$$ **Ans.**

Number of rivets that can be closed per minute

Since the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore, number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\,000} \times 60 = 18 \text{ rivets}$$ **Ans.**

Thank you

AMIRAJ
COLLEGE OF ENGINEERING & TECHNOLOGY