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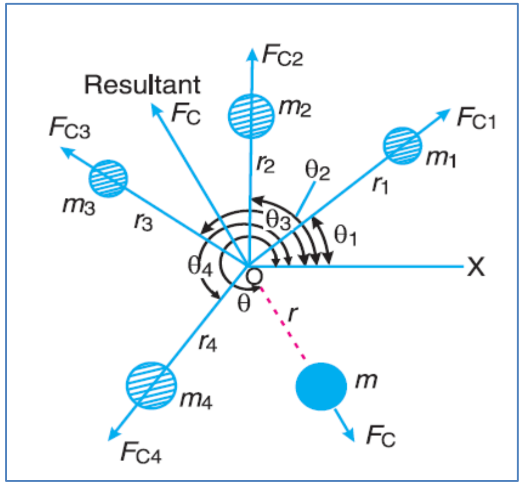
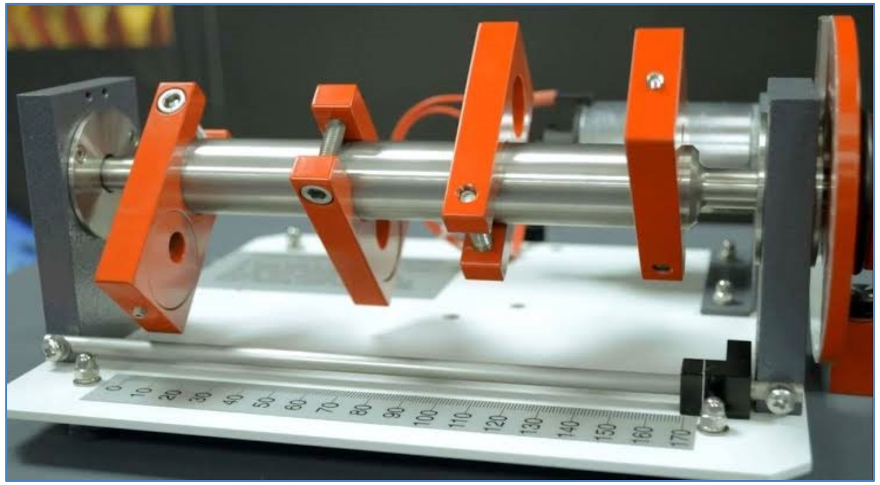
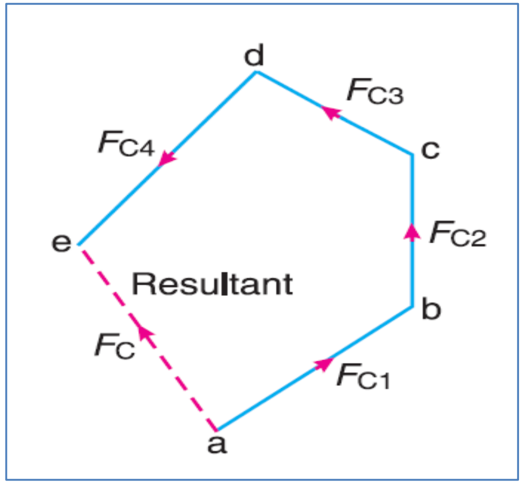
Balancing







Subject:- DOM Code:- 3151911	Prepared by: Asst.Prof.Chirag Mevada (Mechanical Department,ACET)	COLLEGE OF ENGINEERING & TECHNOLOGY

GUJARAT TECHNOLOGICAL UNIVERSITY

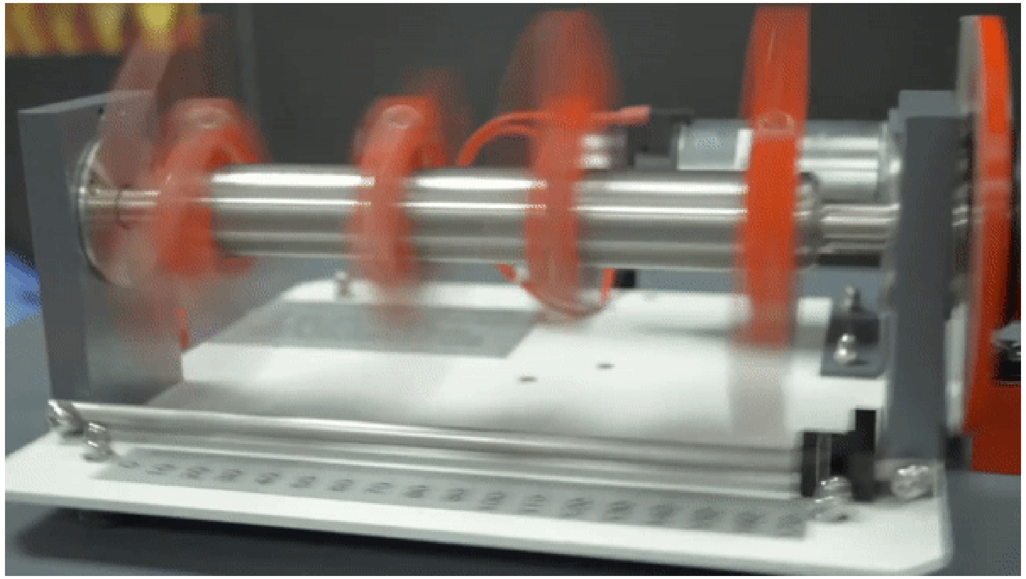
Bachelor of Engineering
Subject Code: 3151911
Semester – V
DYNAMICS OF MACHINERY

3	**Balancing:** Introduction, static balancing, dynamic balancing, transference of force from one plane to another plane, balancing of several masses in different planes, force balancing of linkages, balancing of reciprocating mass, balancing of locomotives, Effects of partial balancing in locomotives, secondary balancing, balancing of inline engines, balancing of v-engines, balancing of radial engines, balancing machines.	11

•S.S. Ratan, Theory of Machines, 3rd

❑ Introduction

- The high speed of engines and other machines is a common phenomenon now-a-days.
- It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible.
- If these parts are not properly balanced, the dynamic forces are set up.
- These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.
- In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimize pressure on the main bearings when an engine is running.



•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Balancing of Rotating Masses

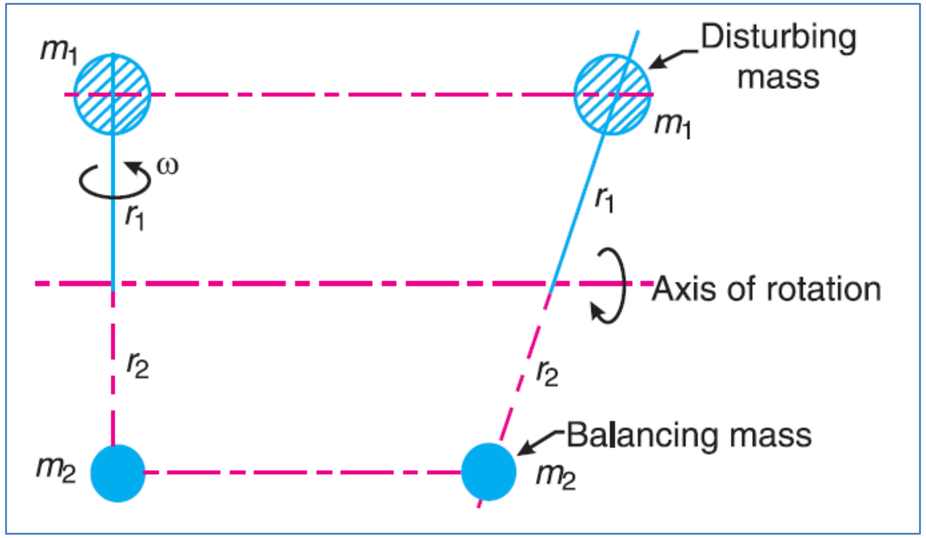
- Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.
- In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass.
- This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite.
- The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

- The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

❑ Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

- Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig.
- Let r_1 be the radius of rotation of the mass m_1 (i.e. distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).
- We know that the centrifugal force exerted by the mass m_1 on the shaft,



$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \quad \ldots(1)$$

- This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

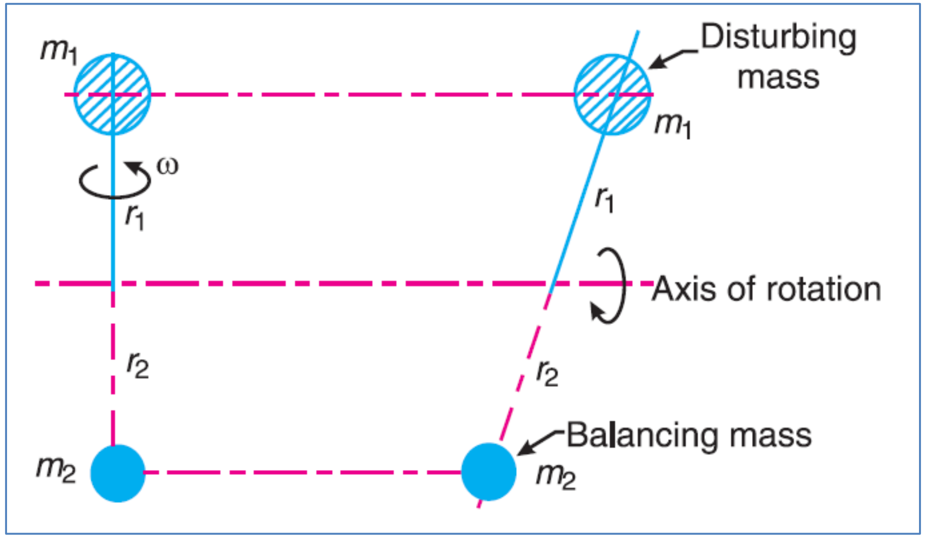
∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \quad \ldots(2)$$

❑ Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Equating equations *(i)* and *(ii)*,

$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$



Notes :

1. The product $m_2.r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.
2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

- We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced.
- In other words, the two forces are equal in magnitude and opposite in direction.
- But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings.
- Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing.***
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

- The conditions (1) and (2) together give dynamic balancing. The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

❑ Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

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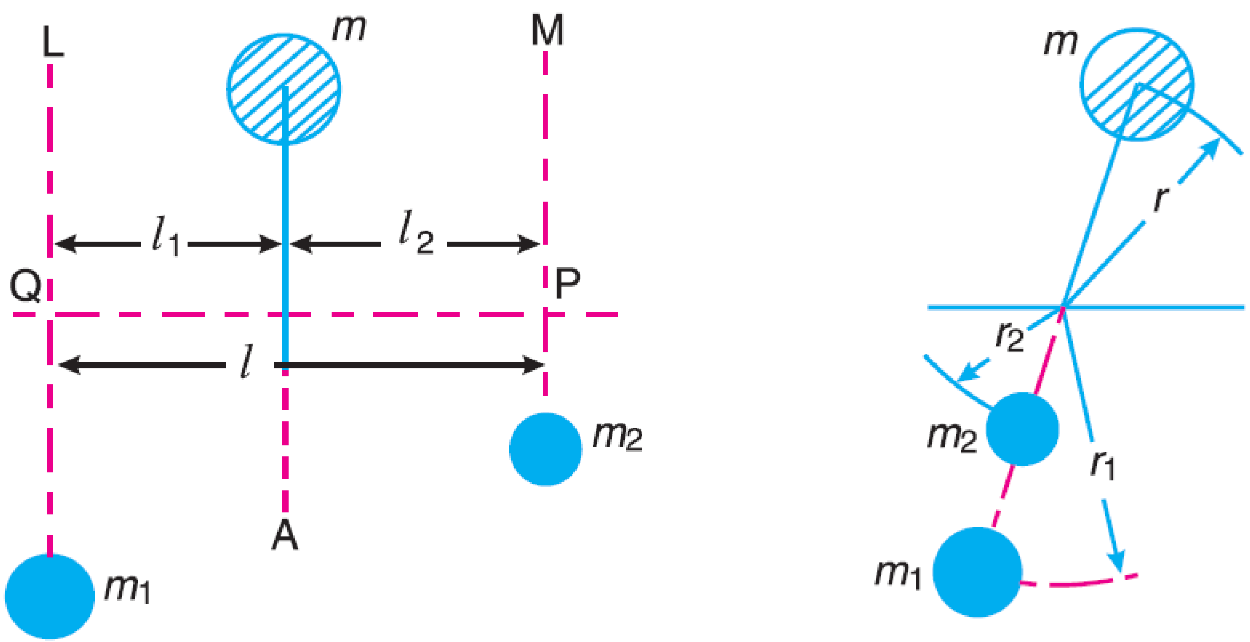
•Khurmi, R. et al.; Theory of Machines, 14th ed.

1. When the plane of the disturbing mass lies in between the planes of the two balancing masses

- Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in Fig.



- Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively.
- Let l_1 = Distance between the planes A and L,
- l_2 = Distance between the planes A and M, and
- l = Distance between the planes L and M.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

- We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

- Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

- and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

- Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad or \quad m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad ...(1)$$

- Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad or \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore m_1 \cdot r_1 \times l = m \cdot r \times l_2 \qquad ...(2)$$

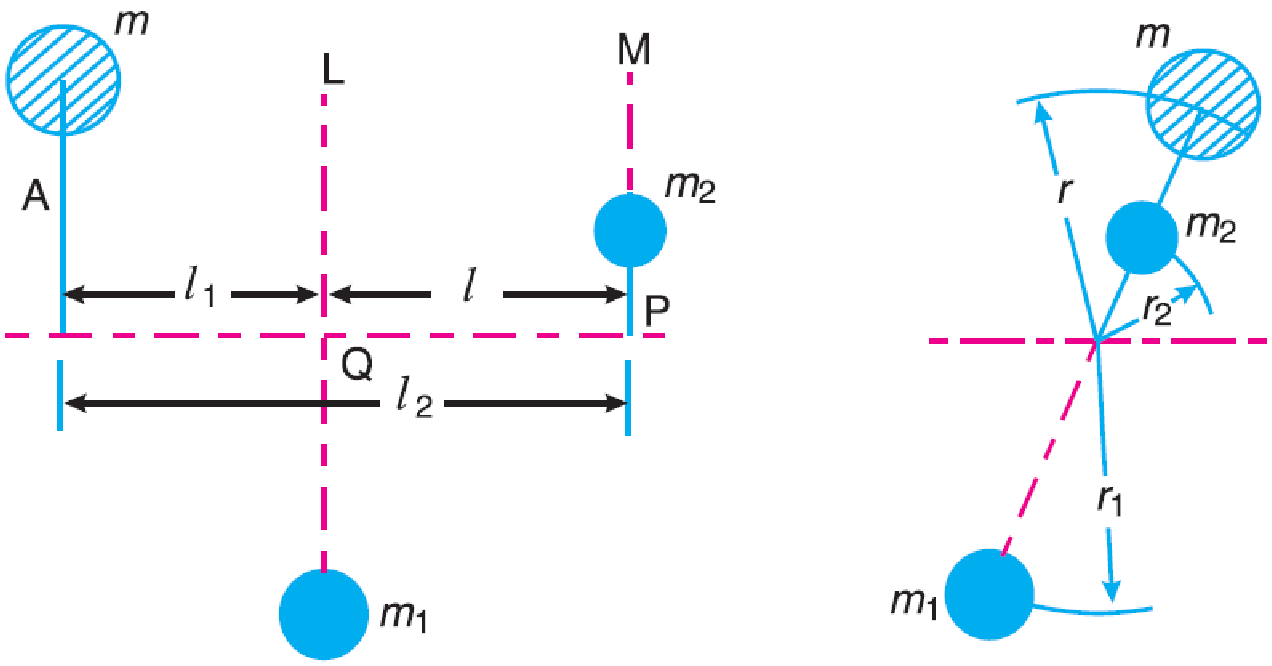
- Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad or \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore m_2 \cdot r_2 \times l = m \cdot r \times l_1 \qquad ...(3)$$

1. When the plane of the disturbing mass lies on one end of the planes of the balancing masses

- In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig.



- As discussed, the following conditions must be satisfied in order to balance the system, i.e.

$$F_C + F_{C2} = F_{C1} \quad or \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \qquad ...(4)$$

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- Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \qquad or \qquad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore m_1 \cdot r_1 \times l = m \cdot r \times l_2 \qquad \ldots(5)$$

- Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \qquad or \qquad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

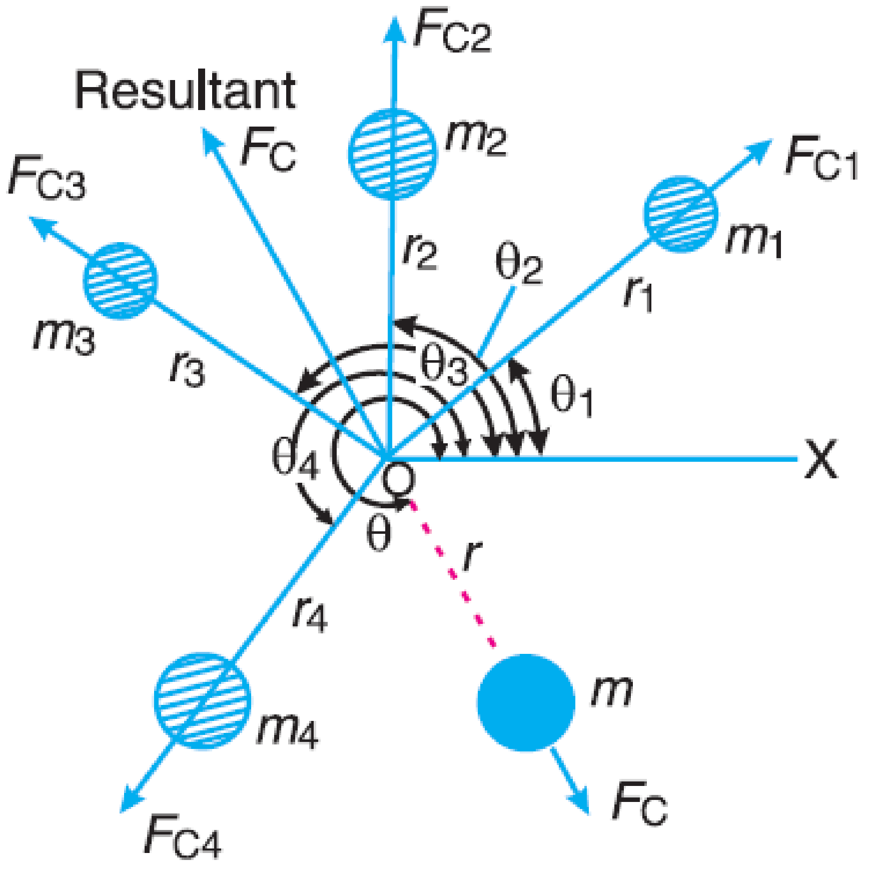
$$\therefore m_2 \cdot r_2 \times l = m \cdot r \times l_1 \qquad \ldots(6)$$

❑ Balancing of Several Masses Rotating in the Same Plane

- Consider any number of masses (say four) of magnitude m_1, m_2, m_3 *and* m_4 at distances of r_1, r_2, r_3 *and* r_4 from the axis of the rotating shaft. Let $\omega_1, \omega_2, \omega_3$ *and* ω_4 be the angles of these masses with the horizontal line OX, as shown in Fig. (a).

- Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

- The magnitude and position of the balancing mass may be found out by:-
  1. Analytical Method
  2. Graphical Method



(*a*) Space diagram.

❑ Balancing of Several Masses Rotating in the Same Plane

1. Analytical Method

- The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :
  1. First of all, find out the centrifugal force (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.
  2. Resolve the centrifugal forces horizontally and vertically and find their sums, i.e. ΣH and ΣV . We know that
  - Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

  - and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

  3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

  4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

  5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
  6. Now find out the magnitude of the balancing mass, such that

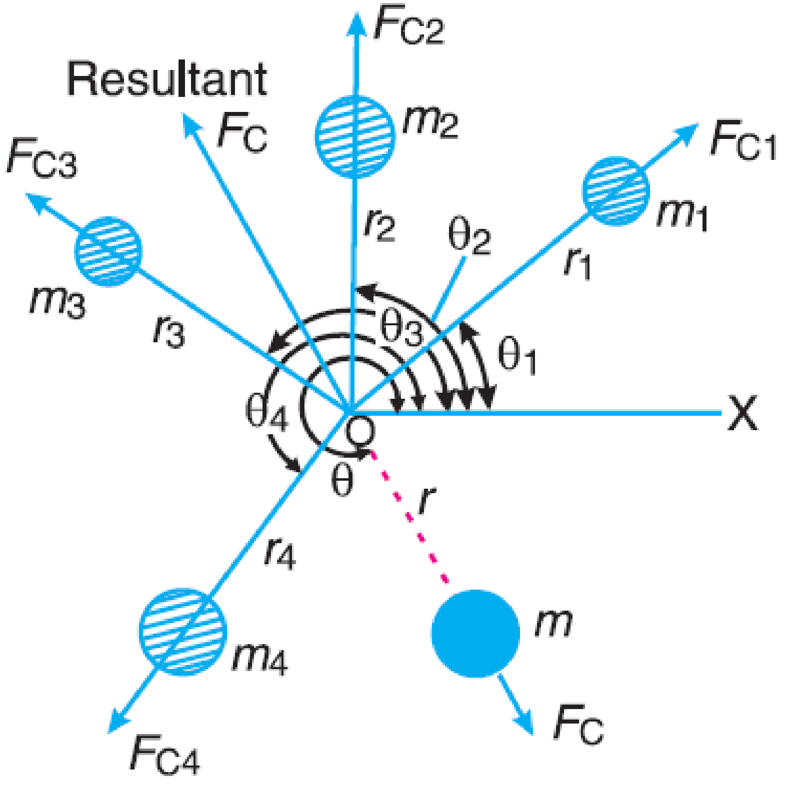
$$F_C = m \cdot r$$

•Khurmi, R. et al.; Theory of Machines, 14th ed.

❑ Balancing of Several Masses Rotating in the Same Plane

2. Graphical Method

- The magnitude and position of the balancing mass may also be obtained graphically as discussed below:
  1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (a).
  2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
  3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that ab represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale.
     - Similarly, draw *bc, cd* and *de* to represent centrifugal forces of other masses m_2, m_3 *and* m_4 *(or* $m_2.r_2$, $m_3.r_3$ *and* $m_4.r_4$*).*



(a) Space diagram.

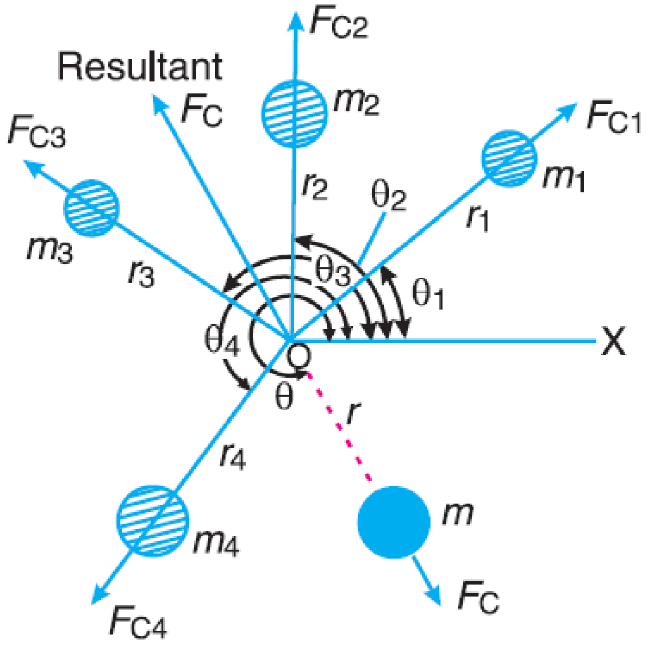
❑ Balancing of Several Masses Rotating in the Same Plane

2. Graphical Method

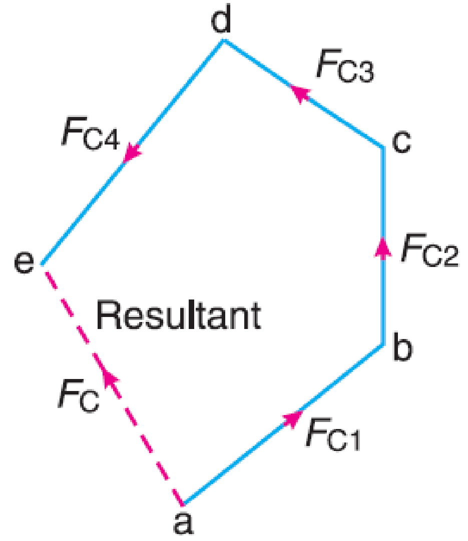
4. Now, as per polygon law of forces, the closing side ae represents the resultant force in magnitude and direction, as shown in Fig. (b).
5. The balancing force is, then, equal to the resultant force, but in opposite direction.
6. Now find out the magnitude of the balancing mass *(m)* at a given radius of rotation *(r)*, such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

or $m.r$ = Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ *and* $m_4.r_4$



(*a*) Space diagram.



(*b*) Vector diagram.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

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- Consider any number of masses (say four) of magnitude m_1, m_2, m_3 *and* m_4 at distances of r_1, r_2, r_3 *and* r_4 from the axis of the rotating shaft. Let $\omega_1, \omega_2, \omega_3$ *and* ω_4 be the angles of these masses with the horizontal line OX, as shown in Fig. (a).

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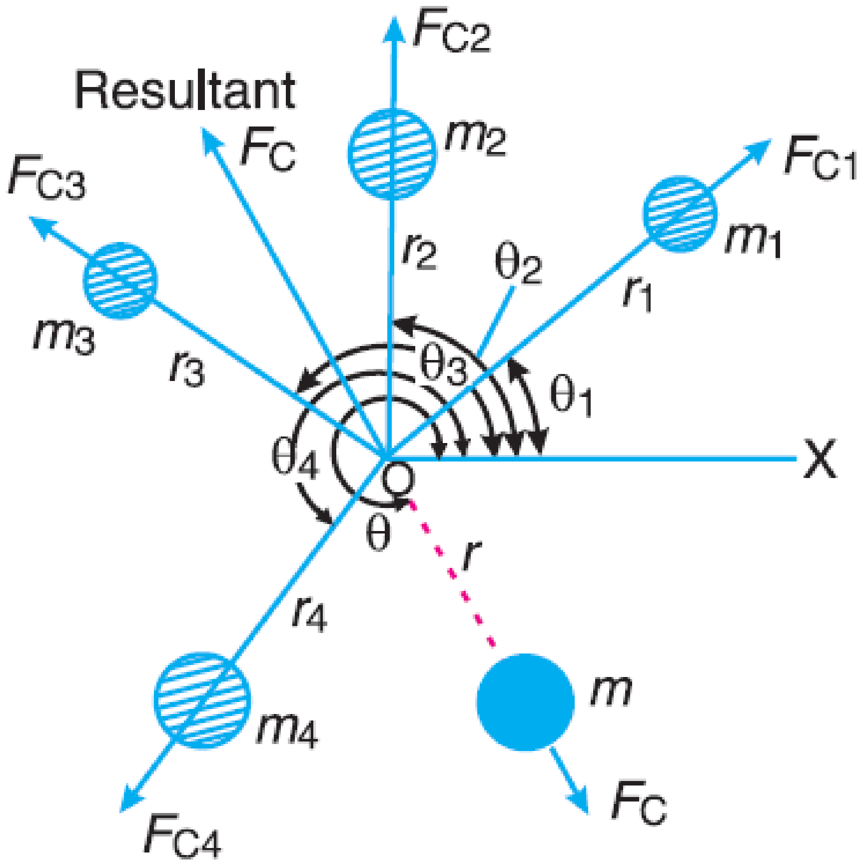

(a) Space diagram.

❑ Balancing of Several Masses Rotating in the Same Plane

1. Analytical Method

➢ The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

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➢ Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

➢ and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass, such that

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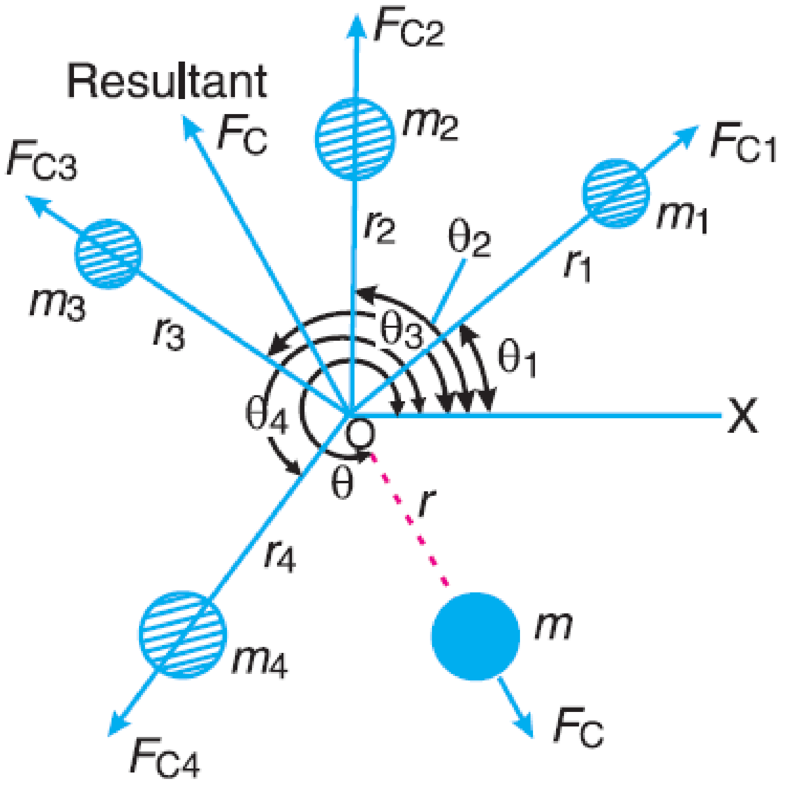
❑ Balancing of Several Masses Rotating in the Same Plane

2. Graphical Method

- The magnitude and position of the balancing mass may also be obtained graphically as discussed below:
  1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (a).
  2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
  3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that ab represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale.
     - Similarly, draw *bc, cd* and *de* to represent centrifugal forces of other masses m_2, m_3 *and* m_4 *(or* $m_2.r_2$, $m_3.r_3$ *and* $m_4.r_4$*).*



(*a*) Space diagram.

❑ Balancing of Several Masses Rotating in the Same Plane

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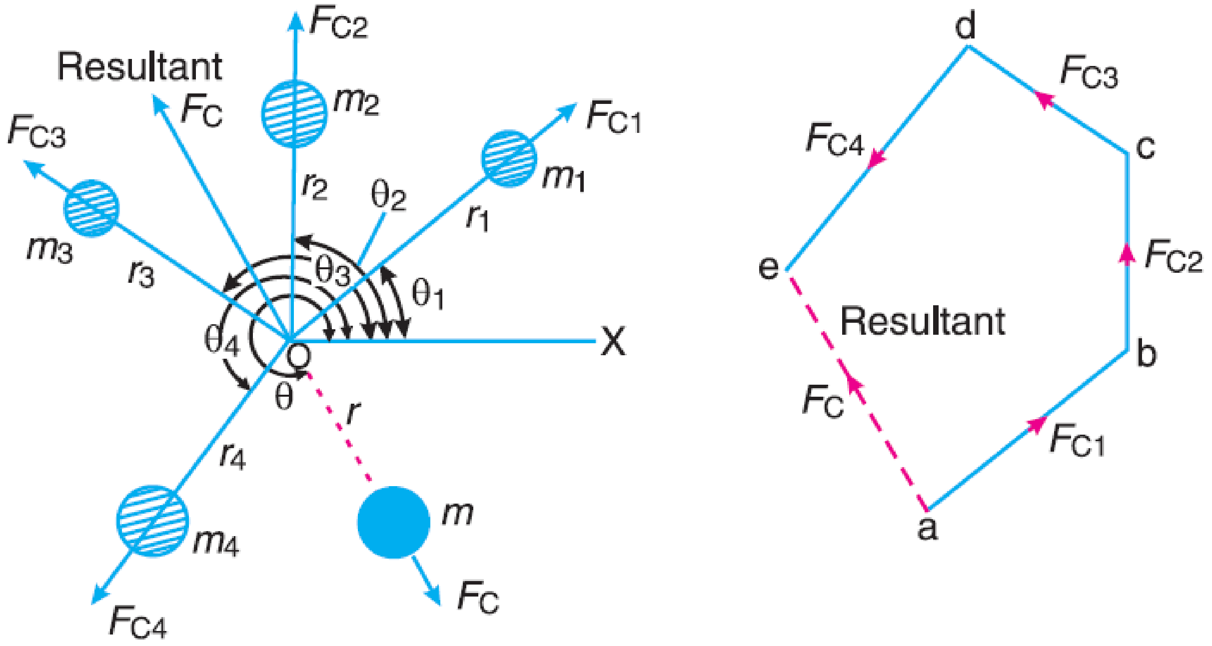
4. Now, as per polygon law of forces, the closing side ae represents the resultant force in magnitude and direction, as shown in Fig. (b).
5. The balancing force is, then, equal to the resultant force, but in opposite direction.
6. Now find out the magnitude of the balancing mass *(m)* at a given radius of rotation *(r)*, such that

$m \cdot \omega^2 \cdot r$ = Resultant centrifugal force

or $m.r$ = Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ *and* $m_4.r_4$



(*a*) Space diagram. (*b*) Vector diagram.

❑ ***Example 1:*** ***Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.***

Solution. Given : $m_1 = 200$ kg ; $m_2 = 300$ kg ; $m_3 = 240$ kg ; $m_4 = 260$ kg ; $r_1 = 0.2$ m ; $r_2 = 0.15$ m ; $r_3 = 0.25$ m ; $r_4 = 0.3$ m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m

Let m = Balancing mass, and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

1. *Analytical method*

The space diagram is shown in Fig.

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$$

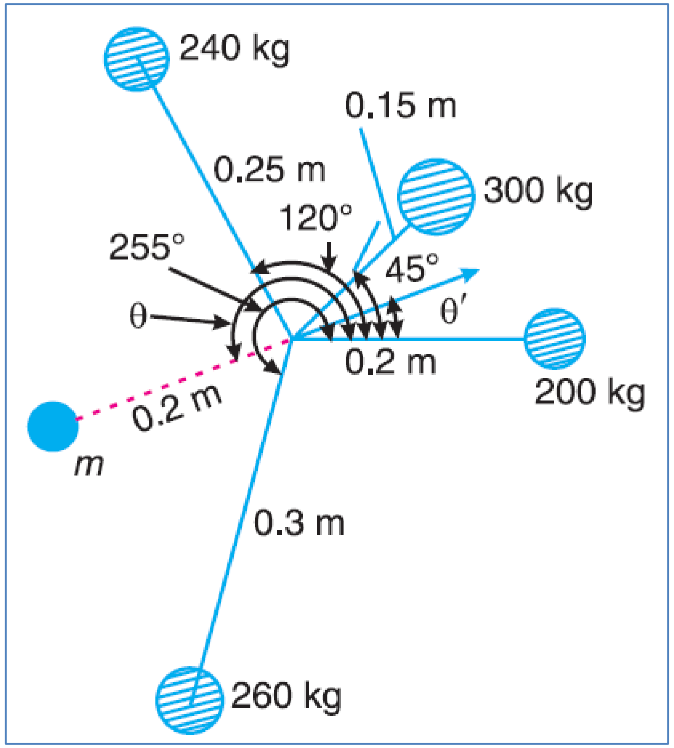
$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$



$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2 / 0.2 = 116 \text{ kg} \quad \textbf{Ans.}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48° \quad \textbf{Ans.}$$

2. *Graphical method*

The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig (*a*).
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

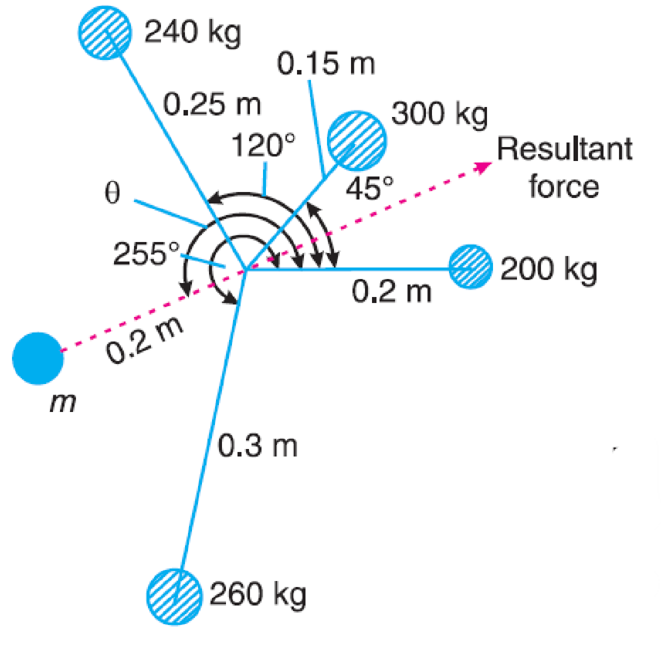
$$m_1.r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$
$$m_2.r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$
$$m_3.r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$
$$m_4.r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

3. Now draw the vector diagram with the above values, to some suitable scale, as shown in Fig. (*b*). The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.
4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. (*a*). Since the balancing force is proportional to *m.r*, therefore

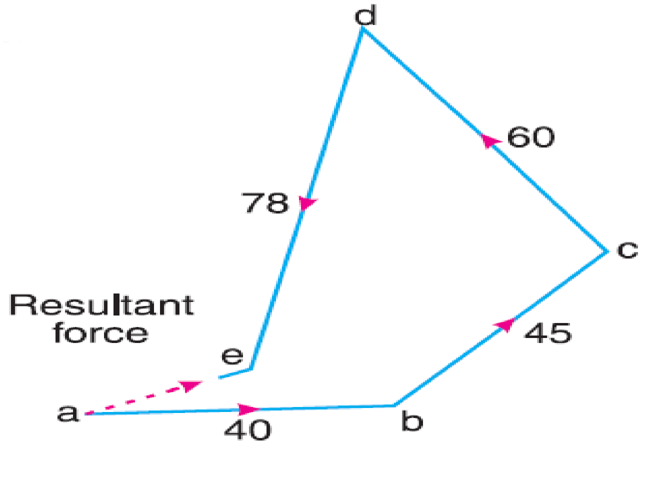
$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m} \quad \text{or} \quad m = 23/0.2 = \textbf{115 kg Ans.}$$

By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal mass of 200 kg,

$$\theta = 201° \textbf{ Ans.}$$



(*a*) Space diagram.



(*b*) Vector diagram

❑ Balancing of several masses rotating in different planes

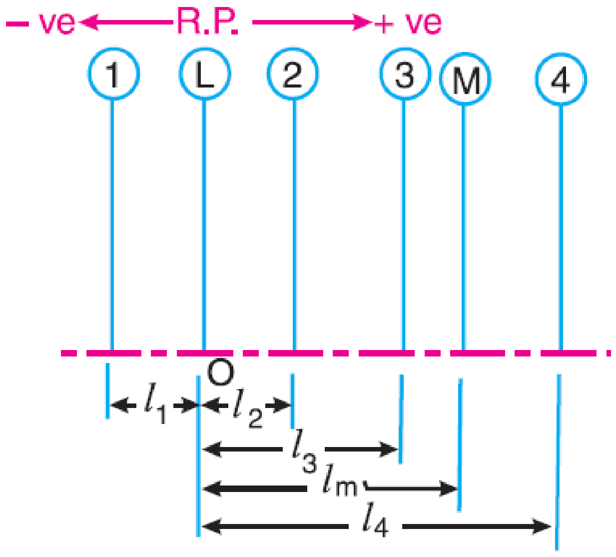
- When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it.
- The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane.
- In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

  1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
  2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

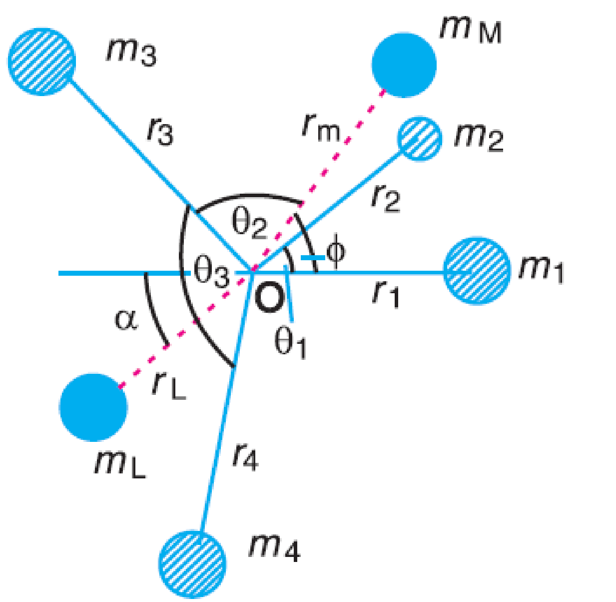
❑ Balancing of several masses rotating in different planes

- Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. (*a*).
- The relative angular positions of these masses are shown in the end view [Fig. (*b*)].
- The magnitude of the balancing masses m_L and m_M in planes *L* and *M* may be obtained as discussed below :

1. Take one of the planes, say *L* as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative*,** and those to the right as ***positive*.**



(*a*) Position of planes of the masses.



(*b*) Angular position of the masses.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

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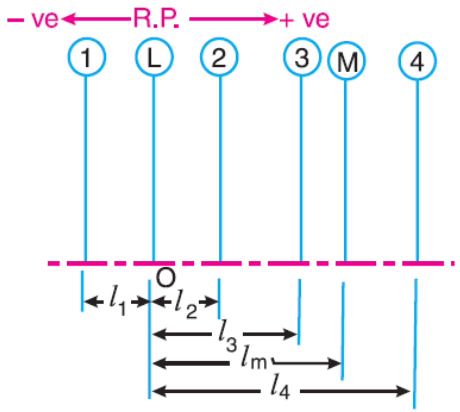
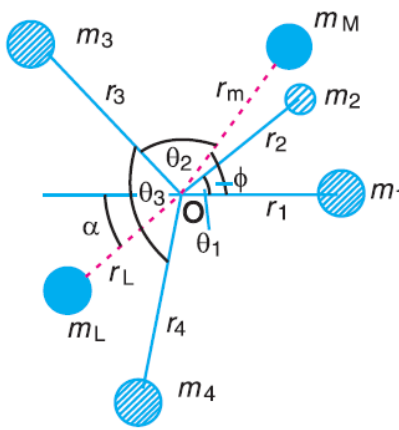
❑ Balancing of several masses rotating in different planes

2. Tabulate the data as shown in Table 1. The planes are tabulated in the same order in which they occur, reading from left to right.

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses.

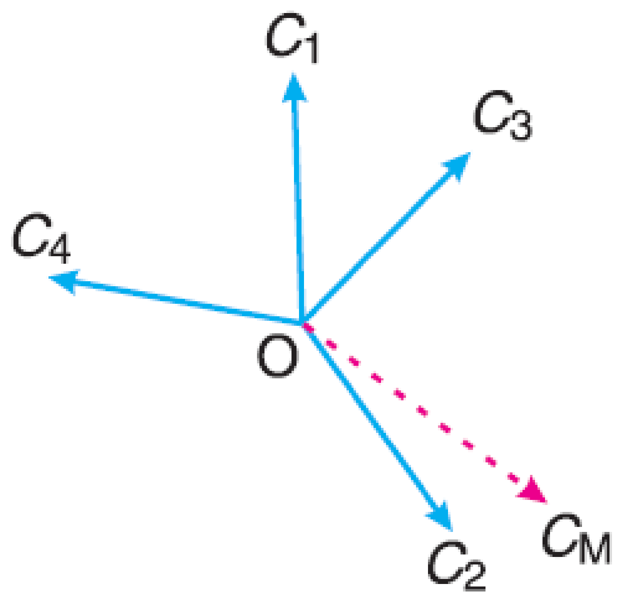


(*b*) Angular position of the masses.

❑ Balancing of several masses rotating in different planes

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is proportional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper.

- The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. (*c*).
- Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

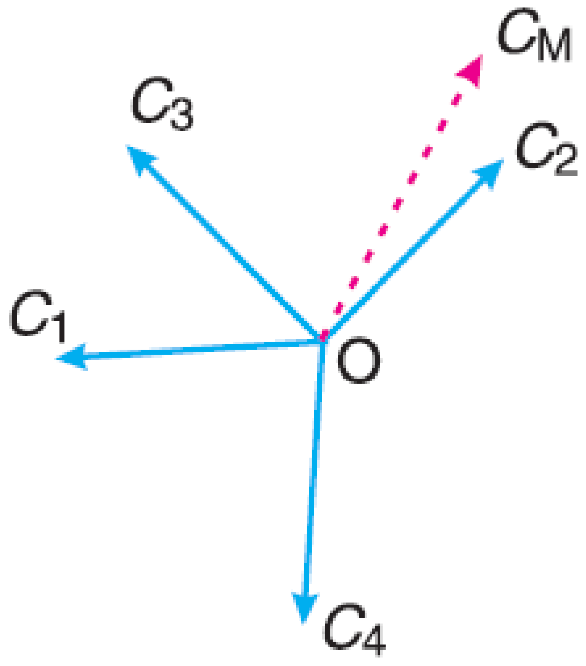


(*c*) Couple vector.

❑ Balancing of several masses rotating in different planes

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. (*d*).

- We see that their relative positions remains unaffected.
- Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in opposite direction.
- Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***



(*d*) Couple vectors turned counter clockwise through a right angle.

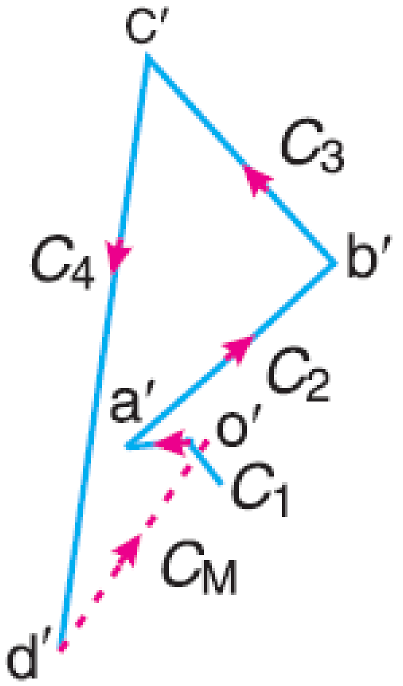
❑ Balancing of several masses rotating in different planes

5. Now draw the couple polygon as shown in Fig. (*e*).

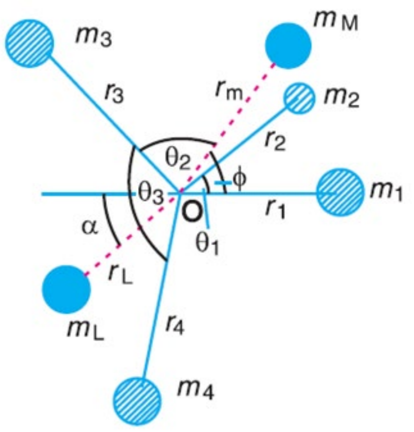
- The vector *d o* represents the balanced couple.
- Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

- From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination of this mass may be measured from Fig. (*b*).



(*e*) Couple polygon.



(*b*) Angular position of the masses.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

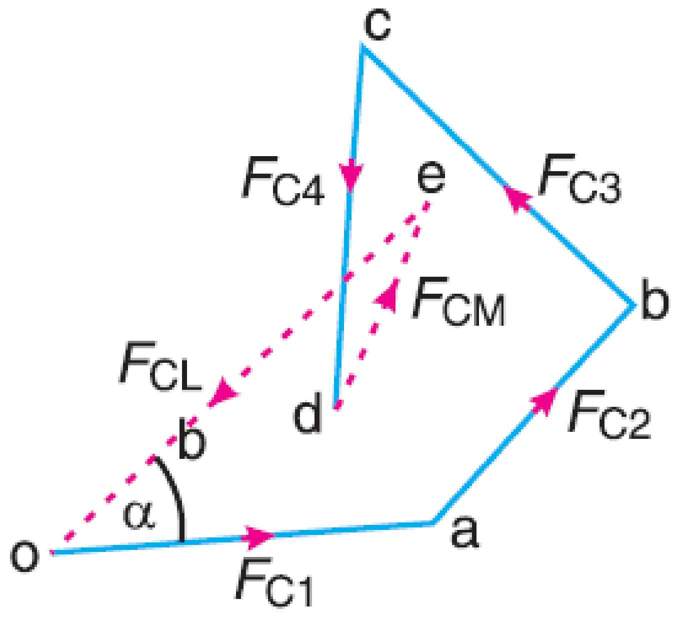
❑ Balancing of several masses rotating in different planes

5. Now draw the force polygon as shown in Fig. (*f*).

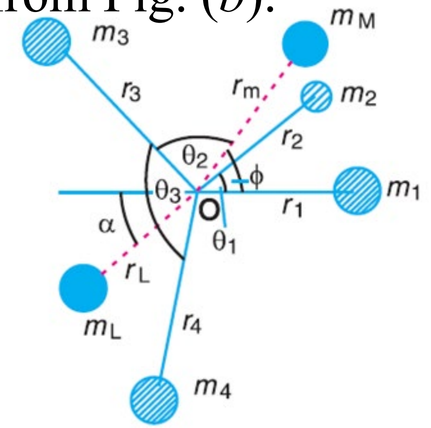
- The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

- From this expression, the value of the balancing mass m_L in the plane *L* may be obtained and the angle of inclination of this mass with the horizontal may be measured from Fig. (*b*).



(*f*) Force polygon.



(*b*) Angular position of the masses.

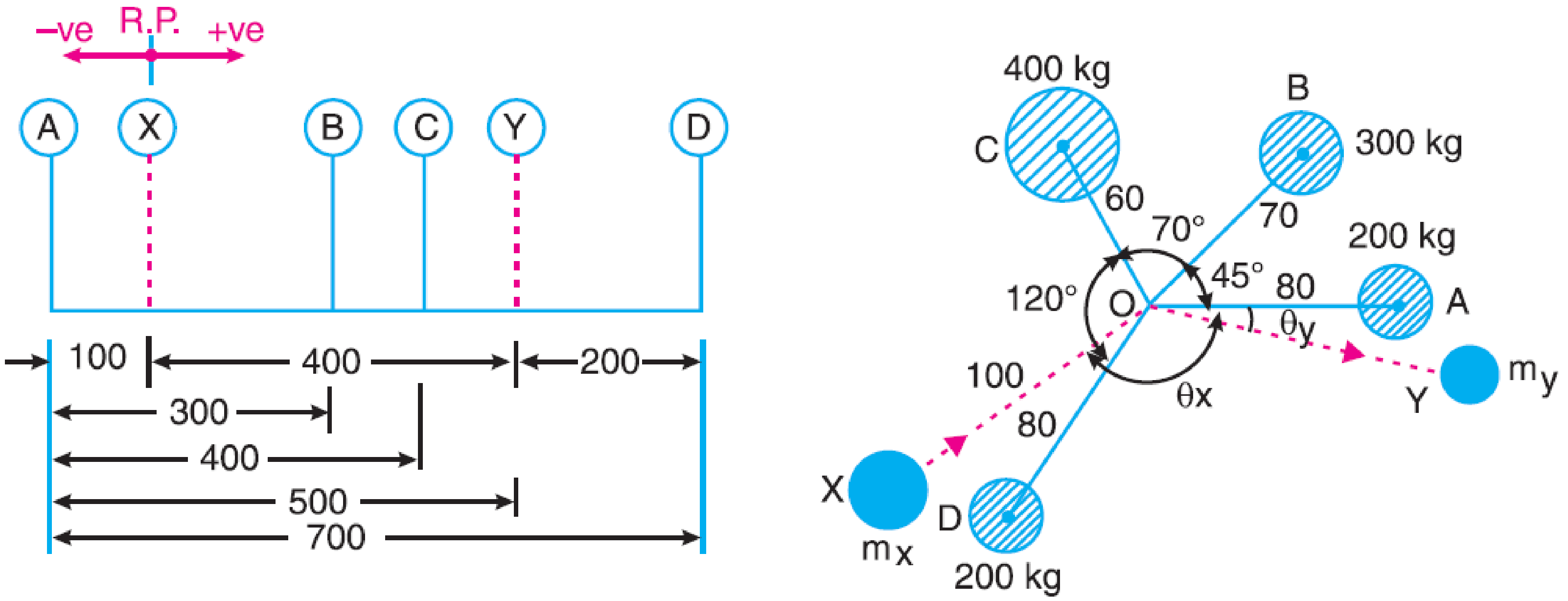
•Khurmi, R. et al.; Theory of Machines, 14th ed.

- **Example :-** *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; $r_X = r_Y$ = 100 mm = 0.1 m

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm = 0.08m ; $r_B = 70$ mm = 0.07 m ; $r_C = 60$ mm = 0.06 m ; $r_D = 80$ mm = 0.08 m ; $r_X = r_Y = 100$ mm = 0.1 m

- Let m_X = Balancing mass placed in plane *X*, and m_Y = Balancing mass placed in plane *Y*.
- The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. (*a*) and (*b*) respectively.



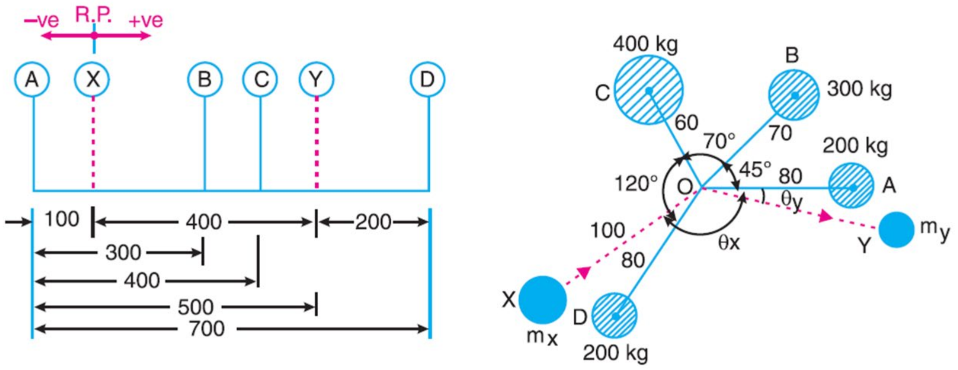
All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

- Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve.

➢ The data may be tabulated as shown in Table

Plane (1)	*Mass (m)* *kg* (2)	*Radius (r)* *m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

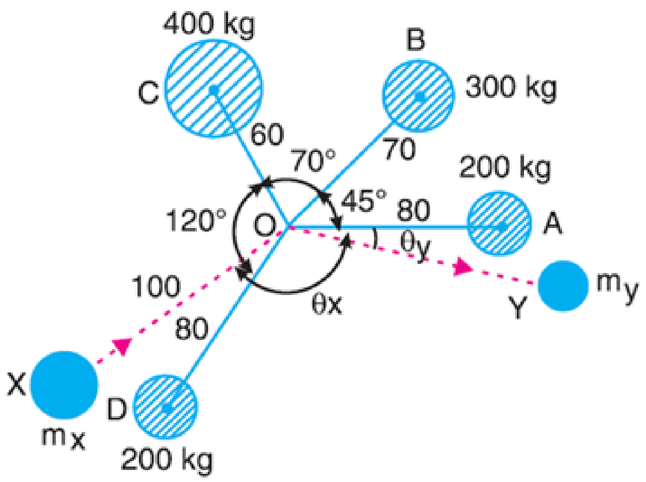
•Khurmi, R. et al.; Theory of Machines, 14th ed.

1. First of all, draw the couple polygon from the data given in Table (column 6) as shown in Fig. (*c*) to some suitable scale.

- The vector *d'o'* represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by

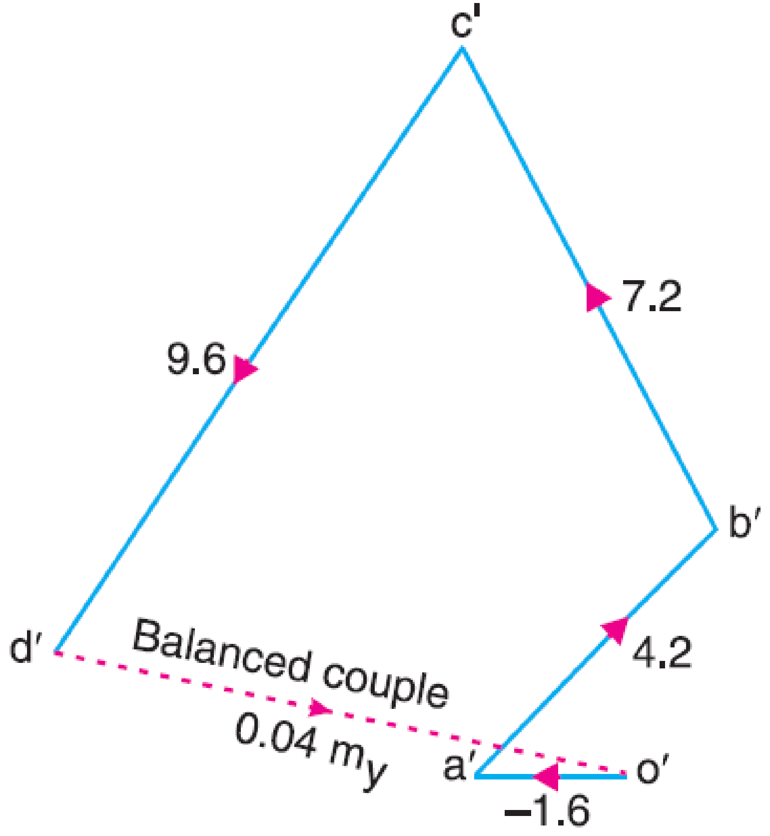
$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg}$$ **Ans.**

- The angular position of the mass m_Y is obtained by drawing m_Y in Fig. (*b*), parallel to vector *d'o'*.
- By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**



ons in mm.

(*b*) Angular position of masses.



(*c*) Couple polygon.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

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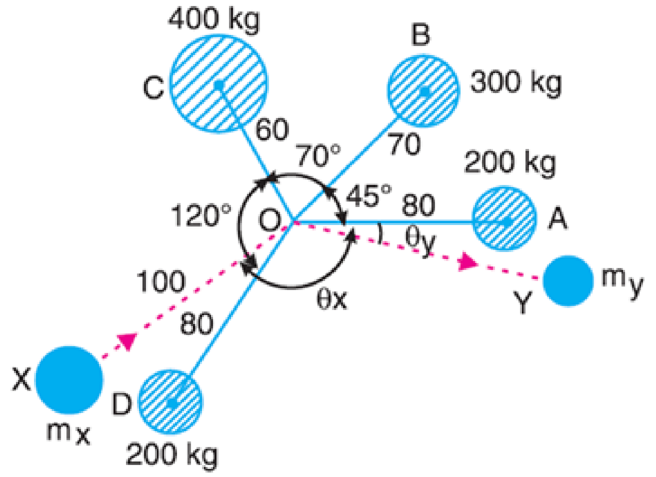
Now draw the force polygon from the data given in Table (column 4) as shown in Fig. (*d*).

The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

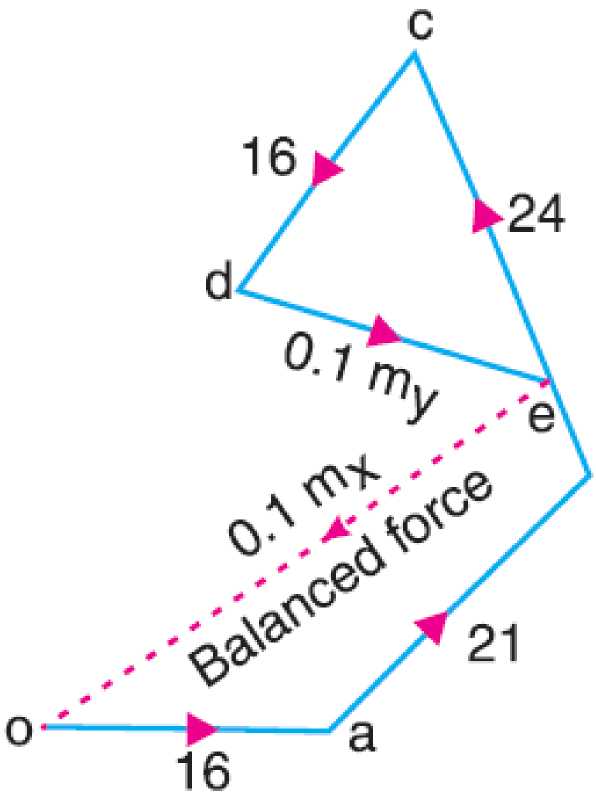
$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = \mathbf{355 \text{ kg Ans.}}$$

- The angular position of the mass m_X is obtained by drawing Om_X in Fig. (*b*), parallel to vector *eo*.
- By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**



ons in mm.

(*b*) Angular position of masses.



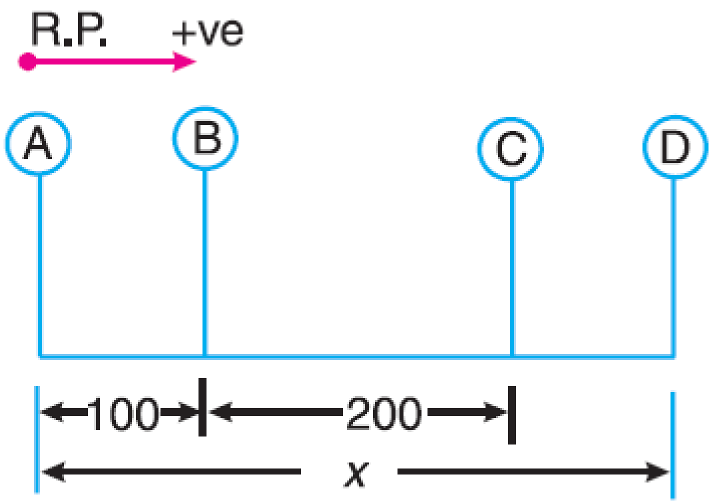
(*d*) Force polygon.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

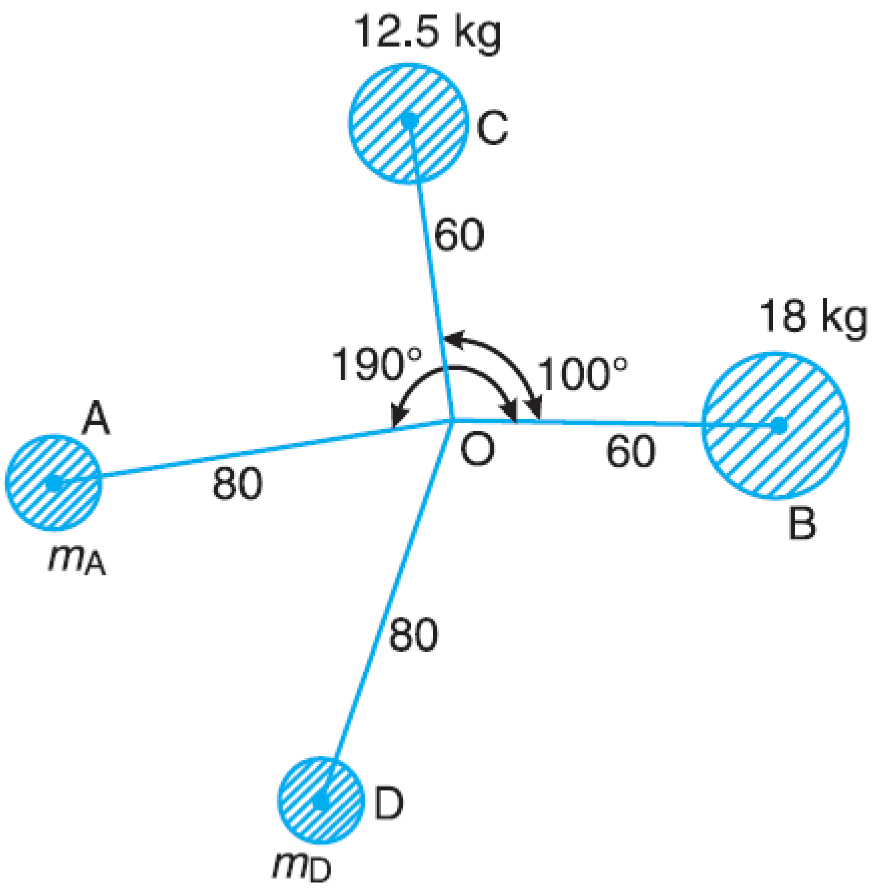
❑ ***Example 2 :-*** *A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :* ***1.*** *The magnitude of the masses at A and D;* ***2.*** *the distance between planes A and D; and* ***3.*** *the angular position of the mass at D.*

Solution. Given : $m_B = 18$ kg ; $m_C = 12.5$ kg ; $r_B = r_C = 60$ mm $= 0.06$ m ; $r_A = r_D = 80$ mm $= 0.08$ m ; $\angle BOC = 100°$; $\angle BOA = 190°$

The position of the planes and angular position of the masses is shown in Fig. (*a*) and (*b*) respectively.



(*a*) Position of planes.



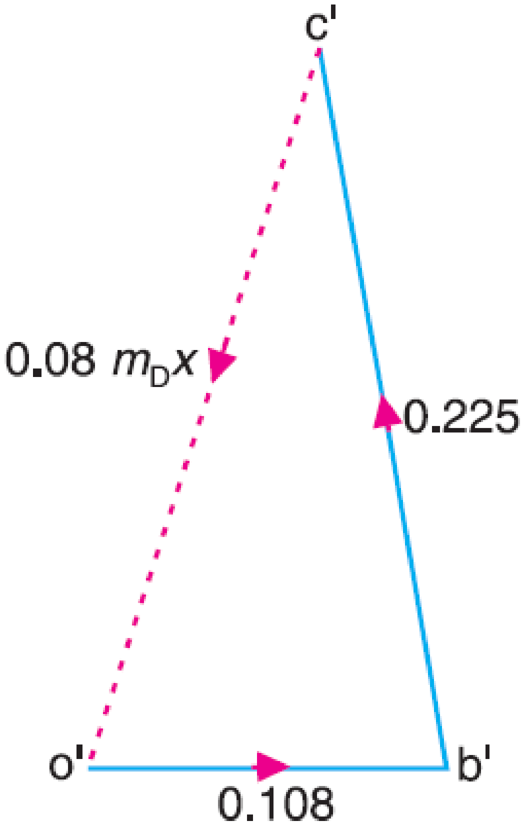
All dimensions in mm.

(*b*) Angular position of masses.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

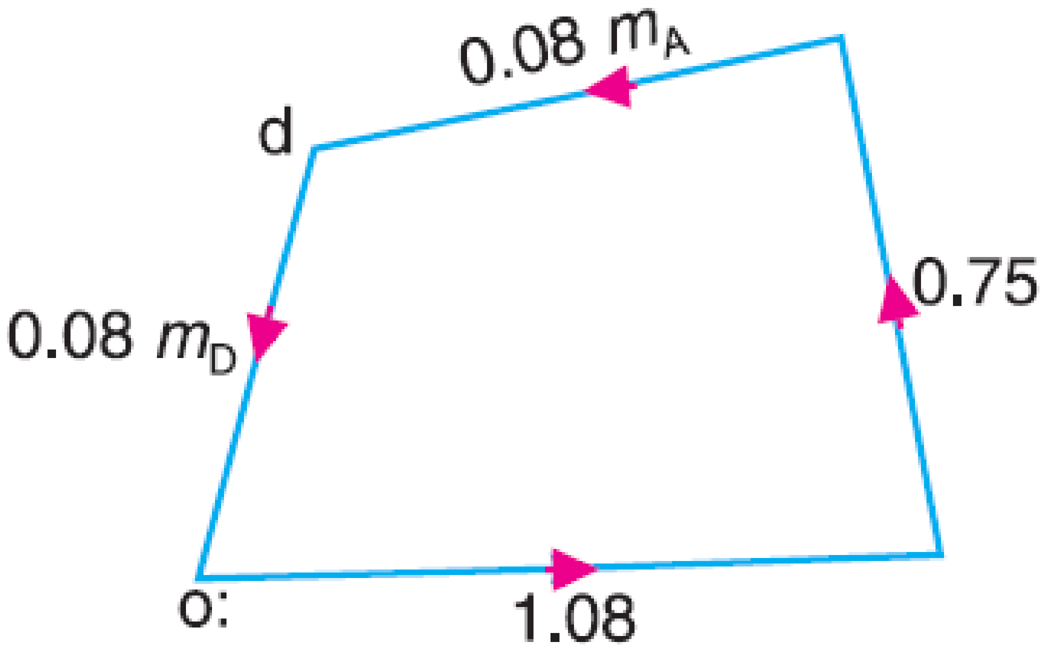
- The data may be tabulated as shown in Table

Plane (1)	*Mass (m) kg* (2)	*Eccentricity (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane A(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A (R.P.)	m_A	0.08	0.08 m_A	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	0.08 m_D	x	0.08 m_D . x



(c) Couple polygon.

$0.08\ m_D.x$ = vector $c'\ o'$ = 0.235 kg-m^2



(*d*) Force polygon.

$0.08\ m_A$ = vector *cd* = 0.77 kg-m or m_A = 9.625 kg **Ans.**

and vector *do* is proportional to $0.08\ m_D$, therefore by measurement,

$0.08\ m_D$ = vector *do* = 0.65 kg-m or m_D = 8.125 kg **Ans.**

•Khurmi, R. et al.; Theory of Machines, 14th ed.

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2. *Distance between planes A and D*

From equation (*i*),

$$0.08\ m_D.x = 0.235\ \text{kg-m}^2$$

$$0.08 \times 8.125 \times x = 0.235\ \text{kg-m}^2 \quad \text{or} \quad 0.65\ x = 0.235$$

$$\therefore \quad x = \frac{0.235}{0.65} = 0.3615\text{m} = 361.5\ \text{mm}$$ **Ans.**

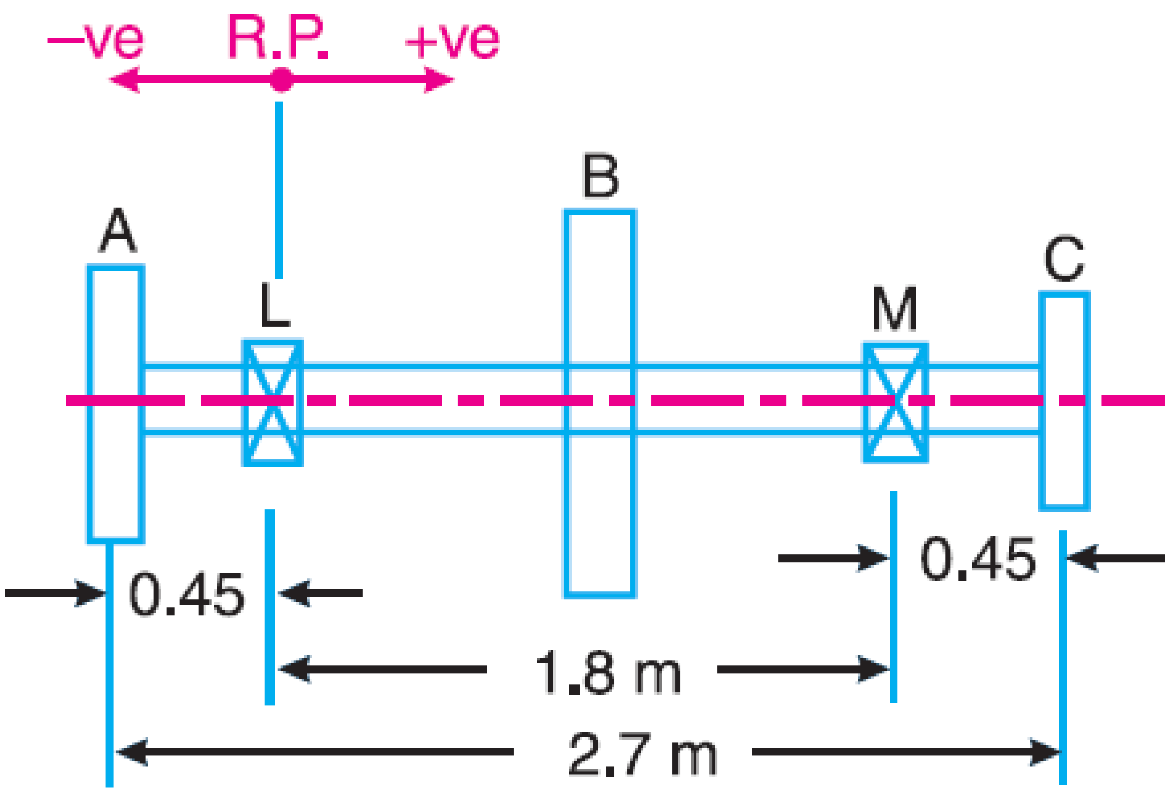
3. *Angular position of mass at D*

By measurement from Fig. (*b*), we find that the angular position of mass at *D* from mass *B* in the anticlockwise direction, *i.e.* $\angle BOD = 251°$ **Ans.**

- ***Example 3 :-*** *A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine :* ***1.*** *relative angular positions of the pulleys, and* ***2.*** *dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.*

Solution. Given : $m_A = 48$ kg ; $m_C = 20$ kg ; $r_A = 15$ mm = 0.015 m ; $r_C = 12.5$ mm = 0.0125 m ; $m_B = 56$ kg ; $r_B = 15$ mm = 0.015 m ; $N = 300$ *r.p.m.* or = 2 × 300/60 = 31.42 rad/s

- The position of the shaft and pulleys is shown in Fig. (*a*).



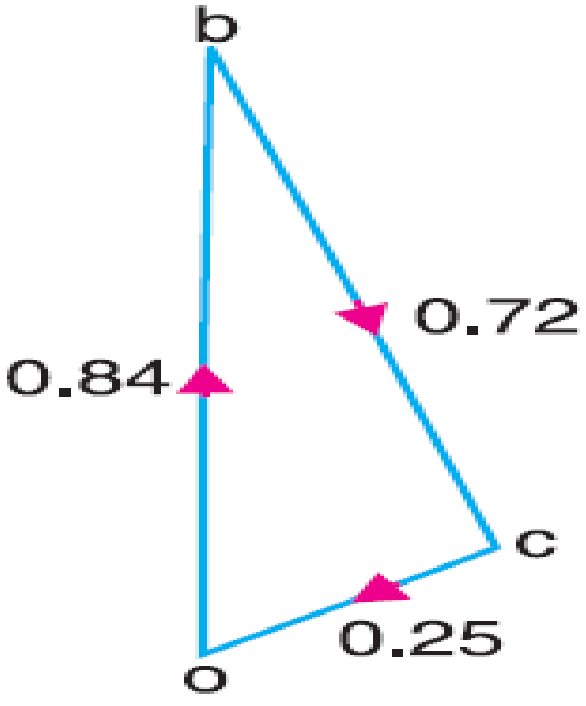
(*a*) Position of shaft and pulleys.

•Khurmi, R. et al.; Theory of Machines, 14th ed.

- The data may be tabulated as shown in Table

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane L(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	48	0.015	0.72	– 0.45	– 0.324
L(R.P)	m_L	r_L	$m_L.r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M.r_M$	1.8	1.8 $m_M.r_M$
C	20	0.0125	0.25	2.25	0.5625

•Khurmi, R. et al.; Theory of Machines, 14th ed.

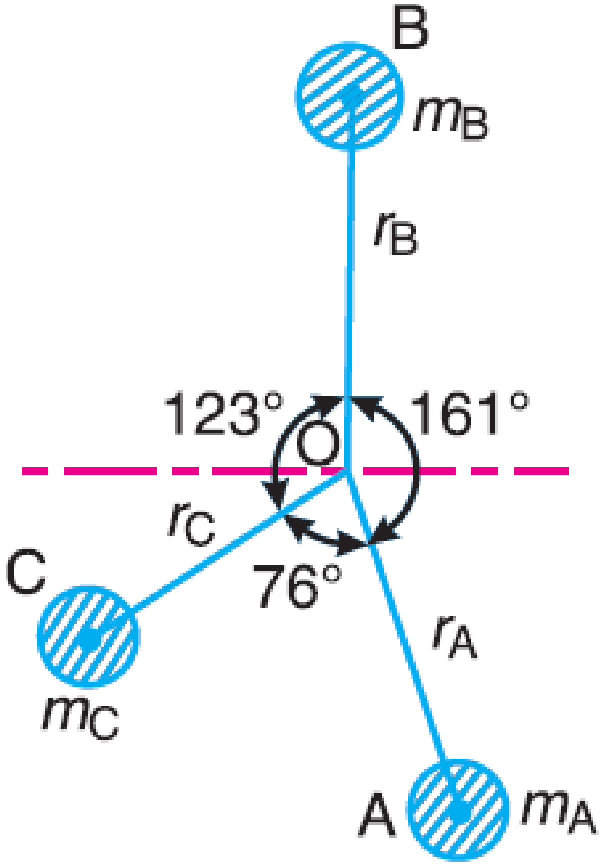


(c) Force polygon.

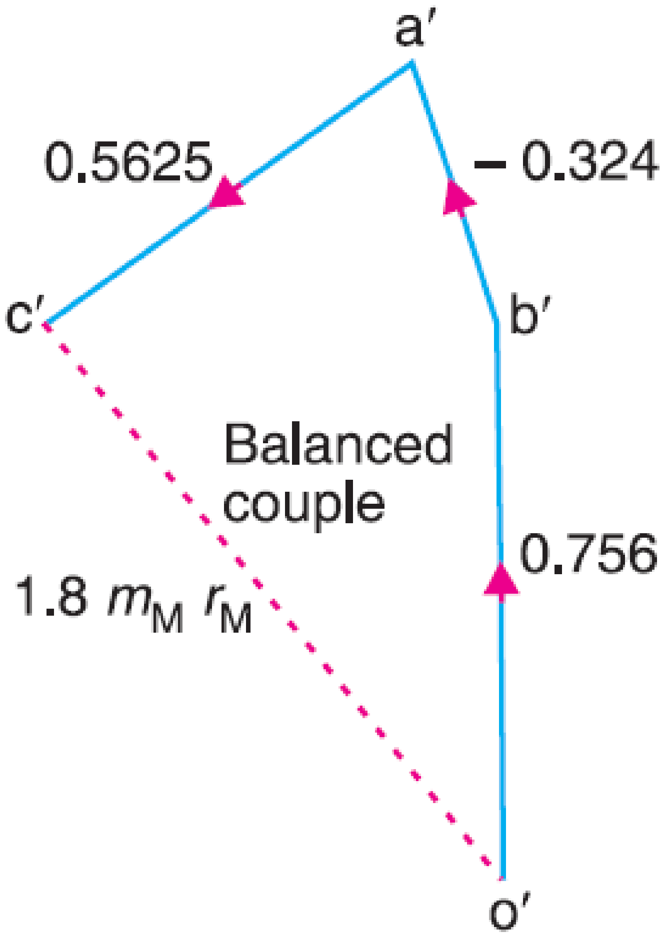
Angle between pulleys B and A = 161° **Ans.**

Angle between pulleys A and C = 76° **Ans.**

Angle between pulleys C and B = 123° **Ans.**



(b) Angular position of pulleys.

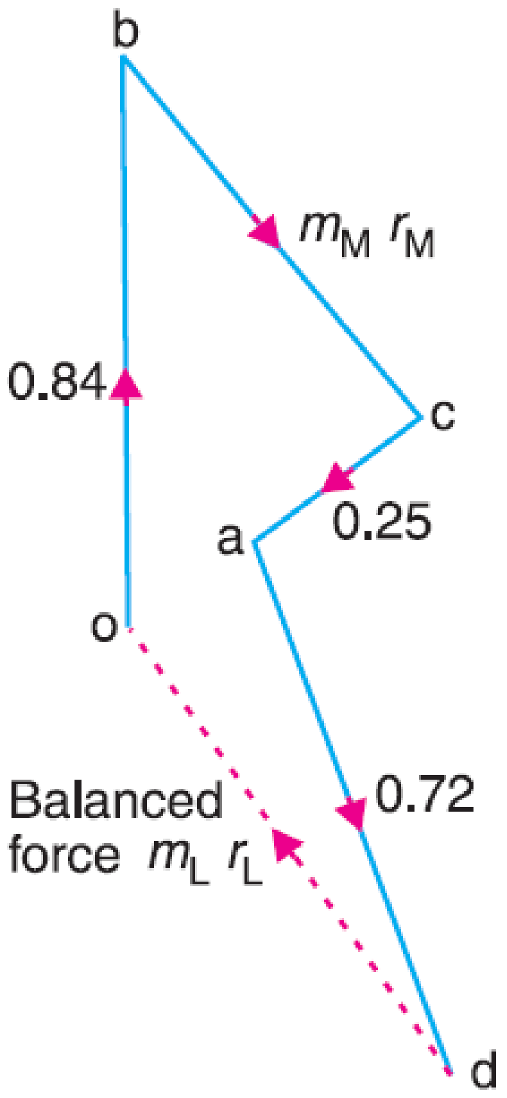


(*d*) Couple polygon.

$1.8\ m_M.r_M$ = vector $c'o'$ = 0.97 kg-m^2 or $m_M.r_M$ = 0.54 kg-m

∴ Dynamic force at the bearing *M*

$$= m_M.r_M.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N}$$ **Ans.**



(*e*) Force polygon.

$$m_L.r_L = 0.54 \text{ kg-m}$$

$\therefore$ Dynamic force at the bearing *L*

$$= m_L.r_L.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N } \textbf{Ans.}$$

Out-of-balance couple

$$= \text{vector } o'\,c' = 0.97 \text{ kg-m}^2$$

$$= 0.97 \times \omega^2 = 0.97\ (31.42)^2 = 957.6 \text{ N-m}$$

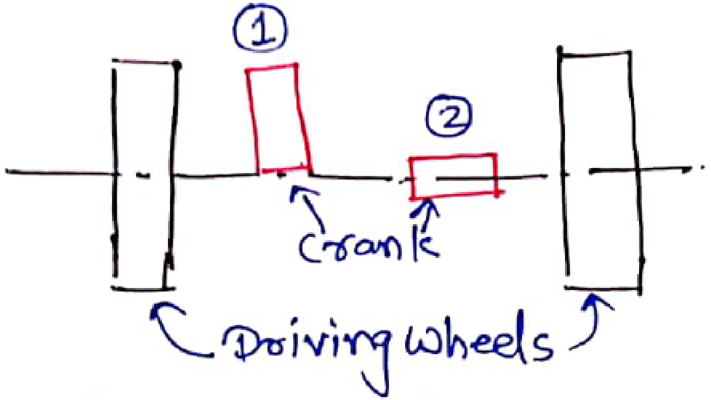
Dynamic force on each bearing

$$= \frac{\text{Out-of-balance couple}}{\text{Distance between bearings}} = \frac{957.6}{1.8} = 532 \text{ N } \textbf{Ans.}$$

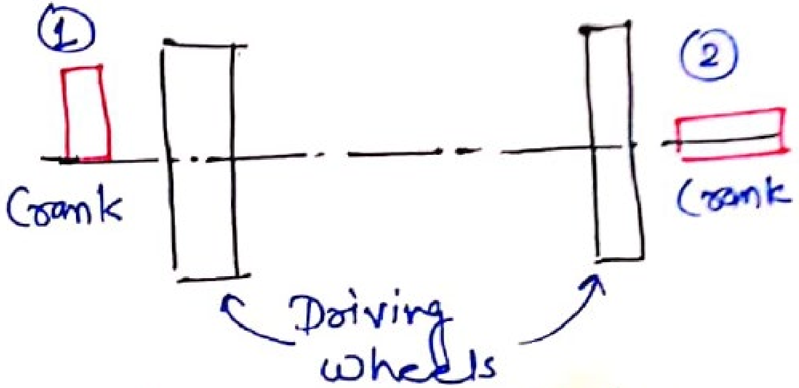
❑ Partial Balancing of Locomotives

- Two cylinder placed at right angle.

* Four types :- 1) Inside cylinder locomotives
2) Outside " "
3) Single or uncoupled locomotives
4) Coupled locomotives



a) Inside Cylinder



b) Outside cylinder.

❑ Effect of Partial Balancing of Locomotives

- Two unbalanced forces:-
  - i) Along the line of stroke. ie. F_H
  - ii) Perpendicular to the line of stroke. ie. F_V
- The effect of F_H produces Variation of Tractive force along the line of stroke and couple of such force is known as Swaying Couple.
- The effect of F_V is to produce variation of pressure on Rails which causes hammering action on rails called as Hammer Blow.

Variation of Tractive Force (F_T)

→ Unbalanced force along line of stroke for Cylinder 1,

$$F_{HU1} = (1-c)\, m\omega^2 r \cos\theta$$

→ Unbalanced force along line of stroke for Cylinder 2,

$$F_{HU2} = (1-c)\, m\omega^2 r \cos(90+\theta)$$

$$F_{HU2} = (1-c)\, m\omega^2 r \sin\theta$$

→ Resultant Unbalance force or Tractive Force F_T,

$$F_T = F_{HU1} + F_{HU2}$$

$$F_T = (1-c)\, m\omega^2 r [\cos\theta - \sin\theta]$$

Variation of Tractive Force (F_T)

→ To have Maximum or Minimum Value.

$$\frac{dF_T}{d\theta} = 0. \Rightarrow \frac{d}{d\theta}\left[(1-c)\,m\omega^2 r\,(\cos\theta - \sin\theta)\right] = 0$$

$$-\sin\theta - \cos\theta = 0.$$

$$\sin\theta = -\cos\theta$$

$$\tan\theta = -1$$

or $\theta = 135^\circ$ and 315°

→ Minimum Value of F_T for $\theta = 135^\circ$

$$F_{T(min.)} = -\sqrt{2}\,(1-c)\,m\omega^2 r$$

→ Maximum Value of F_T for $\theta = 315^\circ$

$$F_{T(max.)} = \sqrt{2}\,(1-c)\,m\omega^2 r$$

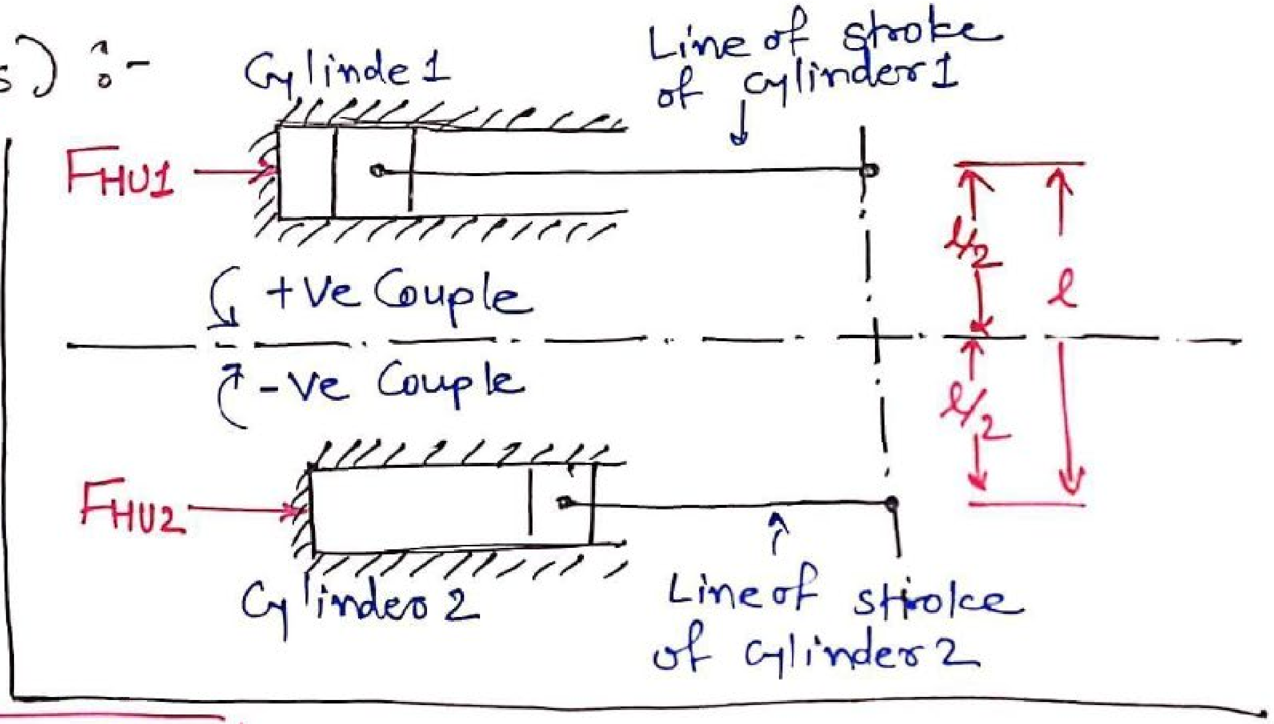
$$F_T = \pm\sqrt{2}\,(1-c)\,m\omega^2 r$$

❑ Swaying Couple (C_S)

* Swaying Couple (C_s) :-

→ Taking moment about Center line, we get swaying couple,

$$C_s = F_{HU1} \cdot \frac{l}{2} - F_{HU2} \cdot \frac{l}{2}$$



$$C_s = (1-c)\, m\omega^2 \cdot r(\cos\theta \mp \sin\theta)\, l/2$$

→ Swaying Couple is maximum or minimum when,

$$\frac{dC_s}{d\theta} = 0 \Rightarrow -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1$$

OR $\theta = 45^\circ$ or 225°

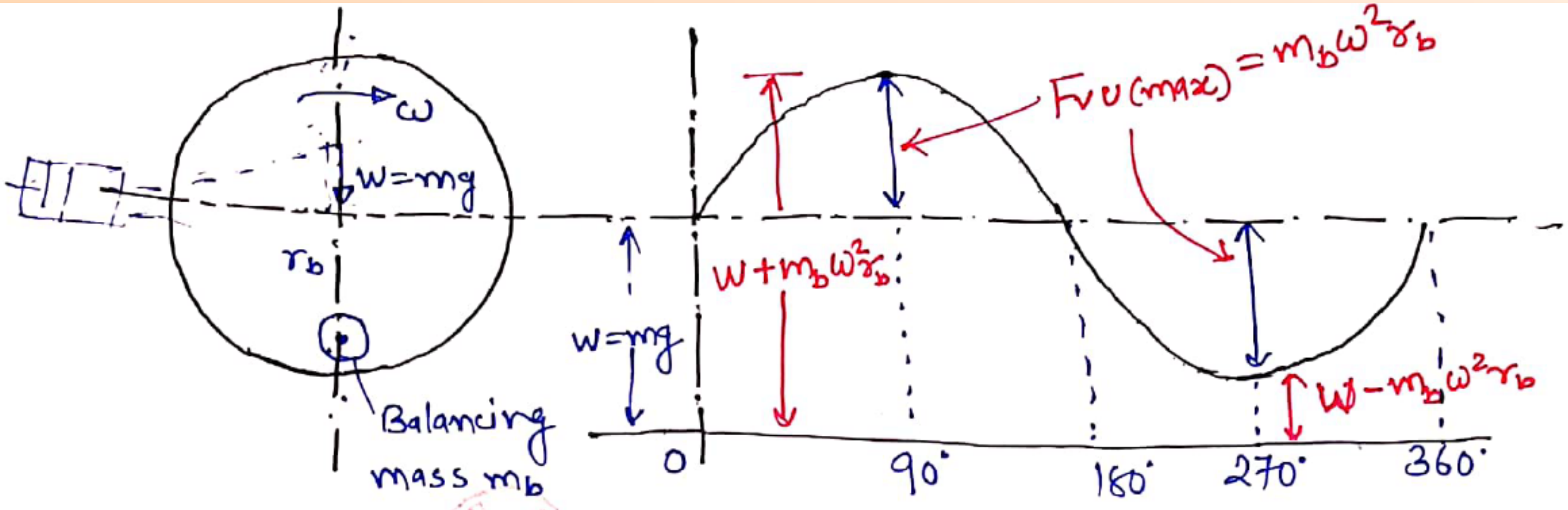
❑ Swaying Couple (C_S)

$$C_{s(min.)} = -\frac{l}{\sqrt{2}}(1-c)m\omega^2 r$$ ($\theta = 225°$)

$$C_{s(max.)} = \frac{l}{\sqrt{2}}(1-c)m\omega^2 \cdot r$$ ($\theta = 45°$)

→ Swaying Couple :- $$C_s = \pm\frac{l}{\sqrt{2}}(1-c)m\omega^2 \cdot r$$

Hammer Blow



- Unbalanced Force along perpendicular to line of stroke,

  $= m_b \omega^2 r_b \sin\theta$

- This Force is maximum for, $\theta = 90°$ or $270°$

  $$F_{VU(max)} = m_b \omega^2 r_b$$

- Let, $W = mg$, Load acting downward on each whe[el]

- Net pressure between wheel and rail is,

  $= W \pm m_b \omega^2 r_b$

Note:- If above equation is negative wheel will be lifted from rail and this will happen when $W < m_b \omega^2 r_b$.

- To avoid this, limiting condition is

  $W = m_b \omega^2 r_b$

  and Limiting value of Angular speed is

  $$\omega = \sqrt{\frac{W}{m_b r_b}}$$

- ***Example 1 :-*** ***A two cylinder locomotive engine has following specifications:***
  Reciprocating masses/cylinder = 300 kg
  Crank Radius = 290 mm
  Angle between crank = 90^0
  Driving wheel diagram = 1780 mm
  Distance between cylinder centres = 640 mm
  Distance between driving wheel plans = 1530 mm
  Determine:
  (1) The fraction of reciprocating masses to be balanced if the hammer blow is not to exceed 45 kN at 95 km/hr speed.
  (2) The variation in the tractive effort.
  (3) The magnitude of swaying couple.

Soln. :

Given data : Mass of reciprocating parts, m = 300 kg

Radius of crank, r = 290 mm = 0.29 m

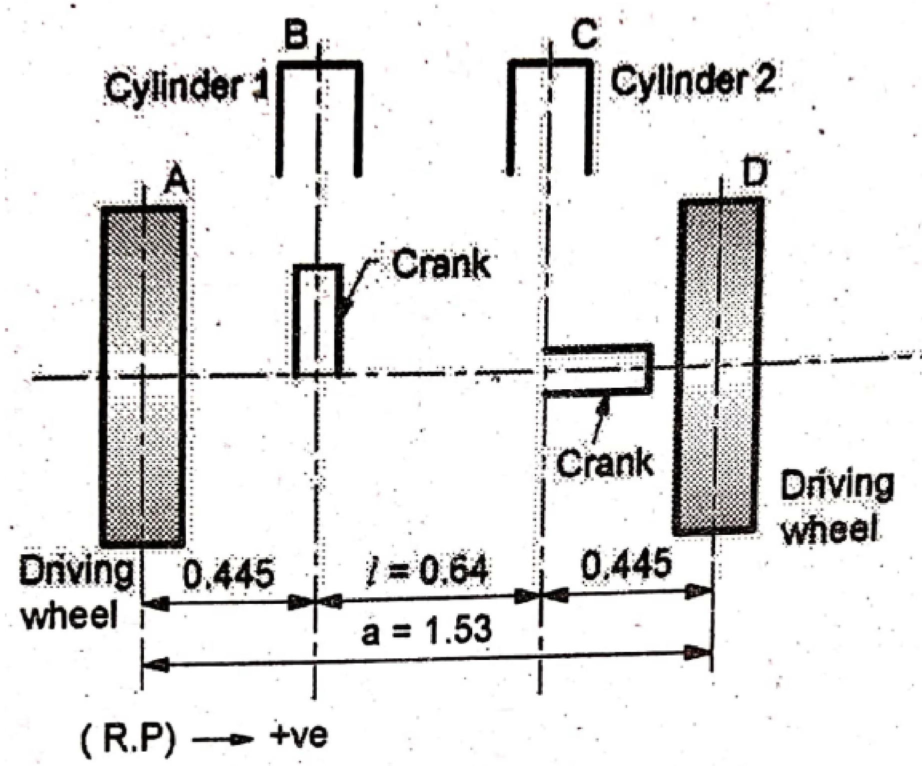
Driving wheel radius, $R = \frac{1780}{2} = 890 \text{ mm} = 0.89 \text{ m}$

Distance between cylinder centers, $l = 640 \text{ mm} = 0.64 \text{ m}$

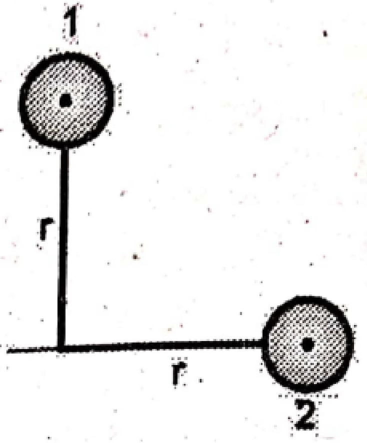
Distance between driving wheel planes, a = 1530 mm = 1.53 m

Hammer Blow, $F_{VU} = 45$ kN at V = 95 km/hr

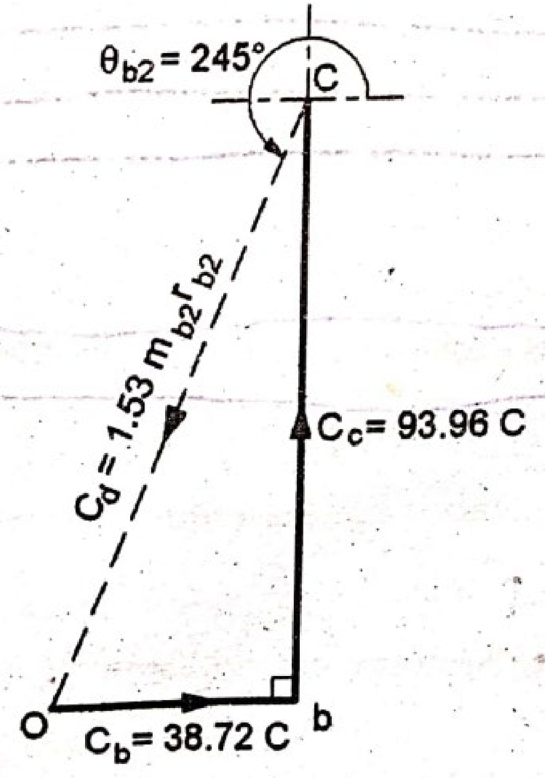
Plane	Mass (m) kg	Radius (r) m	Centrifugal force (mr) kg.m	Distance from Reference plane (l) m	Couple ($m\,r\,l$) kg.m^2	Angular crank Position θ
A	m_{b_1}	r_{b_1}	$m_{b1}\,r_{b1}$	0	0	θ_{b1}
B	300 C	0.29	87 C	0.445	C_b = 38.72 C	0
C	300 C	0.29	87 C	1.08	C_c = 93.96 C	90°
D	m_{b_2}	r_{b_2}	$M_{b2}\,r_{b2}$	1.53	$C_d = m_{b2}\,r_{b2}$ (1.53)	θ_{b2}



(a) Position of planes



(b) Crank position



(c) Couple polygon

Draw couple polygon taking data from column 6 of Table P. 3.1.1.

From Fig. P. 3.1.1(c)

$$C_d = \sqrt{C_b^2 + C_c^2}$$

$$= \sqrt{(38.72\ c)^2 + (93.96\ c)^2}$$

$$= (101.62)\ c$$

but,
$$C_d = m_{b2}\ r_{b2}\ (1.53)$$

$$= (101.62)\ c \qquad ...(i)$$

We know that,

$$\text{Hammer Blow} = F_{VU} = m_{b2}\ \omega^2\ r_{b2} = m_{b1}\ \omega^2\ r_{b1} \qquad ...(ii)$$

$$V = 95\ \text{km/hr} = \frac{95 \times 10^3}{3600}\ \text{m/sec}$$

$$= 26.38\ \text{m/sec.}$$

$$\therefore\ \omega = \frac{V}{R} = \frac{26.38}{0.89} = 29.65\ \text{rad/sec.} \qquad ...(iii)$$

From Equation (ii) and (iii)

$$F_{VU} = m_{b2}\ \omega^2\ r_{b2}$$

$$45 \times 10^3 = m_{b2}\ (29.65)^2\ r_{b2}$$

$$m_{b2}\ r_{b2} = \frac{45000}{(29.65)^2}$$

$$m_{b2}\ r_{b2} = 51.18 \qquad ...(iv)$$

Put Equation (iv) in Equation (i)

$$m_{b2}\ r_{b2}\ (1.53) = (101.62)\ c$$

$$(51.18)\ (1.53) = (101.62)\ c$$

$$c = \mathbf{0.77} \qquad ...\text{Ans.}$$

The variation in tractive effort is,

$$F_T = \pm \sqrt{2}\ m\omega^2\ r\ (1 - c)$$

$$= \pm \sqrt{2}\ (300)\ (29.65)^2 \cdot 0.29\ (1 - 0.77)$$

$$F_T = \pm (1.414)\ (300)\ (879.12)\ (0.29)\ (0.23)$$

$$= \pm\ \mathbf{24873.94\ N} \qquad ...\text{Ans.}$$

The swaying couple is,

$$C_s = \pm \frac{l}{\sqrt{2}}\ m\omega^2\ r\ (1 - c)$$

$$= \pm \frac{0.64}{\sqrt{2}}\ (300)\ (29.65)^2\ (0.29)\ (1 - 0.77)$$

$$C_s = \pm\ \mathbf{7962.06\ Nm} \qquad ...\text{Ans.}$$

- ***Example 2 :-*** *The following data refers to an inside cylinder locomotive:*

Mass of reciprocating parts/cylinder	*: 36 kg*
Revolving masses/cylinder	*: 16 kg*
Pitch of the cylinder	*: 700 mm*
Angle between crank	*: 90^0*
Length of each crank	*: 320 mm*
Wheel tread diameter	*: 1900 mm*
Distance between plans of wheel	*: 1800 mm*
Limiting speed of locomotive	*: 100 kmph*

If total revolving masses and 2/3 of the reciprocating parts are to be balanced, determine :

(i) Variation of tractive force.

(ii) Maximum swaying couple.

Soln. :

Given : $c = \frac{2}{3} = 0.66$

Mass of reciprocating parts, m = 36 kg

Mass of rotating parts, $m_1 = 16$ kg

Total mass to be balanced, $m = m_1 + \frac{2}{3} m = 16 + \frac{2}{3} \times 36 = 40$ kg

Radius of crank, r = 320 mm = 0.32 m

Driving wheel radius, $R = \frac{1900}{2} = 950$ mm = 0.95 m

Distance between cylinder centers, l = 700 mm = 0.7 m

Distance between driving wheel planes, a = 1800 mm = 1.8 m

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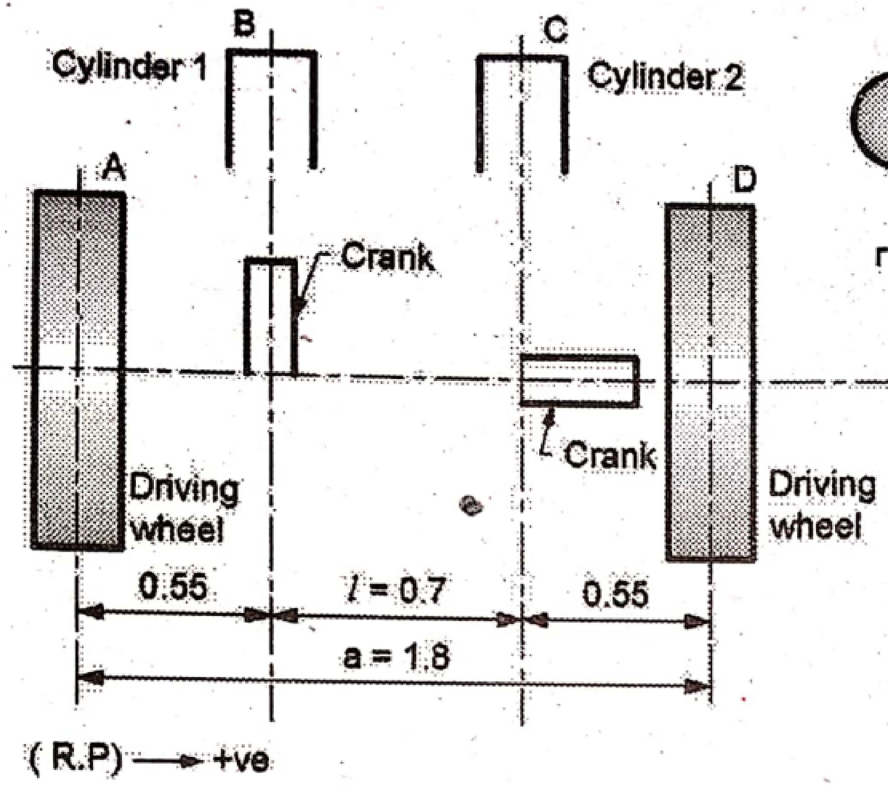
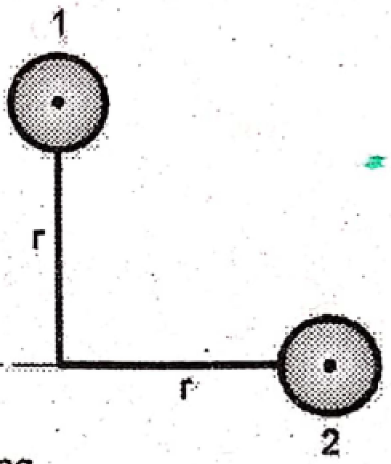
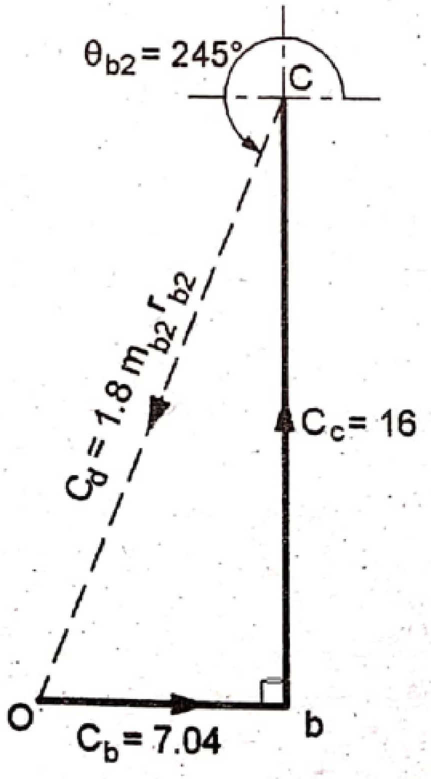
Plane	Mass (m) kg	Radius (r) m	Centrifugal force (mr) kg.m	Distance from Reference plane (*l*) m	Couple (m r *l*) $kg.m^2$	Angular crank Position θ
A	m_{b_1}	r_{b_1}	$m_{b1}\ r_{b1}$	0	0	θ_{b1}
B	40	0.32	12.8	0.55	$C_b = 7.04$	0
C	40	0.32	12.8	1.25	$C_c = 16$	90°
D	m_{b_2}	r_{b_2}	$M_{b2}\ r_{b2}$	1.8	$C_d = m_{b2}\ r_{b2}\ (1.8)$	θ_{b2}



(a) Position of planes



(b) Crank position



(c) Couple polygon

From Fig.

$$C_d = \sqrt{C_b^2 + C_c^2} = \sqrt{(7.04)^2 + (16)^2}$$

$$= 17.48$$

but,

$$C_d = m_{b2}\, r_{b2}\,(1.8) = 17.48$$

$$\therefore\ m_{b2}\, r_{b2} = 9.71$$

Now,

$$V = 100 \text{ km/hr} = \frac{100 \times 10^3}{3600} \text{ m/sec}$$

$$= 27.77 \text{ m/sec.}$$

$$\therefore\ \omega = \frac{V}{R} = \frac{27.77}{0.95} = 29.23 \text{ rad/sec.}$$

$$\therefore \text{ Hammer blow} = m_{b2}\, r_{b2}\, \omega^2 = 9.71 \times (29.23)^2$$

$$= 8301.69 \text{ N}$$

The variation in tractive effort is,

$$F_T = \pm\sqrt{2}\, m\omega^2 r\,(1 - c)$$

$$= \pm\sqrt{2}\,(40)\,(29.23)^2 \cdot 0.32\,(1 - 0.66)$$

$$\mathbf{F_T = \pm\, 5258.49\ N} \qquad \textbf{...Ans.}$$

The swaying couple is,

$$C_s = \pm\frac{l}{\sqrt{2}}\, m\omega^2 r\,(1 - c)$$

$$= \pm\frac{0.7}{\sqrt{2}}\,(40)\,(29.23)^2\,(0.32)\,(1 - 0.66)$$

$$\mathbf{C_s = \pm\, 1840.47\ Nm} \qquad \textbf{...Ans.}$$

Thank you

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COLLEGE OF ENGINEERING & TECHNOLOGY

Prof. Chirag Mevada

Dynamics of Machinery

"Dynamics of Reciprocating Engines"

* Applications :- I.C. Engines, Reciprocating compressors and Reciprocating pumps.

* Balancing of Reciprocating Masses In Single Cylinder Engines.

→ It involves :- i) Determination of Unbalanced Forces (Inertia Forces) due to reciprocating masses.
ii) Balancing of unbalanced Forces by convenient method

* Primary and Secondary Unbalanced Forces Due to Reciprocating Masses :-

→ Acceleration due to reciprocating mass of a slider crank mechanism,

$$f = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

→ Inertia force due to reciprocating mass,

$$F_I = m\omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

→ Unbalanced force due to reciprocating mass,

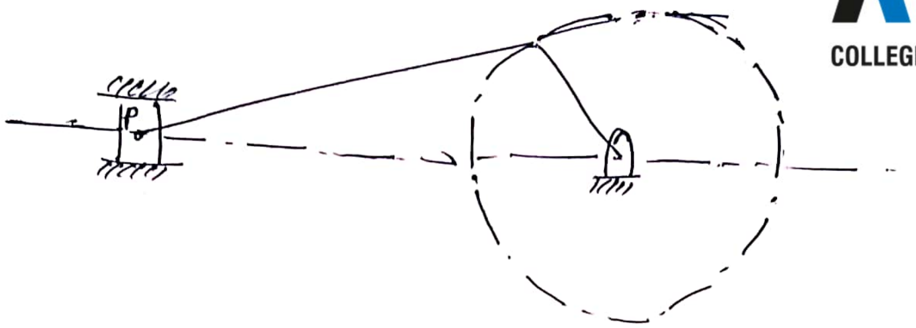
$$F_U = F_I = m\omega^2 r\cos\theta + m\omega^2 r\frac{\cos 2\theta}{n}$$

$$F_U = F_P + F_S$$

F_P ↓ Primary Unbalaced Force

F_S ↓ Secondary Unbalaced Force

→ F_U acts in Line of stroke and direction is opposite of acceleration of reciprocating mass.



* **Primary Unbalanced Force (F_p) :-** It is due to S.H.M. of reciprocating parts.
  - → It is maximum at $\theta = 0°$ and $180°$. twice in one rotation of crank.

* **Secondary Unbalanced Force (F_s) :-** Due to obliquity of arrangement.
  - → It is maximum when $\theta = 0°, 90°, 180°$ and $360°$ i.e. four times in one rotation of crank.
  - → This is twice as that of primary unbalanced force in terms of frequency.
  - → But, Magnitude is $\frac{1}{n}$ times F_p
  - → In case of low and moderate speed engines, F_s is small & generally neglected.

* **Difference Between Unbalanced Force Due to Reciprocating Mass and Rotating Mass :-**

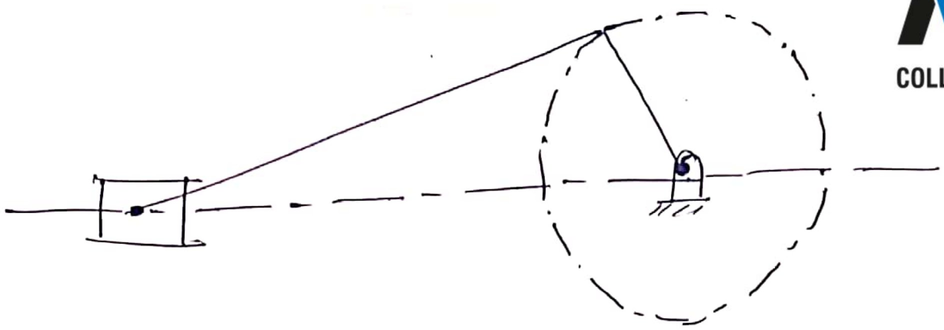
→ Unbalanced force due to reciprocating mass varies in magnitude but constant in direction.

→ While Rotating mass is constant in magnitude ($m\omega^2$) but varies in direction

→ Therefore a single mass can not be used to balance a reciprocating mass completely.

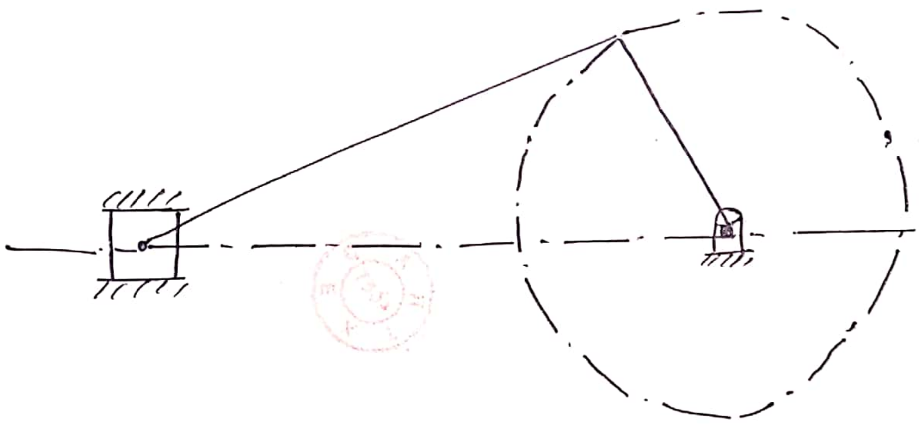
→ However, a single rotating mass can be used to partially balance the reciprocating mass.

* Partial Balancing of Primary Unbalanced Force :-



$$F_P = m\omega^2 r \cos\theta$$

→ **Effect of Balancing Mass :-**



→ **Partial Balancing :-**

$$m_b \omega^2 r_b \cos\theta = C m \omega^2 r \cos\theta$$

$$m_b r_b = C m r \quad \text{where } C < 1$$

$$\text{Balanced Primary Force} = C m \omega^2 r \cos\theta$$

→ Unbalanced Forced due to Partial Balancing.

$$F_H = m\omega^2 r \cos\theta - m_b \omega^2 r_b \cos\theta$$

$$= m\omega^2 r \cos\theta - C m\omega^2 r \cos\theta$$

$$F_H = (1-C)\, m\omega^2 r \cos\theta$$

$F_V = m_b \omega^2 r_b \sin\theta$

$F_V = c m \omega^2 r \sin\theta$

$F_R = \sqrt{F_H^2 + F_V^2} = m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$

→ F_R is minimum, when $\frac{dF_R}{dc} = 0$ ie. $c = \frac{1}{2}$

→ For locomotive, $c = \frac{2}{3}$ to $\frac{3}{4}$

→ $m_b r_b = \underbrace{c m r}_{\text{Partial Balancing}} + \underbrace{m_r r_r}_{\text{Rotating Mass Balance}}$

* **Example:** $N = 240$ rpm, $S = 300$ mm, $m = 50$ kg, $m_r = 30$ kg, $r_r = 150$ mm, $r_b = 400$ mm, $c = 2/3$, $\theta = 60°$ Find Unbalanced Force.

$\omega = \frac{2\pi N}{60} =$ ____ rad/s

→ $m_b r_b = c m r + m_r r_r$

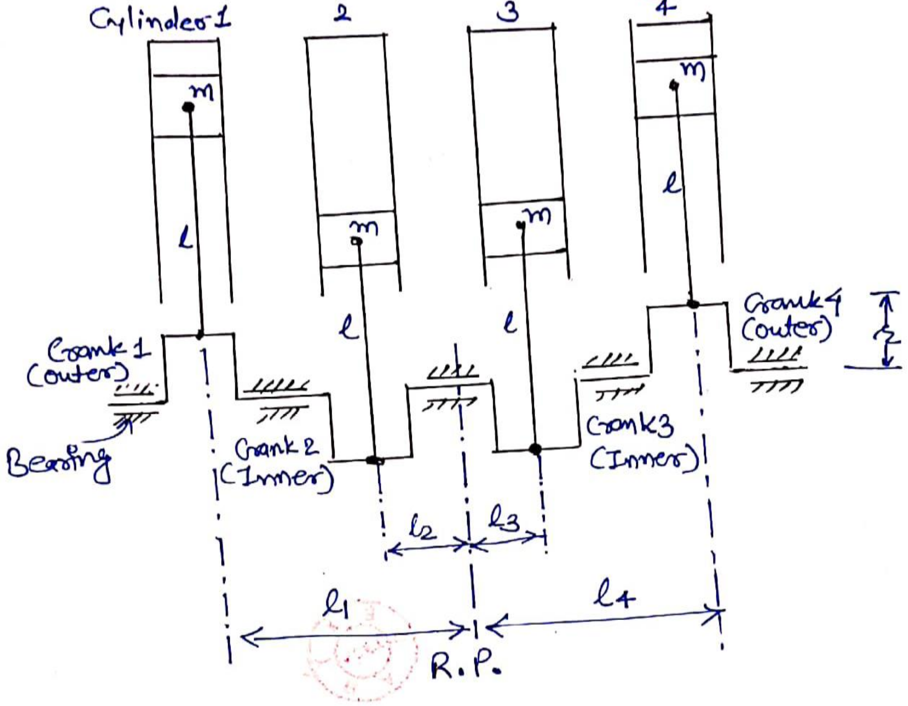
$m_b =$ ____ kg

→ $F_U = m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$

$=$ ____ N.

* Balancing of Reciprocating Masses in Multicylinder Inline Engines :-

→ Multicylinder engines having axes of all cylinders in same plane and on same side of axis of crank shaft, are known as Inline engines.



→ Primary Couple = $C_p = m\omega^2 r l \cos\theta$

Secondary Couple, $C_s = m\omega^2 r l \dfrac{\cos 2\theta}{n}$

→ Condition for complete Balancing :-

1. $\sum F_p = 0$, 2. $\sum C_p = 0$; 3. $\sum F_S = 0$, 4. $\sum C_S = 0$.

* Primary Balancing :-

i) $\sum$ Primary Forces $= 0$ i.e. $\sum m\omega^2 r \cos\theta = 0$. ⇒ $\sum m r = 0$

ii) $\sum$ Primary Couples $= 0$ i.e. $\sum m\omega^2 r \cos\theta \cdot l = 0$ ⇒ $\sum m r l = 0$.

* Method :- 1. Graphical 2. Analytical.

1. Graphical	2. Analytical
1) Primary Force polygon must be closed.	i) $\sum m r \cos\theta = 0$
	ii) $\sum m r \sin\theta = 0$
2) Couple Polygon must be closed	iii) $\sum m r l \cos\theta = 0$
	iv) $\sum m r l \sin\theta = 0$.

* Secondary Balancing :-

i) $\sum$ Secondary Forces $= 0$, $\sum m\omega^2 r \frac{\cos 2\theta}{n} = 0 \Rightarrow \sum \frac{mr}{n} = 0$.

ii) $\sum$ Secondary Couples $= 0$, $\sum m\omega^2 r \cdot l \frac{\cos 2\theta}{n} = 0 \Rightarrow \sum \frac{mrl}{n} = 0$.

n = obliquity Ratio $= l/r$

→ Above conditions can be written as :-

i) $\sum m(2\omega)^2 \left(\frac{r}{4n}\right) \cos 2\theta = 0$

ii) $\sum m(2\omega)^2 \left(\frac{r}{4n}\right) l \cos 2\theta = 0$.

→ These conditions are equivalent to conditions of Primary Balancing for an imaginary crank of length $(r/4n)$, rotating at speed 2ω and inclined at an angle 2θ to i.d.c. This imaginary crank is known as Secondary crank.

– **Primary crank and secondary crank :**

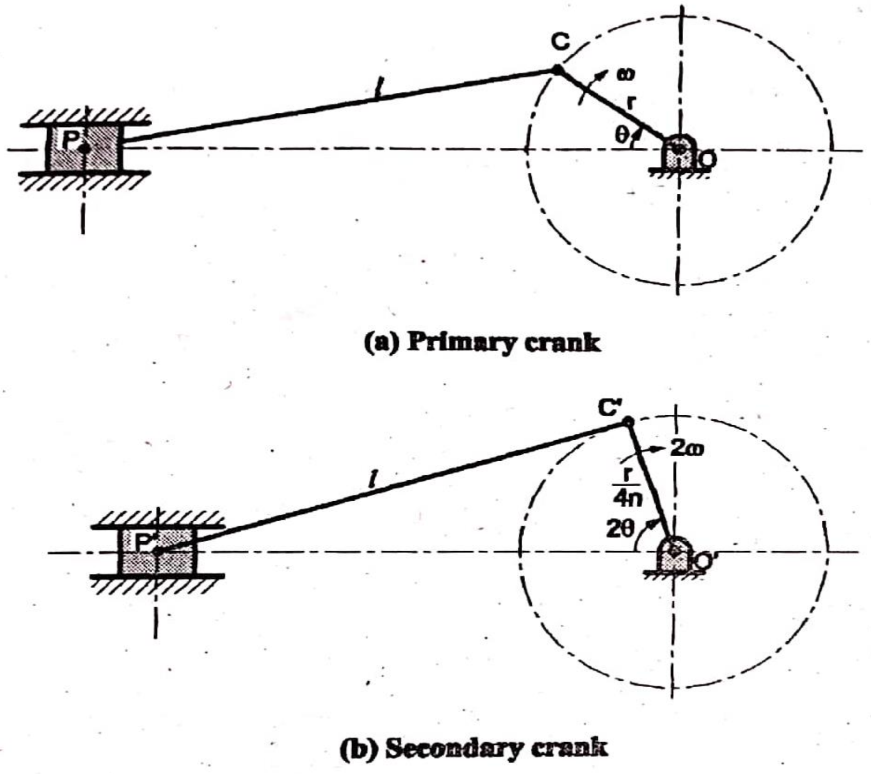


Fig. : Primary and Secondary Cranks

Parameters of Primary Crank	Parameters of Secondary Crank
(i) Crank radius = r, m	(i) Crank radius = r / 4n, m
(ii) Angular speed = ω, rad / s	(ii) Angular speed = 2 ω, rad / s
(iii) Crank position from i.d.c. = θ	(iii) Crank position from i.d.c. = 2 θ

– **Methods of Secondary Balancing :**

The secondary balancing can be carried out by following two methods :

1. Graphical method
2. Analytical method

1. Graphical method

In a graphical method, for a complete secondary balancing :

(i) The secondary force polygon must be closed
(ii) The secondary couple polygon must be closed

2. Analytical method

For a complete secondary balancing, the analytical solution is,

(i) $\Sigma \frac{m\,r}{n} \cos 2\theta = 0$ (ii) $\Sigma \frac{m\,r}{n} \sin 2\theta = 0$

(iii) $\Sigma \frac{m\,r\,l}{n} \cos 2\theta = 0$ (iv) $\Sigma \frac{m\,r\,l}{n} \sin 2\theta = 0$

– If n is same for all cylinders then,

(i) $\Sigma m\,r \cos 2\theta = 0$ (ii) $\Sigma m\,r \sin 2\theta = 0$

(iii) $\Sigma m\,r\,l \cos 2\theta = 0$ (iv) $\Sigma m\,r\,l \sin 2\theta = 0$

– **Unbalance in Engine :**

(i) If engine is not under complete primary balancing, the closing side of primary force polygon gives the maximum unbalanced primary force and the closing side of the primary couple polygon gives the maximum unbalanced primary couple.

(ii) Similarly, if the engine is not under complete secondary balancing, the closing side of secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

Balancing of Four Cylinder Inline Engines

- Consider a four cylinder inline engine having two inner cranks and two outer cranks as shown in Fig.
- The inner cranks 2 and 3 are 180° from the outer cranks 1 and 4. Therefore, the angular positions of the cranks are as follows

Crank 1 $\Rightarrow \theta^\circ$

Crank 2 $\Rightarrow 180^\circ + \theta^\circ$

Crank 3 $\Rightarrow 180^\circ + \theta^\circ$

Crank 4 $\Rightarrow \theta^\circ$

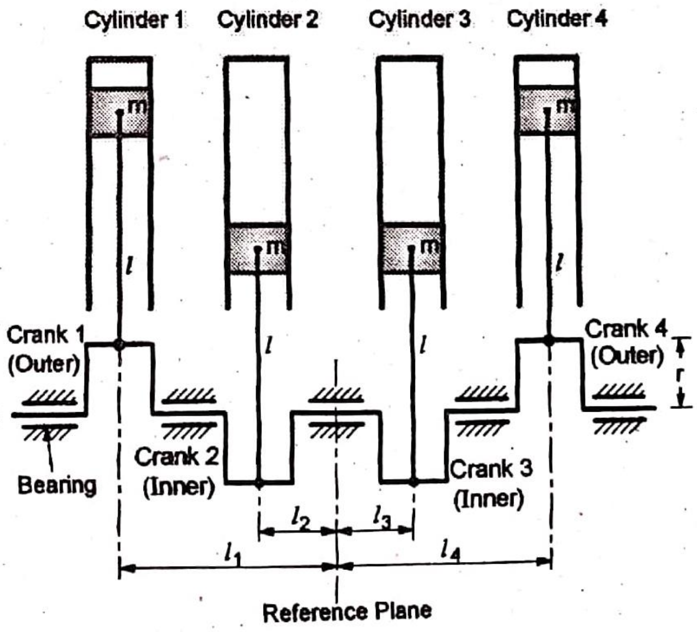


Fig. : Four Cylinder Inline Engine

- For graphical solution, force and couple data is given in Table

Table : Force and Couple Data

Plane	Mass (m), kg	Radius (r), m	Centrifugal Force ÷ ω^2 (mr), kg-m	Distance from R.P. (l), m	Couple ÷ ω^2 (mrl), kg-m²	Primary Crank Position 'θ'	Secondary Crank Position '2 θ'
1	m	r	mr	$-l_1$	$-mr\,l_1$	0°	0°
2	m	r	mr	$-l_2$	$-mr\,l_2$	180° + 0°	360° + 0°
3	m	r	mr	l_3	$mr\,l_3$	180° + 0°	360° + 0°
4	m	r	mr	l_4	$mr\,l_4$	0°	0°

1. Primary crank position

– Fig. shows primary crank position.

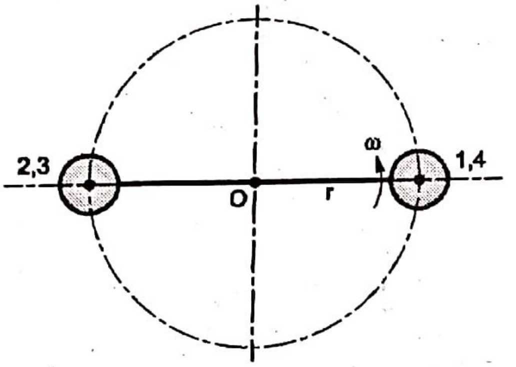


Fig. : Primary Crank Positions

– Assume firing order : 1-4-2-3.

(i) Primary force polygon

$$F_{P1} = \vec{oa} = mr \Rightarrow \text{at } 0^\circ$$

$$F_{P4} = \vec{ab} = mr \Rightarrow \text{at } 0^\circ$$

$$F_{P2} = \vec{bc} = mr \Rightarrow \text{at } 180^\circ$$

$$F_{P3} = \vec{cd} = mr \Rightarrow \text{at } 180^\circ$$

– The magnitude of all primary forces is same (i.e. mr) with two forces acting at 0° and two forces acting at 180°.

– Therefore, the primary force polygon is closed as shown in Fig. and there is no unbalanced primary force.

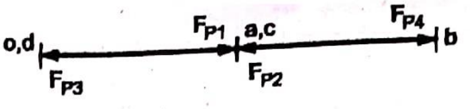


Fig. : Primary Force Polygon

(ii) Primary couple polygon

$$C_{P1} = \vec{o'a'} = -mr\, l_1 \Rightarrow \text{at } 0^\circ$$

$$C_{P4} = \vec{a'b'} = mr\, l_4 \Rightarrow \text{at } 0^\circ$$

$$C_{P2} = \vec{b'c'} = -mr\, l_2 \Rightarrow \text{at } 180^\circ$$

$$C_{P3} = \vec{c'd'} = mr\, l_3 \Rightarrow \text{at } 180^\circ$$

– The system is symmetrical about the reference plane. i.e. $l_1 = l_4$ and $l_2 = l_3$ therefore $C_{P1} = C_{P4}$ and $C_{P2} = C_{P3}$.

– The primary couple polygon is closed as shown in Fig. and there is no unbalanced primary couple.

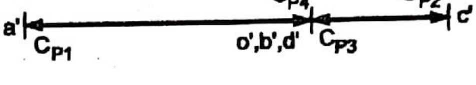


Fig. : Primary Couple Polygon

2. Secondary crank position

– Fig. shows secondary crank position.

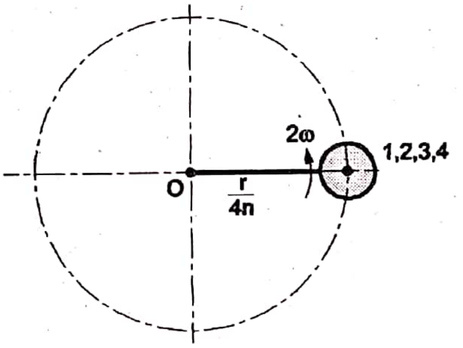


Fig. : Secondary Crank Positions

– Assuming firing order : 1-4-2-3.

(i) Secondary force polygon

$$F_{S1} = \vec{oa} = mr \Rightarrow \text{at } 0°$$

$$F_{S4} = \vec{ab} = mr \Rightarrow \text{at } 0°$$

$$F_{S2} = \vec{bc} = mr \Rightarrow \text{at } 360°$$

$$F_{S3} = \vec{cd} = mr \Rightarrow \text{at } 360°$$

– The magnitude of all secondary forces is same (i.e. mr) and acts in one direction as shown in Fig.

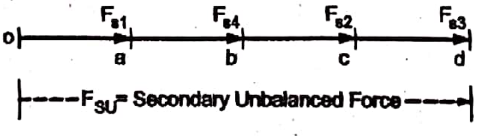


Fig. : Secondary Force Polygon

– The resultant secondary unbalanced force is given by,

$$F_{SU} = \vec{od} \times \text{Scale of secondary force polygon} \times \frac{\omega^2}{n}$$

$$\text{or } F_{SU} = (F_{S1} + F_{S2} + F_{S3} + F_{S4}) \times \frac{\omega^2}{n}$$

$$\text{or } F_{SU} = (mr + mr + mr + mr) \times \frac{\omega^2}{n}$$

$$\text{or } F_{SU} = \frac{4\,m\,r\,\omega^2}{n}$$

(ii) Secondary couple polygon

$$C_{S1} = \overrightarrow{o'a'} = -mr\,l_1 \Rightarrow \text{at } 0°$$

$$C_{S4} = \overrightarrow{a'b'} = mr\,l_4 \Rightarrow \text{at } 0°$$

$$C_{S2} = \overrightarrow{b'c'} = -mr\,l_2 \Rightarrow \text{at } 360°$$

$$C_{S3} = \overrightarrow{c'd'} = mr\,l_3 \Rightarrow \text{at } 360°$$

- The system is symmetrical about the reference plane, i.e. $l_1 = l_4$ and $l_2 = l_3$, therefore $C_{P1} = C_{P4}$ and $C_{P2} = C_{P3}$. The secondary couple polygon is closed as shown in Fig. and there is no unbalanced secondary couple.

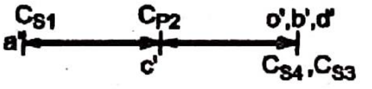


Fig. : Secondary Couple Polygon

- Thus for a given four cylinder inline engine, the primary forces, primary couples and secondary couples are balanced. However, the engine is not balanced for secondary forces.

The cranks and connecting rods of a four-cylinder in-line engine running at 1800 rpm are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. The reciprocating mass corresponding to each cylinder is 1.5 kg. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in order 1-4-2-3. Determine unbalanced primary and secondary forces, if any and unbalanced primary and secondary couples with reference to central plane of the engine.

Soln. :

Given : Speed of engine, $N = 1800$ r.p.m.

$$\therefore \omega = \frac{2\pi \times 1800}{60} = 188.49 \text{ rad/s.}$$

Crank radius, $r = 60 \text{ mm} = 0.06 \text{ m}$

Length of connecting rod,

$$l = 240 \text{ mm} = 0.24 \text{ m} \qquad \therefore n = \frac{l}{r} = \frac{0.24}{0.06} = 4$$

– The central plane of the engine is taken as reference plane. The force and couple data is given in Table

Table : Force and Couple Data

Plane	Mass (m), kg	Radius (r), m	Centrifugal Force ÷ ω^2 (mr), kg-m	Distance from R.P. (*l*), m	Couple ÷ ω^2 (mr*l*), kg-m^2	Primary Crank Position 'θ'	Secondary Crank Position '2 θ'
1	1.5	0.06	0.090	– 0.225	– 0.02025	0°	0°
2	1.5	0.06	0.090	– 0.075	– 0.0067	180°	360°
3	1.5	0.06	0.090	0.075	0.0067	270°	540° i.e. 180°
4	1.5	0.06	0.090	0.225	0.02025	90°	180°

1. **Primary force polygon**
   - Firing order of engine is 1-4-2-3. Hence draw the primary crank positions as shown in Fig.
   - Draw the primary force polygon by taking data from column 4 of Table and considering primary crank positions. As the primary force polygon is closed, there is no unbalanced primary force acting on the engine.

2. **Primary couple polygon**
   - Draw the primary couple polygon by taking data from column 6 of Table and considering primary crank positions, which are shown in Fig.
   - The $\overrightarrow{o'd'}$ vector gives the magnitude of unbalanced primary couple i.e. C_{pU}.

$$C_{PU} = \overrightarrow{o'd'} \times \text{Scale of primary couple polygon} \times \omega^2$$
$$= 18.80 \times 0.001 \times (188.49)^2$$

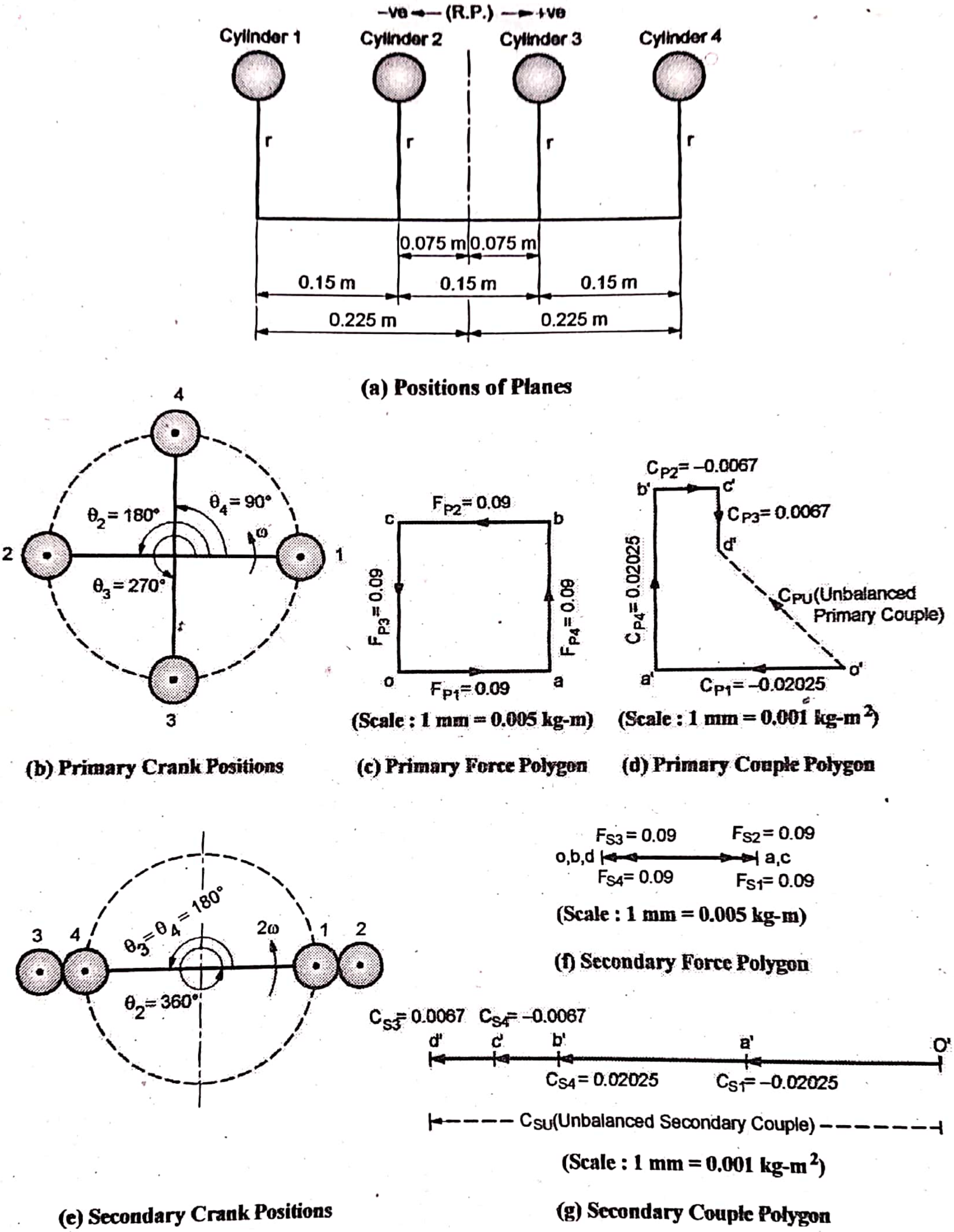
or $\quad C_{PU} = \mathbf{668\ N\text{-}m}$ ...Ans.

3. **Secondary force polygon**
   - Draw the secondary crank positions by taking data form last column of Table as shown in Fig.
   - Draw secondary force polygon by taking data from column 4 of Table and considering secondary crank positions. As the secondary force polygon is closed, there is no unbalanced secondary force acting on the engine.

4. **Secondary couple polygon**
   - Draw the secondary couple polygon by taking data from column 6 of Table and considering secondary crank positions, which are shown in Fig.
   - Since all the secondary couples act in one direction, the unbalanced secondary couple is,

$$C_{SU} = \overrightarrow{o'd'} \times \text{Scale of secondary couple polygon} \times \frac{\omega^2}{n} = 54 \times 0.001 \times \frac{(188.49)^2}{4}$$

or $\quad C_{SU} = \mathbf{479.63\ N\text{-}m}$ ...Ans.



–ve ← (R.P.) → +ve
Cylinder 1
Cylinder 2
Cylinder 3
Cylinder 4
r
0.075 m
0.075 m
0.15 m
0.15 m
0.15 m
0.225 m
0.225 m
(a) Positions of Planes
θ2 = 180°
θ4 = 90°
θ3 = 270°
ω
(b) Primary Crank Positions
FP2 = 0.09
FP3 = 0.09
FP4 = 0.09
FP1 = 0.09
(Scale : 1 mm = 0.005 kg-m)
(c) Primary Force Polygon
CP2 = –0.0067
CP3 = 0.0067
CP4 = 0.02025
CPU (Unbalanced Primary Couple)
CP1 = –0.02025
(Scale : 1 mm = 0.001 kg-m²)
(d) Primary Couple Polygon
θ3 = θ4 = 180°
2ω
θ2 = 360°
(e) Secondary Crank Positions
FS3 = 0.09
FS2 = 0.09
o,b,d
a,c
FS4 = 0.09
FS1 = 0.09
(Scale : 1 mm = 0.005 kg-m)
(f) Secondary Force Polygon
CS3 = 0.0067
CS4 = –0.0067
CS4 = 0.02025
CS1 = –0.02025
CSU (Unbalanced Secondary Couple)
(Scale : 1 mm = 0.001 kg-m²)
(g) Secondary Couple Polygon

Fig.

Concept of Direct and Reverse Cranks

- In a radial engines and V-engines all the connecting rods are connected to a common crank and this crank revolves in one plane. Hence, there is no primary or secondary couple. Only the primary and secondary forces are required to be balanced.
- The method of direct and reverse cranks is used for balancing of the radial engines or V-engines. This method is very useful for determining primary and secondary forces in radial or V-engines.

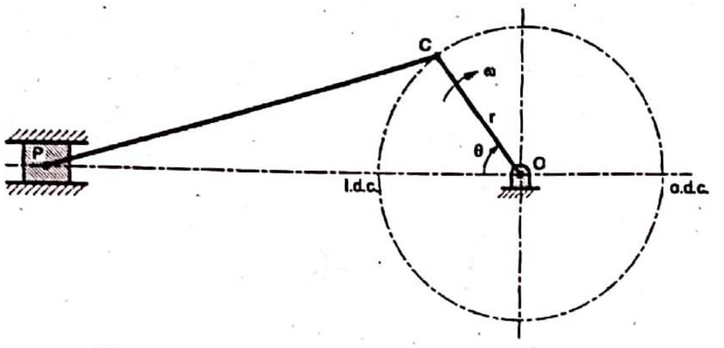


Fig. : Reciprocating Engine Mechanism

Primary Force

- The unbalanced primary force 'F_P' is given by,

$$F_P = m\,\omega^2 r \cos\theta$$

where, m = mass of reciprocating parts, kg

- This unbalanced primary force is equal to the horizontal component of the centrifugal force produced by the imaginary mass 'm' placed at crank pin 'C', as shown in Fig.

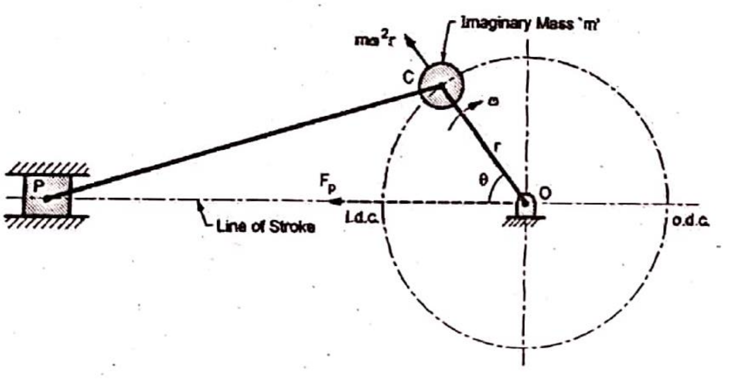


Fig : Reciprocating Engine Mechanism

- The arrangement shown in Fig. can be replaced by another arrangement, shown in Fig. in which OC is called as the **actual crank** or **primary direct crank** and OC′ is called as the **indirect crank** or **primary reverse crank.**
- The primary direct crank OC makes an angle θ with i.d.c. position and is rotating uniformly at 'ω' rad/s in clockwise direction, whereas the primary reverse crank OC′ makes an angle $-\theta$ with i.d.c. position and is rotating uniformly at 'ω' rad/s in anticlockwise direction as shown in Fig. Thus the primary reverse crank is mirror image of the primary direct crank.
- The Parameters of Primary Direct and Reverse Cranks :
- **Primary direct crank**

  Radius of crank = r

  Angular position = θ

  Angular speed = ω rad / s (Clockwise)
- **Primary reverse crank**

  Radius of crank = r

  Angular position = $-\theta$

  Angular speed = ω rad / s (Anticlockwise)

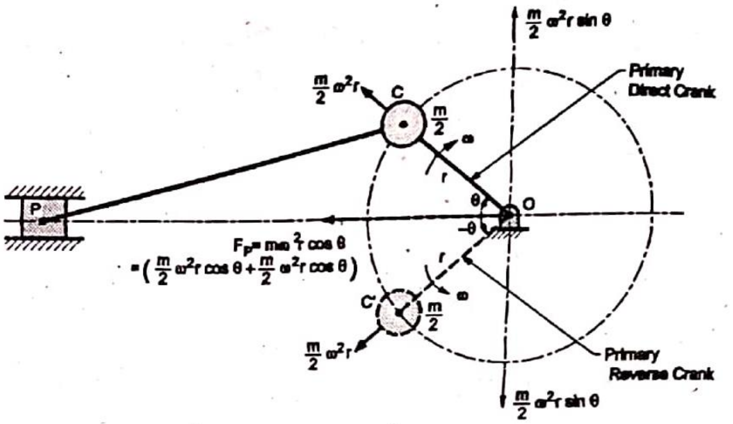


Fig. : Primary Force in Direct and Reverse Cranks

- Let mass **'m'** of the reciprocating parts is divided equally into two parts $\left(\text{i.e.} \frac{m}{2}\right)$. One of the part is placed at direct crank pin **'C'** and the other part is placed at reverse crank pin **'C'** as shown in Fig.
- Centrifugal force acting on each mass placed at direct crank pin C and reverse crank pin **C′** $= \frac{m}{2}\omega^2 r$
- Component of the centrifugal force acting on the mass placed at point C, along the line of stroke $= \frac{m}{2}\omega^2 r\cos\theta$
- Component of the centrifugal force acting on the mass placed at point C′, along the line of stroke $= \frac{m}{2}\omega^2 r\cos\theta$
- Total component of the centrifugal force acting along the line of stroke

$$= \frac{m}{2}\omega^2 r\cos\theta + \frac{m}{2}\omega^2 r\cos\theta$$

$$= m\,\omega^2 r\cos\theta$$

- This total component of centrifugal force acting along the line of stroke, which is equal to primary unbalanced force, $F_P = m\,\omega^2 r \cos\theta$
- Hence, for determining the unbalanced primary force, the mass **'m'** of the reciprocating parts can be replaced by two masses i.e. $\frac{m}{2}$ each at point C and C′ respectively.
- The components of centrifugal forces of masses (m/2) placed at point C and C′ normal to the line of stroke are equal to $\frac{m}{2}\omega^2 r \sin\theta$, but opposite in direction to each other. Hence, these components are balanced.
- Thus, the unbalanced primary force due to reciprocating mass **'m'** can be determined by placing masses **m/2** each at crank pin of primary direct crank and primary reverse crank (i.e. at points C and C′)

Secondary Force

- The unbalanced secondary force 'F_S' is given by,

$$F_S = m\,\omega^2 r \frac{\cos 2\theta}{n}$$

or

$$F_S = m \times (2\omega)^2 \times \frac{r}{4n} \cos 2\theta$$

- The concept of determining unbalanced primary force can be extended to determine the unbalanced secondary force.
  For determining unbalanced secondary force, the mass **'m'** of the reciprocating parts is replaced by two masses equal to $\frac{m}{2}$ at crank pins of secondary direct crank and secondary reverse crank (i.e. at points **C** and **C′**) such that secondary direct crank is making an angle 2θ and secondary reverse crank is making an angle − 2θ with i.d.c. position as shown in Fig.

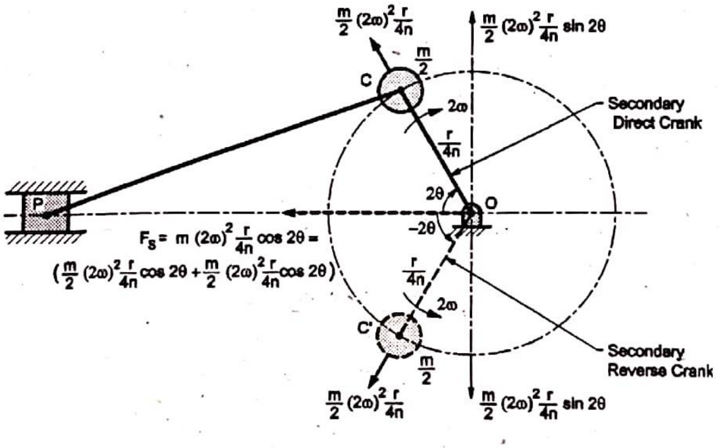


Fig. : Secondary Direct and Reverse Cranks

- Parameters of Secondary Direct and Reverse Cranks :
- **Secondary direct crank**

Radius of crank = r/4n
Angular position = 2θ
Angular speed = 2ω rad/s (clockwise)

Secondary reverse crank

Radius of crank = r/4n
Angular position = − 2θ
Angular speed = 2ω rad/s (Anticlockwise)

Thus, the unbalanced secondary force due to reciprocating mass **'m'** can be determined by placing masses **'m/2'** each at cranks pin of secondary direct crank and secondary reverse crank (i.e. at points **C** and **C′**)

For a twin V-engine the cylinder centerlines are set at 90°. The mass of reciprocating parts per cylinder is 2.5 kg. Length of crank is 100 mm and length of connecting rod is 400 mm. determine the primary and secondary unbalanced forces when the crank bisects the lines of cylinder centerlines. The engine runs at 1000 rpm.

Soln. :

Given : Mass of reciprocating parts, $m = 2.5$ kg

Crank radius, $r = 100 \text{ mm} = 0.1 \text{ m}$

Length of connecting rod, $l = 400 \text{ mm} = 0.4 \text{ m}$

Obliquity ratio, $n = \frac{l}{r} = \frac{0.4}{0.1} = 4$

Speed of engine, $N = 1000$ r.p.m.

$\therefore \quad \omega = \frac{2\pi \times 1000}{60} = 104.71$ rad.

Consider two cylinder V-engine located at 90° from each other, as shown in Fig.

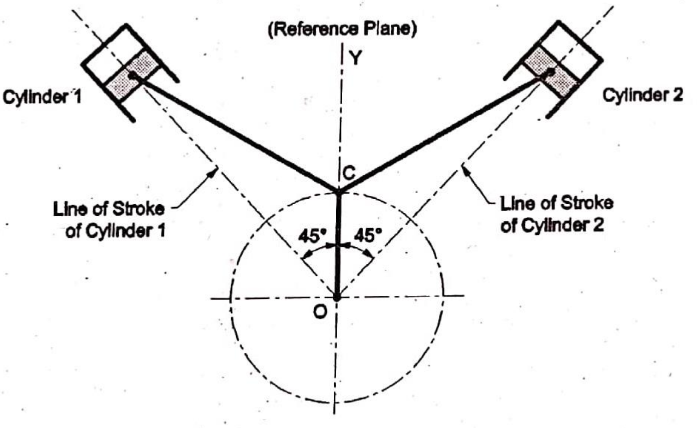


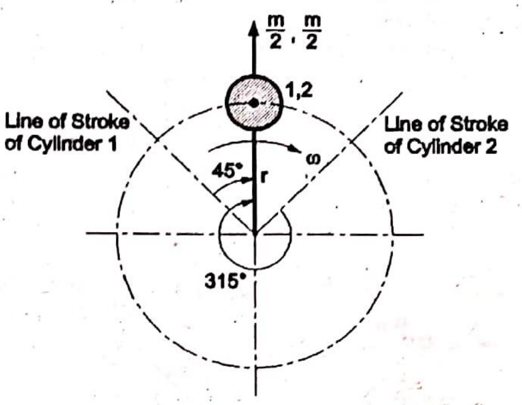
Fig. (a) : Two Cylinder V-engine

Consider **OY** as reference position. The primary and secondary crank positions are given in Table

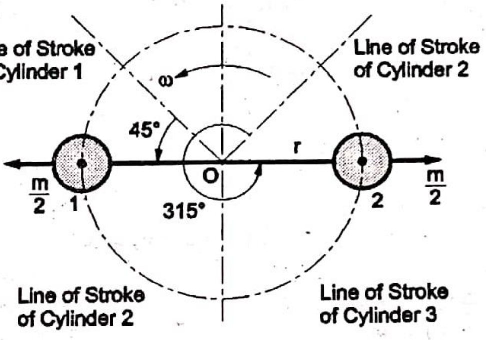
Table

Position	Primary Crank Position 'θ'		Secondary Crank Position '2θ'	
	Direct	Reverse	Direct	Reverse
1	45°	– 45°	90°	– 90°
2	315°	– 315°	630°	– 630°

1. **Primary Forces**



(i) Direct Crank Positions



(ii) Reverse Crank Positions

Fig. **(b) : Primary Forces**

(i) For cylinder 1, $\theta = \pm 45°$, hence rotate the crank 1 in clockwise direction by 45° from its line of stroke for direct crank position and rotate the crank 1 in anticlockwise direction by 45° from its line of stroke for reverse crank position, as shown in Fig. (b).

(ii) For cylinder 2, $\theta = \pm 315°$, hence rotate the crank 2 in clockwise direction by 315° from its line of stroke for direct crank position and rotate the crank 2 in anticlockwise direction by 315° from its line of stroke for reverse crank position.

(iii) From Fig. (b) it is seen that, for reverse crank position the system is balanced and unbalanced force is only due to direct crank position. Therefore,

The unbalanced primary force is,

$$F_P = \left(\frac{m}{2}+\frac{m}{2}\right)\omega^2 r$$

$$= m\,\omega^2 r = 2.5 \times (104.71)^2 \times 0.1$$

or $$F_P = \mathbf{2741.55\ N}$$ **...Ans.**

2. **Secondary Forces**

1. For cylinder 1, $\theta = \pm 90°$, hence rotate the crank 1 in clockwise direction by 90° from its line of stroke for direct crank position and rotate the crank 1 in anticlockwise direction by 90° from its line of stroke for reverse crank position, as shown in Fig. (c).

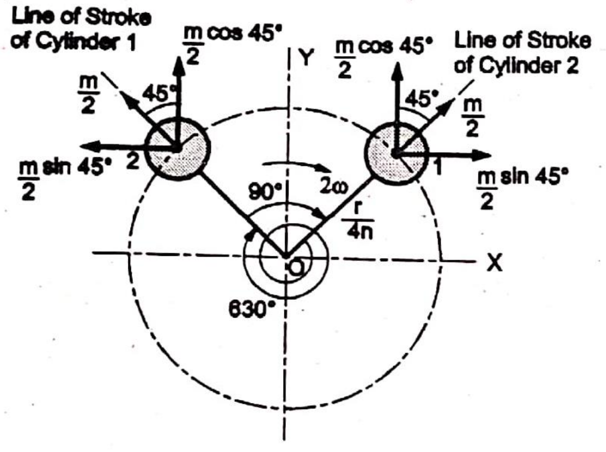
2. For cylinder 2, $\theta = \pm 630°$, hence rotate the crank 2 in clockwise direction 630° from its lines of stroke for direct crank position and rotate the crank 2 in

anticlockwise direction 630° from its line of stroke for reverse crank position.

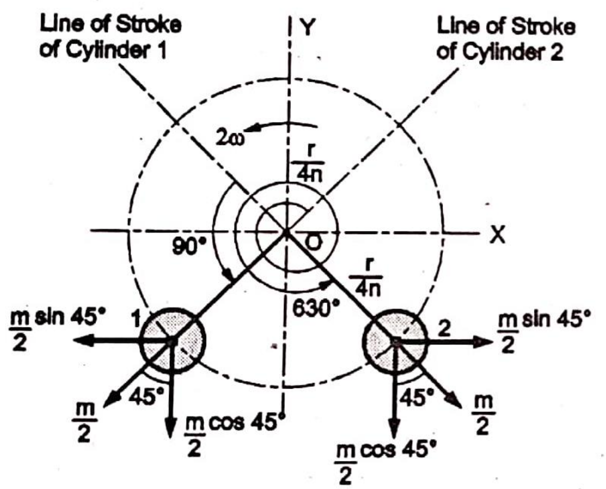
The component of unbalanced secondary force due to direct crank, along OY (upward direction) is,

$$F_{SD} = 2 \times \left[\frac{m}{2}\cos 45°\right] \cdot (2\omega)^2 \frac{r}{4n}$$

The components of unbalanced secondary force due to direct crank, along **OX** (horizontal direction) are balanced,



(i) Direct Crank Positions



(ii) Reverse Crank Positions

Fig. (c) : Secondary Forces

The component of unbalanced secondary force due to reverse crank along **OY** (downward direction) is,

$$\therefore \; F_{SR} = 2 \times \left[\frac{m}{2}\cos 45°\right] \cdot (2\omega)^2 \frac{r}{4n}$$

The components of unbalanced secondary forces due to reverse crank along **OX** (horizontal direction) are balanced,

The total unbalanced secondary force is,

$$F_S = F_{SD} - F_{SR}$$

...[Both are acting in opposite in direction]

$$= 2\left[\frac{m}{2}\cos 45°\right] \cdot (2\omega)^2 \frac{r}{4n} - 2\left[\frac{m}{2}\cos 45°\right] \cdot (2\omega)^2 \frac{r}{4n}$$

or $\mathbf{F_S = 0}$ **...Ans.**

Thus, there is no unbalanced secondary force acting on the engine.

The total unbalanced force acting on the engine is,

$$F_U = F_P + F_S = 2741.55 + 0$$

$$= \mathbf{2741.55\ N}$$ **...Ans.**

Balancing of V-Engines

- A V-engine is a two cylinder radial, engine in which the connecting rods are fixed to the common crank.
- In such engines, the center lines of the cylinders form a letter 'V', therefore these engines are called as **V-engines.**
- In V-engines, the cylinders have a common crank and this crank revolves in one plane, so there is no primary or secondary couple acting on the engine.
- Consider a V-engine, shown in Fig. 3.2.1 having common crank OC and two connecting rods CP and CQ. The lines of stroke OP and OQ are inclined to vertical axis OY at an angle 'α'.

Let, m = mass of reciprocating parts per cylinder, kg

l = length of connecting rod, m

r = radius of crank, m

n = obliquity ratio = l / r

θ = crank angle, measured from vertical axis OY, at any instant

ω = angular velocity of crank, rad/s

2α = V-angle i.e. angle between lines of

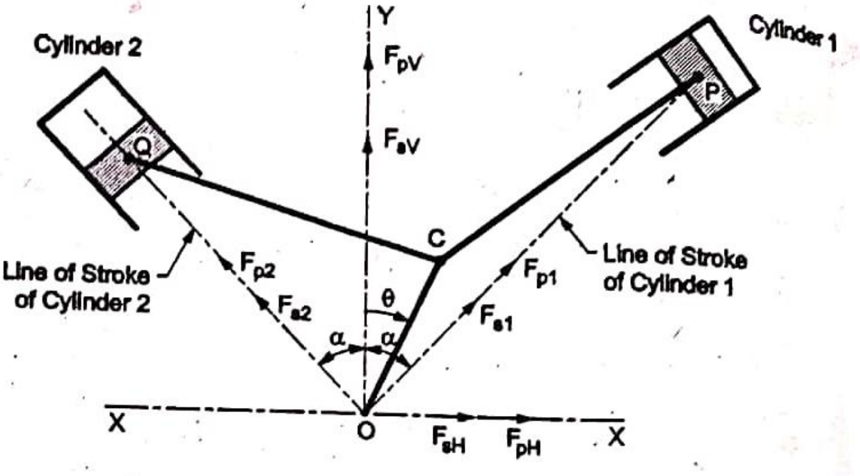


Fig. 3.2.1 : Balancing of V-Engine

We know that,

– Primary unbalanced force in a single cylinder engine is,

$F_P = m\,\omega^2 r \cos\theta$ ...[From Equation (2.10.3)]

– Secondary unbalanced force in a single cylinder engine is,

$F_S = m\,\omega^2 r \frac{\cos 2\theta}{n}$ ...[From Equation (2.10.4)]

1. Primary forces

(i) Primary forces in individual cylinders

– The primary unbalaced force acting along the line of stroke of cylinder 1 is,

$$F_{P1} = m\,\omega^2 r \cos(\alpha - \theta)$$

– The primary unbalanced force acting along the line of stroke of cylinder 2 is,

$$F_{P2} = m\,\omega^2 r \cos(\alpha + \theta)$$

(ii) Total primary force along vertical line OY

– The total primary force along vertical axis OY is,

$$\begin{aligned} F_{PV} &= F_{P1} \times \cos\alpha + F_{P2}.\cos\alpha \\ &= m\,\omega^2 r \cos(\alpha - \theta)\cos\alpha \\ &\quad + m\,\omega^2 r \cos(\alpha + \theta)\cos\alpha \\ &= m\,\omega^2 r \cos\alpha [\cos(\alpha - \theta) + \cos(\alpha + \theta)] \\ &= m\,\omega^2 r \cos\alpha \cdot 2\cos\alpha \cdot \cos\theta \end{aligned}$$

or $F_{PV} = 2\,m\,\omega^2 r \cos^2\alpha \cos\theta$ ...(a)

(iii) Total primary force along horizontal line OX

– The total primary force along horizontal axis OX is,

$F_{PH} = F_{P1}\sin\alpha - F_{P2}\sin\alpha$

...[∵ Both forces $FP_1 \sin\alpha$ and $FP_2 \sin\alpha$ are acting opposite to each other]

$$\begin{aligned} &= m\,\omega^2 r \cos(\alpha - \theta)\sin\alpha - m\,\omega^2 r \cos(\alpha + \theta)\sin\alpha \\ &= m\,\omega^2 r \sin\alpha [\cos(\alpha - \theta) - \cos(\alpha + \theta)] \\ &= m\,\omega^2 r \sin\alpha \cdot 2\sin\alpha \sin\theta \end{aligned}$$

or $F_{PV} = 2\,m\,\omega^2 r \sin^2\alpha \cdot \sin\theta$ ...(b)

(iv) Resultant primary force

– The resultant Primary force is,

$$\begin{aligned} F_P &= \sqrt{(F_{PV})^2 + (F_{PH})^2} \\ &= \sqrt{(2\,m\,\omega^2 \cos^2\alpha \cdot \cos\theta)^2 + (2\,m\,\omega^2 r \sin^2\alpha \cdot \sin\theta)^2} \end{aligned}$$

$$F_P = 2\, m\, \omega^2 r \sqrt{(\cos^2 \alpha \cdot \cos\theta)^2 + (\sin^2 \alpha \cdot \sin\theta)^2} \quad \text{...(3.2.1)}$$

– The angle made by resultant force F_P with vertical axis OY (measured in clockwise direction) is given by,

$$\beta_P = \tan^{-1}\left[\frac{F_{pH}}{F_{pV}}\right]$$

$$= \tan^{-1}\left[\frac{2\, m\, \omega^2 r \sin^2 \alpha \cdot \sin\theta}{2\, m\, \omega^2 \cos^2 \alpha \cdot \cos\theta}\right]$$

or $$\beta_P = \tan^{-1}[\tan^2 \alpha \cdot \tan\theta] \quad \text{...(3.2.2)}$$

2. Secondary Forces

(i) Secondary forces in individual cylinders

– The secondary force acting along the line of stroke of cylinder 1 is,

$$F_{S1} = m\, \omega^2 r \frac{\cos 2(\alpha - \theta)}{n}$$

– The secondary force acting along the line of stroke of cylinder 2 is,

$$F_{S2} = \frac{m\, \omega^2 r \cos 2(\alpha + \theta)}{n}$$

(ii) Total secondary force along vertical line OY

– The total secondary force along vertical axis OY is,

$$F_{SV} = F_{S1} \cos\alpha + F_{S2} \cos\alpha$$

$$= m\, \omega^2 r \frac{\cos 2(\alpha - \theta)}{n} \cdot \cos\alpha + m\, \omega^2 r \frac{\cos 2(\alpha + \theta)}{n} \cdot \cos\alpha$$

$$= \frac{m\, \omega^2 r \cos\alpha}{n} [\cos 2(\alpha - \theta) + \cos 2(\alpha + \theta)]$$

$$= \frac{m\, \omega^2 r \cos\alpha}{n} \cdot 2 \cos 2\alpha \cdot \cos 2\theta$$

or $$F_{SV} = \frac{2}{n} m\, \omega^2 r \cos\alpha \cdot \cos 2\alpha \cdot \cos 2\theta \quad \text{...(c)}$$

(iii) Total secondary force along horizontal line OX

– The total secondary force along horizontal axis OX is,

$$F_{SH} = F_{S1} \sin\alpha - F_{S2} \sin\alpha$$

...[Both forces $F_{S1} \sin\alpha$ and $F_{S2} \sin\alpha$ are acting opposite to each other]

$$= m\, \omega^2 r \frac{\cos 2(\alpha - \theta)}{n} \cdot \sin\alpha - m\, \omega^2 r \frac{\cos 2(\alpha + \theta)}{n} \sin\alpha$$

$$= \frac{m\, \omega^2 r \sin\alpha}{n} [\cos 2(\alpha - \theta) + \cos 2(\alpha + \theta)]$$

$$= \frac{m\, \omega^2 r \sin\alpha}{n} 2 \sin 2\alpha \cdot \sin 2\theta$$

or $$F_{SH} = \frac{2}{n} m\, \omega^2 r \sin\alpha \cdot \sin 2\alpha \cdot \sin 2\theta \quad \text{...(d)}$$

(iv) Resultant secondary force

– The resultant secondary force is,

$$F_S = \sqrt{(F_{SV})^2 + (F_{SH})^2}$$

$$F_S = \sqrt{\left(\frac{2}{n} m\, \omega^2 r \cos\alpha \cdot \cos 2\alpha \cos 2\theta\right)^2 + \left(\frac{2}{n} m\, \omega^2 r \sin\alpha \cdot \sin 2\alpha \sin 2\theta\right)^2}$$

or $$F_S = \frac{2}{n} \cdot m\, \omega^2 r \sqrt{(\cos\alpha \cdot \cos 2\alpha \cdot \cos 2\theta)^2 + (\sin\alpha \cdot \sin 2\alpha \cdot \sin 2\theta)^2}$$

...(3.2.3)

– The angle made by resultant secondary force F_S with vertical axis OY (measured in clockwise direction) is given by,

$$\beta_S = \tan^{-1}\left[\frac{F_{SH}}{F_{SV}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{2}{n} m\,\omega^2 r \cdot \sin\alpha \cdot \sin 2\alpha \cdot \sin 2\theta}{\frac{2}{n} m\,\omega^2 r \cdot \cos\alpha \cdot \cos 2\alpha \cdot \cos\theta}\right]$$

$$\beta_S = \tan^{-1}[\tan\alpha \cdot \tan 2\alpha \cdot \tan\theta] \qquad \ldots(3.2.4)$$

3.2.1 Variation of Resultant Primary and Secondary Forces with Crank Angle

– The V-engines are normally built with total V-angle '**2 α**' as : 60°, 90° or 120°.

– The resultant primary and secondary forces in V-engine are :

$$F_P = 2m\,\omega^2 r\sqrt{[\cos^2\alpha \cdot \cos\theta]^2 + [\sin^2\alpha \sin\theta]^2} \qquad \ldots(e)$$

$$F_S = \frac{2}{n} m\,\omega^2 r\sqrt{[\cos\alpha \cdot \cos 2\alpha \cdot \cos 2\theta]^2 + [\sin\alpha \sin 2\alpha \sin 2\theta]^2} \qquad \ldots(f)$$

1. For 2α = 60° :

$$2\alpha = 60^\circ \qquad \therefore\ \alpha = 30^\circ$$

– From Equation (e),

$$F_P = 2m\,\omega^2 r\sqrt{[\cos^2 30\cos\theta]^2 + [\sin^2 30\sin\theta]^2}$$

$$= 2m\,\omega^2 r\sqrt{\left[\left(\frac{\sqrt{3}}{2}\right)^2\cos\theta\right]^2\left[\left(\frac{1}{2}\right)^2\sin\theta\right]^2}$$

$$= \frac{m\,\omega^2 r}{2}\sqrt{9\cos^2\theta + \sin^2\theta}$$

or $$F_P = \frac{m\,\omega^2 r}{2}\sqrt{1 + 8\cos^2\theta}$$

– F_P is maximum when, $\frac{dF_P}{d\theta} = 0$

i.e. when $\theta = 0^\circ$ or 180°

$$\therefore\ F_{Pmax} = \frac{m\,\omega^2 r}{2}\sqrt{1+8}$$

or $$\mathbf{F_{Pmax} = \frac{3\,m\,\omega^2 r}{2}} \quad \text{at } \theta = 0^\circ \text{ or } 180^\circ \qquad \ldots(g)$$

– From Equation (f),

$$F_S = \frac{2}{n} m\,\omega^2 r\sqrt{[\cos 30 \cdot \cos 60 \cdot \cos 2\theta]^2 + [\sin 30\sin 60\sin 2\theta]^2}$$

$$F_S = \frac{2}{n} m\,\omega^2 r\sqrt{\left[\frac{\sqrt{3}}{2}\frac{1}{2}\cos 2\theta\right]^2 + \left[\frac{1}{2}\frac{\sqrt{3}}{2}\sin 2\theta\right]^2}$$

or $$F_S = \frac{\sqrt{3}\,m\,\omega^2 r}{2n}$$

$$\therefore\ \mathbf{F_{Smax} = F_S = \frac{\sqrt{3}\,m\,\omega^2 r}{2n}}$$

(F_S is independent of θ) …(h)

2. For 2α = 90°

$$2\alpha = 90 \qquad \therefore\ \alpha = 45^\circ$$

– From Equation (e),

$$F_P = 2m\,\omega^2 r\sqrt{[\cos^2 45 \cdot \cos\theta]^2 + [\sin^2 45 \cdot \sin\theta]^2}$$

$$= 2m\,\omega^2 r\sqrt{\left[\left(\frac{1}{\sqrt{2}}\right)^2\cos\theta\right]^2 + \left[\left(\frac{1}{\sqrt{2}}\right)^2\sin\theta\right]^2}$$

or $$F_P = m\omega^2 r$$

$$\therefore \mathbf{F_{Pmax} = F_P = m\,\omega^2 r}$$

(F_P is independent of θ) ...(

– From Equation (f),

$$F_S = \frac{2\,m\,\omega^2 r}{n}\sqrt{[\cos 45 \cdot \cos 90 \cdot \cos 2\theta]^2 + [\sin 45 \cdot \sin 90 \cdot \sin 2\theta]^2}$$

$$= \frac{2}{n} m\,\omega^2 r \sqrt{\left[\frac{1}{\sqrt{2}} \times 0 \times \cos 2\theta\right]^2 + \left[\frac{1}{\sqrt{2}} \times 1 \times \sin 2\theta\right]^2}$$

or $$F_S = \frac{\sqrt{2}\,m\,\omega^2 r}{n}\sin 2\theta$$

– F_S is maximum when, $\frac{dF_S}{d\theta} = 0$

i.e. when $\theta = 45°$ or $135°$

$$\therefore F_{S\,max} = \frac{\sqrt{2}\,m\,\omega^2 r}{n} \text{ at } \theta = 45° \text{ or } 135° \quad \text{...(j)}$$

3. For $2\alpha = 120°$:

$$2\alpha = 120°$$

$$\therefore \alpha = 60°$$

– From Equation (e),

$$F_P = 2m\,\omega^2 r\sqrt{[\cos^2 60 \cdot \cos\theta]^2 + [\sin^2 60 \cdot \sin\theta]^2}$$

$$= 2m\,\omega^2 r\sqrt{\left[\left(\frac{1}{2}\right)^2 \cos\theta\right]^2 + \left[\left(\frac{\sqrt{3}}{2}\right)^2 \sin\theta\right]^2}$$

$$= m\,\omega^2 r\sqrt{\cos^2\theta + 9\sin^2\theta}$$

or $$F_P = m\,\omega^2 r\sqrt{1 + 8\sin^2\theta}$$

– F_P is maximum when, $\frac{dF_P}{d\theta} = 0$

i.e. when $\theta = 90°$ or $270°$

$$\therefore F_{P\,max} = \frac{m\,\omega^2 r}{2}\sqrt{1+8}$$

or $$\mathbf{F_{P\,max} = \frac{3\,m\,\omega^2 r}{2}} \quad \textbf{at } \theta = 90° \textbf{ or } 270° \quad \text{...(k)}$$

– From Equation (f),

$$F_S = \frac{2}{n} m\,\omega^2 r\sqrt{[\cos 60 \cdot \cos 120 \cdot \cos 2\theta]^2 + [\sin 60 \cdot \sin 120 \cdot \sin 2\theta]^2}$$

$$= \frac{2}{n} m\,\omega^2 r\sqrt{\left[\frac{1}{2} \cdot \frac{1}{2}\cos 2\theta\right]^2 + \left[\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}\sin 2\theta\right]^2}$$

$$= \frac{m\,\omega^2 r}{2n}\sqrt{\cos^2 2\theta + 9\sin^2 2\theta}$$

or $$F_S = \frac{m\,\omega^2 r}{2n}\sqrt{1 + 8\sin^2 2\theta} \quad \text{...}(l)$$

– F_S is maximum when, $\frac{dF_S}{d\theta} = 0$

i.e. when $\sin 2\theta = \pm 1$

i.e. when $2\theta = 90°$ or $270°$

i.e. when $\theta = 45°$ or $135°$

$$\mathbf{F_{S\,max} = 3\frac{m\,\omega^2 r}{2n}} \quad \text{at } \theta = 45° \text{ or } 135° \quad \text{...(m)}$$

Ex. 3.2.4 **GTU - Dec. 11, 7 Marks**

The reciprocating mass per cylinder in a 60° twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 r.p.m. Determine the maximum and minimum values of the primary forces. Also find out the resultant secondary force.

Soln. :

$$m = 1.5 \text{ kg}, S = 2r = 100 \text{ mm},$$

$$\therefore \quad r = \frac{S}{2} = \frac{100}{2} = 50 \text{ mm}$$

$$l = 250 \text{ mm} = 0.25 \text{ m},$$

$$\therefore \quad n = \frac{l}{r} = \frac{0.25}{0.05} = 5$$

$$N = 2500 \text{ r.p.m},$$

$$\therefore \quad \omega = \frac{2\pi \times 2500}{60} = 261.29 \text{ rad/s}$$

$$2\alpha = 60°,$$

$$\therefore \quad \alpha = 30°$$

The resultant primary force is,

$$F_p = 2\, m\omega^2 r \sqrt{(\cos^2\alpha \cdot \cos\theta)^2 + (\sin^2\alpha \cdot \sin\theta)^2}$$

$$F_p = 2\, m\omega^2 r \sqrt{(\cos^2 30 \cdot \cos\theta)^2 + (\sin^2 30 \cdot \sin\theta)^2}$$

$$= \frac{m\omega^2 r}{2}\sqrt{9\cos^2\theta + \sin^2\theta}$$

For maximum and minimum values of F_p,

$$\frac{dF_P}{d\theta} = 0$$

$$\therefore \quad 0 = \frac{m\omega^2 r}{2}\left[\frac{-9 \times 2\cos\theta\sin\theta + 2\sin\theta \cdot \cos\theta}{2\sqrt{9\cos^2\theta + \sin^2\theta}}\right]$$

$$= \frac{m\omega^2 r}{4}\left[\frac{-18\sin\theta\cos\theta + 2\sin\theta \cdot \cos\theta}{2\sqrt{9\cos^2\theta + \sin^2\theta}}\right]$$

$$\therefore \quad 0 = \frac{m\omega^2 r}{4} \times \frac{-16\sin\theta\cos\theta}{\sqrt{9\cos^2\theta + \sin^2\theta}}$$

$$= \frac{m\omega^2 r}{4} \times \frac{-8\sin 2\theta}{\sqrt{9\cos^2\theta + \sin^2\theta}}$$

$$-8\sin 2\theta = 0 \quad \text{or} \quad \sin 2\theta = 0$$

$$\therefore \quad 2\theta = 0 \text{ or } \pi,$$

$$\therefore \quad \theta = 0 \text{ and } \frac{\pi}{2}$$

The maximum resultant primary force at $\theta = 0$ is,

$$F_{p\,(max)} = \frac{m\omega^2 r}{2}\sqrt{9\cos^2 0° + \sin^2 0°} = \frac{3}{2} m\omega^2 r$$

$$= \frac{3}{2} \times 1.5\,(261.79)^2 \times 0.05 = 7710.07 \text{ N}$$

The minimum resultant primary force at $\theta = \pi/2$ is,

$$F_{p\,(min)} = \frac{m\omega^2 r}{2}\sqrt{9\cos^2\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{\pi}{2}\right)}$$

$$= \frac{m\omega^2 r}{2} = \frac{1.5 \times (261.79)^2 \times 0.05}{2} = 2570.02 \text{ N}$$

The resultant secondary force is,

$$F_s = \frac{2}{n} m\omega^2 r \sqrt{(\cos\alpha \cdot \cos 2\alpha \cdot \cos 2\theta)^2 + (\sin\alpha \cdot \sin 2\alpha \cdot \sin 2\theta)^2}$$

$$= \frac{2}{n} m\omega^2 r \sqrt{(\cos 30° \times \cos 60° \cdot \cos 2\theta)^2 + (\sin 30° \cdot \sin 60° \cdot \sin 2\theta)^2}$$

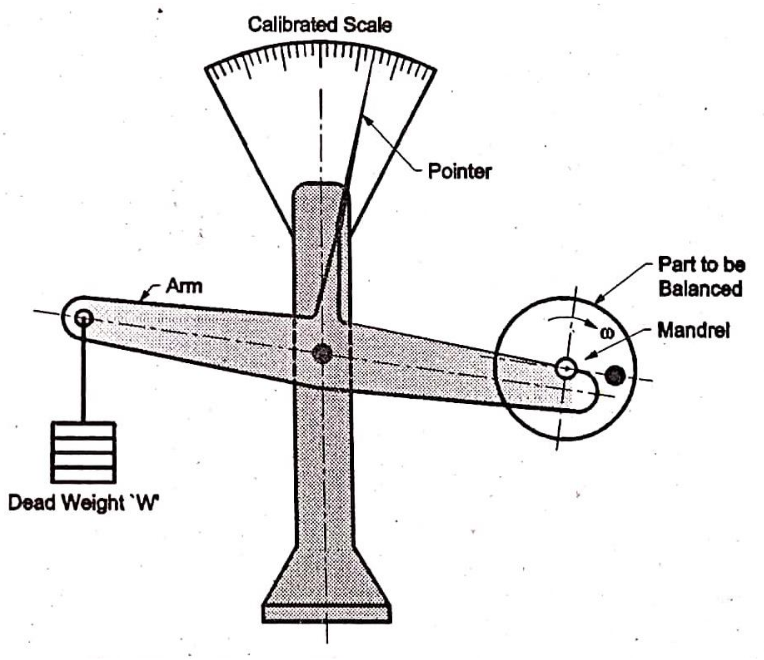
$$= \frac{2}{n} m\omega^2 r \sqrt{(0.43\cos 2\theta)^2 + (0.43\sin 2\theta)^2} = \frac{0.43 \times 2 \times m \times \omega^2 r}{n}$$

$$= \frac{0.43 \times 2 \times 1.5\,(261.79)^2 \times 0.05}{5}$$

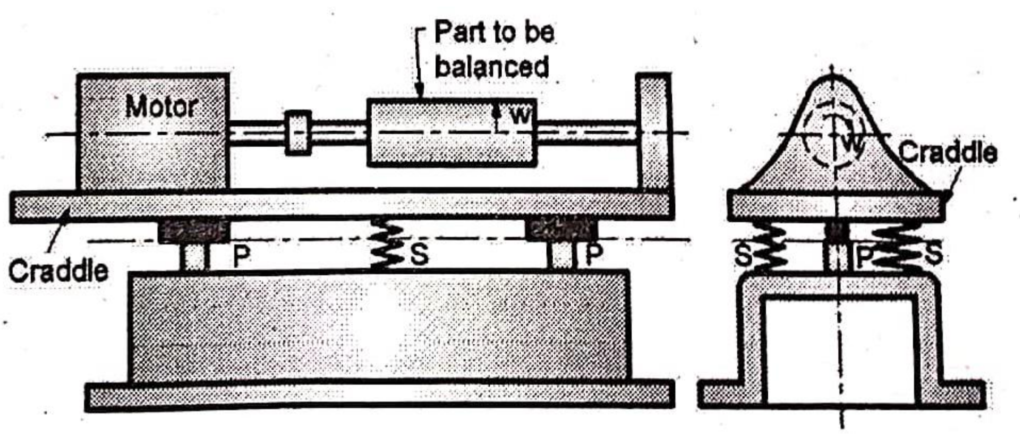
$$F_s = 890.28 \text{ N}$$

Static Balancing Machines

1. Gravity Type Static Balancing Machine



2. Oscillating Type Static Balancing Machine



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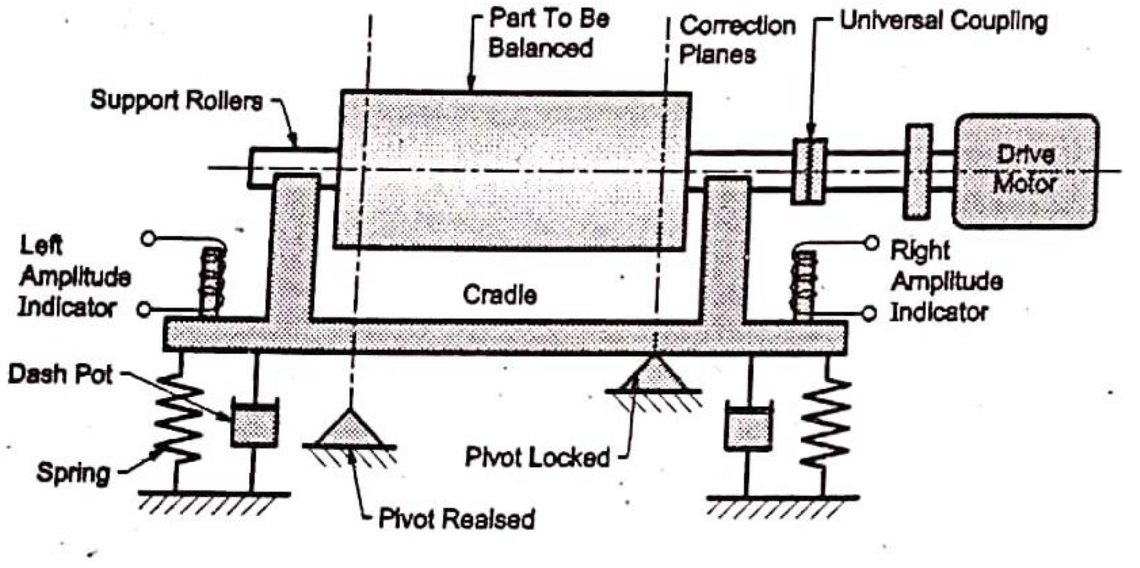
Dynamic Balancing Machines

Pivoted Cradle Type Dynamic Balancing Machine



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